eV. This mixing of the single-particle states into one or more compound nucleus state is introduced by the residual nucleon-nucleus interactions. Thus, the clustering of narrow levels in the vicinity of a single-particle state is to be expected.

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KINETIC ENERGY AND PROMPT NEUTRON DISTRIBUTIONS IN FISSION

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Refined experiments in the last few years have established a number of unexpected facts. The kinetic energy of a fission pair as a function of the mass ratio of the two fragments shows a drastic dip in the symmetric fission region.[1] The average number of prompt neutrons per fragment as a function of the fragment mass increases almost linearly with mass up to the symmetric fission region and then suddenly drops to near zero; after this it increases almost linearly again (the so-called saw-tooth structure).[2] These features are difficult to explain in any theory. The difficulty is particularly serious for the statistical theory because these results are in contradiction to the predictions of this theory as carried out in an earlier article[3] (referred to as paper I). The purpose of this note is to point out that the statistical theory is still in a position to explain these phenomena provided that all effects of the nuclear shells are taken into account, not only the effect on nuclear masses considered in paper I but also the effect on nuclear deformability.

When we compare the experimental kinetic energy with the predicted energy curve (essentially due to Coulomb repulsion) in paper I, we notice that the experimental value is consistently higher than the Coulomb energy value in the neighborhood of fragment mass 133 while in the symmetric fission region it is lower. Since the experimental result in the latter region is less accurate than in the former, it is probably wiser to emphasize the abnormal "peak" of the kinetic energy around mass 133 over and above the Coulomb energy curve rather than the dip in the symmetric fission region. This is more so when we recognize that the peaks for isotopes from U^{233} up all fall in the same region where the primary fission fragments are near the 82-neutron shell. The above point of view is further strengthened by the recent work of Britt, Wegner, and Gursky[4] on the fission of isotopes far below U^{233}, which shows no dip in the symmetric fission region (Bi^{209}, Au^{197}, and Ra^{226}) but also shows the peak in the same mass region as before (Ra^{226}), only far removed from the symmetric fission region (too far to be observed in Bi^{209} and Au^{197}). Similarly, Niday's curve[1] for U^{235} shows the peak and also a flat region in symmetric fission. The prompt-neutron distribution curves for different isotopes seem to coincide; this leads to the idea of the Terrell universal curve. These results show that the kinetic energy and prompt neutron distributions are determined by the properties of the fission products, particularly those in the closed-shell region; in other words, the process is determined by the final condition instead of by the initial condition.

In paper I we have pointed out a similar situation in mass distribution (asymmetric fission). The mass distribution curves of U^{233}, U^{235}, and Pu^{239} are such that their heavy-fragment peaks coincide, indicating that this group of fragments has a special property (which turns out to be the large change of nuclear mass due to the closing of the nuclear shells) that makes their corresponding fission modes particularly favorable. This was made a strong point for the statistical theory because the only way the final condition can determine the process is that the process proceeds in a sequence of quasiequilibrium so that the past is forgotten and the future final condition may exercise its control by the statistical weight. The evidence on energy and neutron distributions discussed above thus again calls for a statistical theory, and the problem left to be solved is to find the particular property of the final products that is responsible, through its effect on the statistical weight, for the appearance of the observed distributions.

In the shell effect on nuclear deformability we find the necessary property of the fission products to explain these distributions. In paper I the statistical theory of energy and neutron distributions was developed quantitatively in addition to the mass and charge distributions. In the detailed calculation the deformability of the fission fragments enters as a parameter. In paper I the parameters for deformation energy calculations are taken from the liquid drop model and thus are smooth functions of the mass number. On the experimental side, however, there is evidence from Coulomb excitation data that the deformability changes drastically according to the closing of the shells.[5] A closed-shell nucleus is stiff and difficult to deform. As more pairs of nucleons are added to it, it becomes easier to deform so that eventually the nucleus may exist in a permanently deformed shape in its ground state. Using the terminology of reference 5, the deformation energy of a nucleus with radius represented by $R = R_0[1 + \beta_2 \times P_2(\cos\theta)]$ is $\frac{1}{2}C_2\beta_2^2$, where P_2 is the second

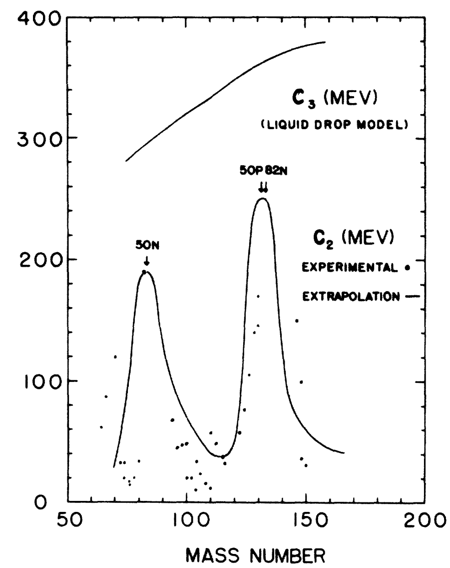

FIG. 1. Deformation energy constant C_2 as a function of mass number. The liquid-drop-model value used in paper I is plotted for comparison.

Legendre polynomial and C_2 is a constant. The experimental value of C_2 taken from reference 5, plotted as points in Fig. 1, increases by a factor of 4 to 10 at a closed shell. The corresponding liquid-drop-model parameter used in the calculation of paper I is also plotted for comparison. Obviously the latter does not show the marked shell effect, and therefore the calculation has to be repeated using more realistic parameters. In paper I the radius of the deformed fission fragment at scission is assumed to be given by $R = R_0[1 + \beta_3 P_3(\cos\theta)]$, and the deformation energy is $\frac{1}{2}C_3\beta_3^2$, the constant C_3 being plotted in Fig. 1. The use of P_3 instead of P_2 in paper I has been explained. At that time this choice gave better agreement in the magnitude of the kinetic energy, but this is no longer true now due to the change of the nuclear radius constant r_0 from 1.5 fermis to 1.2 fermis. For the additional reason that we have experimental information only on C_2, we now assume the deformation of the fission fragment at scission is due to P_2 and the deformation energy is $\frac{1}{2}C_2\beta_2^2$. The values of C_2 for primary fission fragments, which are beta-unstable, are obtained by extrapolation (smooth curve in Fig. 1) based on data points of stable nuclei. The peaks of the smooth curve are not the same as those of the points because of the extrapolation.

The results of calculation for neutron fission of U^{235} are shown in Fig. 2. The calculated

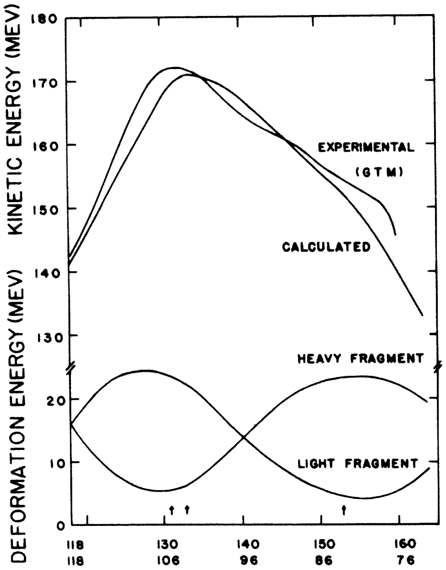

FIG. 2. Calculated kinetic energy and deformation energy for neutron fission of U^{235} as functions of mass number. Experimental kinetic-energy curve of Gibson, Thomas, and Miller is plotted for comparison.

kinetic energy agrees well with the experimental curve. The large dip in the symmetric fission region is reproduced. To correlate the deformation energy curves with the Terrell curve, the curve for the light fragment should be reflected with respect to the vertical axis; then the sawtooth shape becomes evident.

Similar calculations are carried out for two extreme cases, the deuteron fission of Ra^{226} and the spontaneous fission of Cf^{252}, using the same set of extrapolated C_2 values. The last procedure is not justified because the primary fission products in the three cases have different values of neutron excess. The results are not expected to agree with experiments exactly. Nevertheless, they do exhibit the general tendency that as the mass of the fissioning nucleus increases, the peak of the kinetic-energy curve moves towards the symmetric fission region and reaches it near Cf^{252} (dip disappears). The calculated deformation energy curves of the three cases (after reflection) coincide with one another fairly closely in the heavy-fragment region, though not quite so closely in the light-fragment region. This coincidence is related to the approximate universality of the Terrell curve.

The results may be understood qualitatively. The stiffness of the fragments near the 82-neutron shell makes them less deformed than others and therefore their deformation energy is smaller and the corresponding Coulomb energy between the two fragments is greater. The latter explains the peak of the kinetic-energy curve in the 82-neutron shell region. The former explains the small number of prompt neutrons from this region. The complementary light fragment emits a large number of neutrons because it is easier to deform and the deformation energy is proportional to the square of the deformation parameter β_2. Since in the explanation we make use of the property of the fission products, the variation of the kinetic energy and prompt neutron distributions with respect to the fissioning nucleus is easily accounted for. The qualitative relation between the neutron distribution and the variation of the nuclear stiffness due to closed shells has been suggested by Terrell.[2]

The connection with the asymmetric fission problem will be discussed briefly. If in the mass distribution calculation of paper I we replace the kinetic and deformation energy values with the new values of this work, we do not obtain asymmetric fission. This is rather a defect of the approximation employed than one of the statistical theory. Since the symmetric fission fragments are heavily deformed, we expect the total deformation energy of the two fragments in the symmetric fission region D_s to be larger than that in the asymmetric region D_a. If D_s is so large that the energy loss in the dip of the kinetic energy is taken up by the deformation energy, then the important quantity excitation energy will be essentially determined by the mass of the fission fragments which favors asymmetric fission.[6] In the present calculation, however, D_s is only about 5 MeV larger than D_a. This is due to the quadratic approximation of the deformation energy here employed. The value of β_2 for symmetric fission fragments is of the order of unity, and therefore to neglect the terms higher than the second in the deformation energy gives rise to serious error in underestimating the deformation energy. The value of β_2 for fragments in the 82-neutron shell region is of the order of 0.2; the quadratic term in deformation energy is the leading term. Thus the present approximation underestimates D_s more than D_a. With better approximations it is possible that D_s may be substantially larger than D_a, and we may have asymmetric fission.

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data of C_2 before publication, though the intended calculations were not carried out in detail until years later after conclusive experimental information had become available. The support by the National Science Foundation is kindly acknowledged.

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TOTAL RADIATION WIDTHS FOR s-WAVE AND p-WAVE NEUTRON CAPTURE IN Nb^{93}†

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The recent successful use of resonant-capture γ-ray spectra to assign parities to resonances[1] has made possible for the first time a systematic study of resonance parameters for s-wave and p-wave neutron capture by direct measurement of individual resonances. Of these parameters the total radiation widths for s-wave and p-wave resonances, which we denote as $(\Gamma_\gamma)_s$ and $(\Gamma_\gamma)_p$, are of particular interest as a source of information about the dependence of the electric-dipole radiation strength on the parity of highly excited radiating states. Furthermore, accurate values of these widths can be used in analyzing measurements of total capture cross sections to obtain more reliable estimates of the s-wave and p-wave neutron strength functions. Until now, the usual assumption[2,3] has been that $(\Gamma_\gamma)_s = (\Gamma_\gamma)_p$; but the results of measurements on resonances in Nb^{93}, presented in this note, demonstrate a marked difference between these quantities.

An accurate measurement of Γ_γ is possible for many resonances in Nb^{93} because in each case the neutron width Γ_n is much smaller than the total width Γ. Consequently, Γ_γ can be determined precisely by assuming $g = 0.5$ and subtracting $2g\Gamma_n$ from measured value of the total width. The values of Γ and $2g\Gamma_n$ for resonances in Nb^{93} are shown in Table I. They were obtained from transmission measurements on niobium plates of thicknesses 0.00148, 0.0709, 0.0178, and 0.284 atom per barn. The samples, 99.9% pure, contained a 0.1% impurity of tantalum which produces resonance structure of importance only near the 35.9-eV resonance. Effects of this structure were included in the analysis. The resonances at 119 eV and 194 eV were measured with a time-of-flight resolution width of 12 nsec/m and the remaining cases measured with a resolution of 24 nsec/m by use of the Argonne fast chopper. Typical thick-sample transmission data are shown in Fig. 1 for two resonances whose parameters are dis-

Table I. Parameters for neutron resonances in niobium. The results for resonances at 35.9 eV and 42.2 eV are in agreement with those of Saplakoglu, Bollinger, and Coté.[a] Because their results for resonances at higher energies are based on an assumed value $\Gamma_\gamma = 0.22$ eV for the radiation width, they will be subject to large corrections. The value of Γ_γ for the level at 194 eV is in strong disagreement with an older value of 0.34 ± 0.06 eV obtained in an indirect measurement by Rae.[b]

E_0 (eV)	Parity	Γ (10^{-3} eV)	$2g\Gamma_n$ (10^{-3} eV)	Γ_γ (10^{-3} eV)
35.9	···	215 ± 40	0.12 ± 0.01	215 ± 40
42.2	···	260 ± 20	0.09 ± 0.01	260 ± 20
94.3	···	215 ± 50	0.34 ± 0.03	215 ± 50
106	+	96 ± 50	0.05 ± 0.01	96 ± 50
119	+	113 ± 20	0.58 ± 0.06	113 ± 20
194	+	175 ± 25	41.7 ± 4	133 ± 30

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