[6]The measured χ is believed to include a contribution of the order of 140×10^{-6} emu/g-atom from the orbital paramagnetism of the conduction electrons plus other smaller terms as discussed previously [D. J. Lam, D. O. Van Ostenburg, M. V. Nevitt, H. D. Trapp, and D. W. Pracht, Phys. Rev. 131, 1428 (1963)]. These terms must be subtracted from the measured susceptibility in order to obtain an estimate of χ_{Pauli}.

EVIDENCE FOR QUANTIZED VORTICES IN A SUPERCONDUCTING STRIP*

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(Received 21 August 1963; revised manuscript received 16 September 1963)

In this Letter we report on the preliminary results of a study of the properties of extremely narrow superconducting film strips (of the order of 1 μ wide and 1000Å thick) in a perpendicular magnetic field. Resistivity versus magnetic field curves obtained in the intermediate state at the transition temperature T_c indicate that quantized current vortices are present; and that in the limit of very narrow strips, their size depends upon the width of the strip.

In the absence of a more exact theory, we have made use of Tinkham's approximate theory[1] to predict and interpret our experimental results. Tinkham's theory leads to the conclusion that in a thin superconductor in a perpendicular magnetic field near T_c there are quantized current vortices which completely fill the space. If one assumes that the vortices are circular, their over-all radius is determined by the relation,

$$\pi R^2 H = n\varphi_0, \quad (1)$$

where H is the applied, perpendicular magnetic field, n is an integer, and $\varphi_0 = hc/2e = 2.07 \times 10^{-7}$ G-cm^2 is the flux quantum for pairs. As the magnetic field is increased the vortices become smaller and crowd together, allowing space for a new vortex of the same size to form. If the plane of the thin superconductor is of large extent, compared to the size of the vortices, this will happen even for incremental changes of the field. Thus as the magnetic field is increased the vortices always completely fill the space.

The situation is markedly different for a very narrow superconducting strip whose width is of the order of the size of the vortices determined by Eq. (1) for $n=1$. Figure 1(a) describes the vortex arrangement when the width of the strip is exactly equal to $2R$, for $n=1$, in Eq. (1). Now, when the magnetic field is increased, if the vortices become smaller, the arrangement is that shown in Fig. 1(b) in which case it is impossible for the vortices to completely fill the space. We suggest that it may be more favorable energetically for the vortex arrangement in Fig. 1(a) to persist as the magnetic field is increased until a value of H corresponding to the vortex arrangement in Fig. 1(c) is reached. Then the large vortices will break upon into smaller ones which again completely fill the space. Since in this picture the vortices can exist only in discrete sizes, this should lead to periodic effects in the free energy. We also expect that the vortices will not be exactly circular but some shape intermediate between that shown in Fig. 1(a) and Fig. 1(d) in order to fill the space more completely.

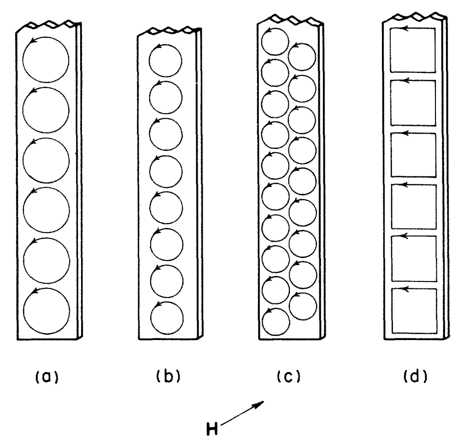

FIG. 1. Various vortex configurations in a narrow superconducting strip in a perpendicular magnetic field.

In order to obtain the free energy versus magnetic field relation for a narrow superconducting strip, we use Tinkham's model in which he assumes that in a thin superconductor at $T \sim T_c$ in a perpendicular magnetic field H, the kinetic energy of a vortex is given by

$$T(r) = [\omega(r)/2\Lambda_0][(n\varphi_0 - \pi r^2 H)/2\pi rc]^2 + O(r), \quad (2)$$

where $\omega(r)$ is the order parameter of the Ginzburg-Landau theory, $\Lambda_0 = m/n_s{}^0 e^2$ is the London parameter evaluated at 0°K, n is an integer, and

r is the distance from the center of the vortex. $O(r)$ represents the field-independent terms and n_s^0 is the number of superconducting electrons at 0°K. After Tinkham, for $\omega(r)$ we choose the trial function

$$\omega(r)=\omega_0(r/R)^{\alpha}, \tag{3}$$

where R is the over-all radius of the vortex and ω_0 is the value of $\omega(r)$ at the edge of the vortex. We then integrate Eq. (2) over the volume element $2\pi r dr$ to obtain the kinetic energy per unit volume, E_K/V, of a circular vortex of over-all radius R. We obtain

$$\frac{E_K}{V}=\frac{\omega_0 H}{4\pi c^2\Lambda_0}\left(\frac{-2n\varphi_0}{\alpha+2}+\frac{\pi R^2 H}{\alpha+4}\right)+O(R). \tag{4}$$

We now assume that R is fixed in a narrow strip as discussed above and minimize Eq. (4) with respect to H. We obtain, in addition to a minimum at $H=0$, minima at the following values of H:

$$H_{\min}=(n\varphi_0/\pi R^2)[(\alpha+4)/(\alpha+2)]. \tag{5}$$

Using Tinkham's variational analysis one finds a minimum in the free energy at $\alpha\sim 1.3$ for $n=1$, $\alpha\sim 2.1$ for $n=2$, etc. However, the free energy is very insensitive to the choice of α. Because of this and the crudity of the trial function [Eq. (3)], the above result could easily be shifted in a real superconducting film which has physical inhomogenieties (holes, dislocations, etc.). The above analysis shows that the minima occur at higher magnetic fields than in the case of a ring or hollow cylinder (multiply connected case). In the latter case $\omega(r)=\omega_0$ and $r=R_a$ (the radius of the ring or cylinder) in Eq. (2), and minima occur at $n\varphi_0/\pi R_a^2$. In the simply connected case, according to Eq. (5), the first minimum, which corresponds to $n=1$ and $\alpha=1.3$, occurs at $1.6\varphi_0/\pi R^2$, 60% higher than in the multiply connected case for $R=R_a$.

In an attempt to observe such a minimum (or minima) in the kinetic energy, and therefore, the free energy of a thin superconductor, we examined thin superconducting Sn strips approximately 1 micron wide (see Fig. 1) very near the transition temperature T_c.[4] In strips this narrow the size of the vortices should be limited by the width of the strip rather than by physical inhomogeneities in the film. The reduced resistivity r/r_N in the intermediate state was measured as a function of the applied, perpendicular field H. As discussed previously by Little and Parks,[2] a decrease in r/r_N corresponds to an increase in T_c. This, in turn, corresponds to a decrease in the free energy of the superconducting state.[3] Thus a minimum in the free energy versus H curve should lead to a minimum in the r/r_N versus H curve.

The superconducting film strips were prepared in the following way. An organic fiber of the order of 1 micron in diameter was prepared from GE-7031 varnish and mounted over a hole on a Pyrex glass slide. Then spectroscopically pure Sn was evaporated onto one side of the fiber and the slide at a pressure of 10^{-6} to 10^{-5} mm Hg. A film strip prepared in this way is illustrated in Fig. 2(b). This is an approximation to the "ideal" geometry shown in Fig. 2(a). The film thickness of the strips was measured with an accuracy of ±20 Å by the Tolansky interference method. The diameters of the fibers were determined by interpreting the diffraction pattern in a conventional optical microscope. The diameter of one fiber was determined by electron microscopy and this measurement was used to calibrate the above technique. In the resistive transition region of the strip (a region up to 0.01°K in width just below T_c for bulk Sn), both dc and ac measurements of the resistivity of the strip were made. A 100-cps phase-sensitive, lock-in Wheatstone bridge was used for the ac measurements. The ac and dc measurements gave identical results, and in both cases current densities of the order of 10^4 A/cm^2 were used.

The r/r_N versus H curves for a superconducting strip 1.5 μ wide and 860Å thick, and for one 4.9 μ wide and 1480Å thick, for various temperatures in the intermediate state, are shown in Fig. 3. A minimum in the r/r_N versus H curves of the 1.5-μ strip [Fig. 3(a)] occurs approximate-

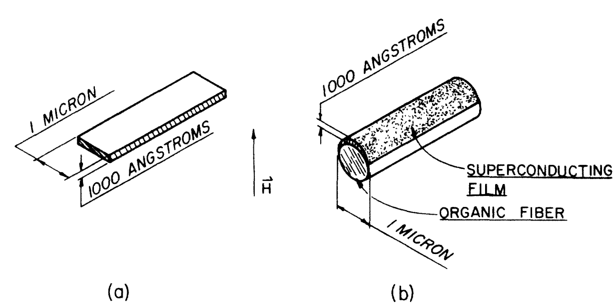

FIG. 2. (a) "Ideal" geometry of superconducting film strip with the approximate dimensions indicated. (b) Geometry of film strip used in experiment (approximate dimensions). The orientation of the strip in the magnetic field H is indicated.

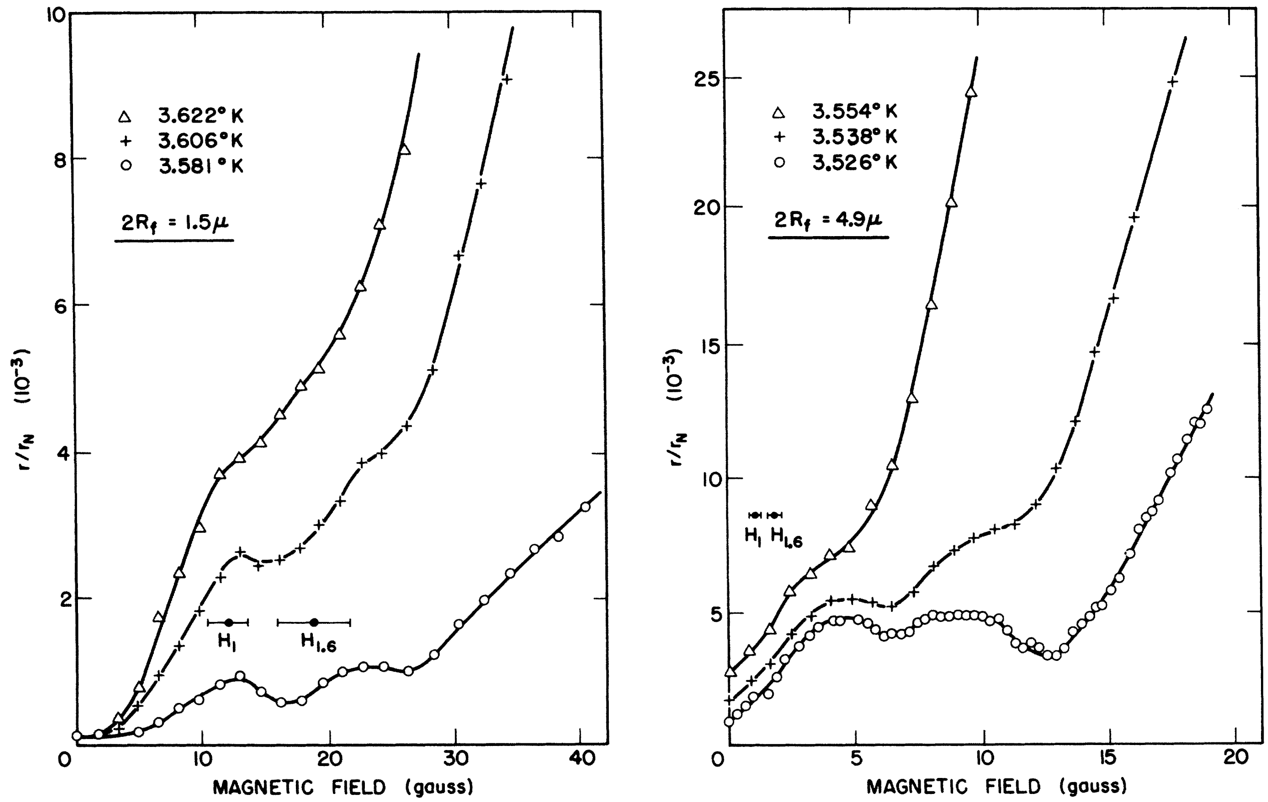

FIG. 3. (a) Reduced resistivity (r/r_N) versus H curves at various temperatures for a superconducting strip 1.5 μ wide at 860Å thick. H_1 is determined by $H_1 = \varphi_0/\pi R_f^2$ and $H_{1.6}$ by $1.6\varphi_0/\pi R_f^2$. The error flags on the values of H_1 and $H_{1.6}$ reflect the errors in the measurement of the diameters of the fibers, which for the smallest fibers is approximately ±7% of the diameter. This corresponds to errors in H_1 and $H_{1.6}$ of ±14%. (b) Similar series for a strip 4.9 μ wide and 1480Å thick.

ly at the predicted value of H, $H_{\min} = H_{1.6} = 1.6\varphi_0/\pi R_f^2$, where R_f is one-half the projected width of the strip (actually the radius of the fiber). This agrees with our predictions and indicates that quantized vortices containing one flux quantum, $n = 1$, and limited in size by the width of the strip are present. The second dip in the two lower-temperature curves corresponds, perhaps, to some or all of the vortices remaining the same size as the width of the strip and n switching from 1 to 2 as the magnetic field is increased. The curves for the wider strip [Fig. 3(b)] have the same general characteristics as those of Fig. 3(a) except the minima in the curves occur at fields several times higher than the predicted values. A qualitative explanation for this discrepancy will follow.

In order to relate the first minimum in the r/r_N versus H curve to the width of the superconducting strip, representative curves for four different samples are shown in Fig. 4. The shapes of the curves of Fig. 4 depend upon the temperature, the film thickness, and the quality of the film. Therefore, only the qualitative nature of the curves, specifically, the location on the abscissa of the minima should be considered. For the 1.5-μ, 2.0-μ, and 2.4-μ strips the first minimum in the curve occurs very close to, but slightly lower than, the predicted value of the field $H_{\min} = H_{1.6}$. If we assume that the vortices are not quite circular as discussed before, this brings the experimental and predicted values of the location of the minima into almost perfect agreement. For the very wide film (4.9 μ) the results are anomalous as discussed above.

In all of the films studied the resistive anomalies, which we have explained in terms of vortices in the film strips, were observed only at the lower end of the resistive transition at values of r/r_N of the order of 0.01 or less. Our qualitative explanation for this is the following. At the higher-temperature end of the resistive transition we imagine that the intermediate state

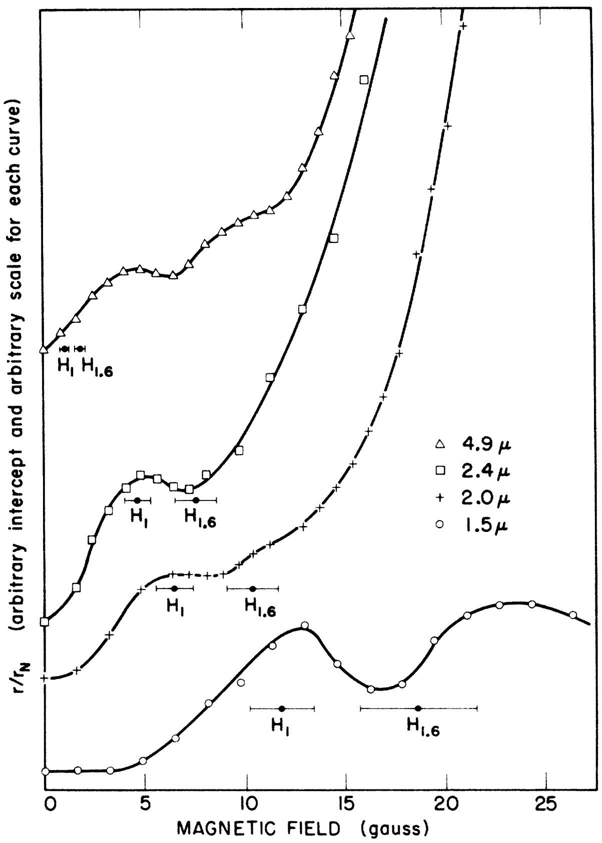

FIG. 4. Reduced resistivity curves for four superconducting strips of various widths. Since the curves for the various strips were obtained at different temperatures, the ordinate intercepts and scale are arbitrary, and different for each sample.

of the film consists of small "islands" of superconducting metal in a "sea" of normal metal.[5] This will have two detrimental effects. Firstly, it is improbable that supercurrents of the order of the diameter of the strip—and therefore, vortices— can form until the "islands" are quite large and almost completely fill the space of the strip. Secondly, the size of the "islands" will be sensitive to the magnetic field. This will give rise to a monotonically varying r/r_N versus H background which obscures the presence of the vortices. This background will be steepest for the largest surface-to-volume ratio of the "islands," which corresponds to the higher-temperature end of the resistive transition. Both of these effects become less important at the lower-temperature end of the resistive transition. This picture may also serve to explain the anomalous results obtained for the wide (4.9-μ) strip. In the presence of physical inhomogeneities it would be much more difficult for a large vortex [Fig. 1(a)] to form than a small vortex [Fig. 1(c)], because a persistent current over a much longer distance is required. This could result in a stability against vortex formation until a magnetic field corresponding to the vortex arrangement in Fig. 1(c) is reached. The locations of the minima in the r/r_N versus H curves are in good agreement with the calculated values based upon this explanation.

In summary we state the following results and conclusions from these experiments.

(1) Evidence has been obtained for the existence of quantized current vortices in a thin superconductor in a perpendicular magnetic field.

(2) In very narrow superconducting strips (less than approximately 3 μ wide) the first appearance of vortices was found to occur at the value of the magnetic field at which vortices containing one flux quantum can form in the strip.

We concede that the agreement between the experimental results and the predictions based upon Tinkham's theory may be fortuitous because of the approximate nature of the theory. Also some of the anomalous results which we have attributed to physical inhomogeneities in the films may, in fact, be intrinsic effects. Further experiments should clarify this question. In spite of these concessions, there seems to be little question that the resistive anomalies observed are associated with vortices in the film strips. Certainly, these results call for further theoretical investigations.

We wish to thank B. Brandt for measuring the film thickness of the samples and M. Foster, P. G. de Gennes, and M. Tinkham for stimulating discussions.

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*Work supported in part by the National Science Foundation and United Aircraft Corporation.

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[1]M. Tinkham, Phys. Rev. 129, 2413 (1963).

[2]W. A. Little and R. D. Parks, Phys. Rev. Letters 9, 9 (1962); Proceedings of the Eighth International Conference on Low-Temperature Physics, London, 1962 (to be published).

[3]Since the free energy of the normal state is independent of the magnetic flux, a decrease in the free energy of the superconducting state corresponds also to an increase in the free-energy difference between the normal and superconducting states, which determines T_c.

[4]At $T \sim T_c$ the penetration depth is extremely long

and, therefore, the magnetic field is uniform in the film and equal to the applied magnetic field.

[5]One expects that the size of the superconducting "islands" must be at least as large as the superconducting coherence length. The thin films studied here are type-II superconductors and the coherence length ξ is determined from the relation $\xi = \frac{1}{3}(l v_F \hbar/\Delta)^{1/2}$, where l is the mean free path, v_F the velocity of the electrons at the Fermi surface, and Δ the energy gap [P. G. de Gennes (private communication)]. Using this relation we obtain $\xi \sim 200$-400 Å for the films studied. Since this is much shorter than the width of the strips, the "island" picture is feasible at least with respect to this consideration.

CONDUCTION BAND MINIMA IN AlAs AND AlSb

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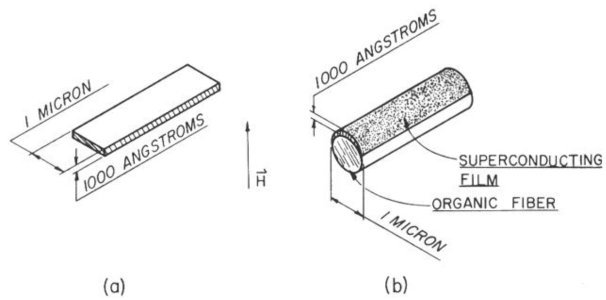
The photoresponse of surface barrier rectifiers[1] made by evaporating a metal such as gold or platinum on a cleaved surface of AlAs and AlSb has been measured in the front wall configuration. The photoresponse of such units for $h\nu > E_g$, where E_g is the energy gap, will be proportional to the absorption coefficient as long as the optical attenuation length is large compared to both the width of the space-charge region and the minority carrier diffusion length. The analysis is essentially the same as that for p-n junctions with the exception that the barrier is at the surface and hence more sensitive to photons of high absorption coefficient. Photoinjection of carriers from the metal into the semiconductor for photon energies where $h\nu < E_g$ can also be observed.[1]

If for indirect band-to-band transitions[2] the absorption coefficient α is proportional to $(h\nu - h\nu_t)^2$, where $h\nu_t$ is the threshold energy for the absorption and if the assumptions made above are valid, then the (photoresponse)$^{1/2}$ should be proportional to $h\nu - h\nu_t$. For direct band-to-band transitions the absorption rises to large values much more rapidly near the direct threshold, and the photoresponse should also show an abrupt rise. As in the case of the absorption measurements, it can be anticipated that indirect processes which occur for higher energies than the direct one will be largely obscured. Photomeasurements of this type have been used to determine the dependence on composition of both the direct and indirect transitions in the $Ga(As_{1-x}P_x)$ system.[3]

The room-temperature energy gap[4] of AlAs is apparently close to 2.2 eV. The value 2.16 eV is frequently given, but the present authors were unable to locate the source of this value in the literature. Paul[5] has pointed out that systematic extrapolation from silicon would suggest that the deepest minima in the conduction band are in the (100) directions (Δ_1 states). To the best of the authors' knowledge, no other experimental information concerning either the (100) minima or any of the higher lying minima in this material is available.

The room-temperature energy gap[6,7] of AlSb is approximately 1.5 eV and again extrapolation from silicon suggests (100) conduction-band minima. Pressure measurements[8] and electron effective-mass studies[9] tend to confirm this prediction. It has been suggested that the (000) minimum is ~0.3 eV above the (100) minima. This conclusion was drawn from optical and electrical measurements[7] of the $(Ga_{1-x}Al_x)Sb$ system and was supported by infrared measurements[10] which showed an absorption band in n-type material with a threshold of 0.29 eV. The absorption was attributed to interband transitions between the two types of minima. However, this interpretation has been questioned recently[5] for similar infrared evidence in the case of GaP.

The results of the measurements on AlAs are illustrated in Fig. 1(a), where the (photovoltage)$^{1/2}$ vs $h\nu$ is plotted for gold on an n-type sample. Very similar results were obtained for several samples. The low-energy response, $h\nu < E_g$, arises from photoinjection from the metal. Extrapolation of this response to $h\nu > E_g$ and subtracting from the measured values shows clearly both the indirect and direct transitions. Again extrapolation of the indirect response and subtraction gives just the direct edge. The values of $h\nu_t$ are 2.1 eV for the indirect process and 2.9 eV for the direct threshold. The results confirm the indirect nature of the absorption edge the $h\nu_t$



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