1616 (1960).
[2]F. Varsanyi and G. H. Dieke, Phys. Rev. Letters 7, 442 (1961).
[3]G. H. Dieke and E. Dorman, Phys. Rev. Letters 11, 17 (1963).
[4]Similar observations about Pr^{3+} levels were recently made by R. D. McLaughlin and J. G. Conway, J. Chem Phys. 38, 1037 (1963).
[5]F. Varsanyi and G. H. Dieke, J. Chem. Phys. 36, 835 (1962).
[6]B. R. Judd, Proc. Roy. Soc. (London) A241, 414 (1957).

THEORY OF THE PARALLEL FIELD MAGNETOACOUSTIC EFFECT

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Mackinnon, Taylor, and Daniel[1] have recently reported the observation of magnetoacoustic resonances in the attenuation of longitudinal sound waves propagating parallel to the magnetic field in cadmium. At present, no theoretical treatment[2-4] of this geometrical arrangement seems applicable to the experimental results. In fact, the semiclassical theory of Cohen, Harrison, and Harrison predicts that the attenuation should be completely independent of magnetic field. A possible explanation of the effect, based on rather intuitive reasoning, has been suggested by Mackinnon, Taylor, and Daniel. This explanation associates the effect with deviations from a spherical Fermi surface. The concepts involved are illustrated in Fig. 1, where the Fermi surface (or part of the Fermi surface) is taken to be an ellipsoid tilted at an angle away from the direction of the magnetic field $\vec{B}_0$. The effect of the field $\vec{B}_0$ is to cause electrons on the Fermi surface to move around the surface on a plane perpendicular to the direction of the field. The velocity of an electron in real space is normal to the Fermi surface. Thus electrons on the orbit marked A will oscillate back and forth in real space in the direction of $\vec{B}_0$ during each cyclotron period. Electrons on an orbit such as B will move a certain distance in the direction of $\vec{B}_0$ every cyclotron period. (The arrows indicate the direction of the velocity in real space at the points where they are drawn.) The possibility of spatial resonances between the wavelength of the sound wave and the "oscillation distance" in case A is obvious.

The object of this note is to present a fundamental theoretical treatment of this effect which points out the mathematical origin of the oscillations. A detailed analysis[5] of the dependence of the amplitude and period of the oscillations on the components of the effective-mass tensor, and on the detailed experimental conditions, will be presented elsewhere. The results obtained here confirm the qualitative picture suggested by Mackinnon, Taylor, and Daniel as the origin of the effect, although the detailed conditions for obtaining maxima and minima are somewhat different.

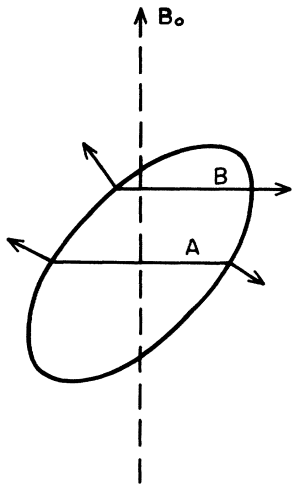

FIG. 1. A schematic of two possible electron orbits in k space in the presence of the field $\vec{B}_0$. In the case of orbit A, the electrons oscillate back and forth parallel to $\vec{B}_0$. This can easily be seen from the directions of the velocity in real space (normal to the constant energy surface) which are drawn at the ends of the orbit. The cross-sectional area of this orbit is a maximum, and we believe the resonances arise primarily from this orbit. In case B, the electrons will drift parallel to the magnetic field. They move with nonuniform velocity, but cover a given distance each cycle.

We start with the Hamiltonian H_0 for a single electron in the field B_0:

$$H_0 = \frac{1}{2m}\left\{\alpha_1 p_x^2 + \alpha_2\left(p_y - \frac{e}{c}B_0 x\right)^2 + \alpha_3 p_z^2 + \alpha_4\left[\left(p_y - \frac{e}{c}B_0 x\right)p_z + p_z\left(p_y - \frac{e}{c}B_0 x\right)\right]\right\}. \quad (1)$$

In the absence of the dc magnetic field B_0, Eq. (1) yields ellipsoidal energy surfaces oriented at an angle to the k_y and k_z axis, the exact shape and orientation depending on the parameters α_1 to α_4. We have chosen a Cartesian coordinate system with B_0 in the z direction, and we have taken the vector potential of the dc magnetic field as $(0, B_0 x, 0)$. The eigenvalues and eigenfunction of H_0 are given by

$$|\nu\rangle = |nk_y k_z\rangle = L^{-1}\exp(ik_y y + ik_z z) \times \mu_n\{x + [\hbar(\alpha_1\alpha_2)^{1/2}/m\omega_0][k_y + (\alpha_4/\alpha_2)k_z]\}, \quad (2)$$

$$E_\nu = E_n(k_z) = \hbar\omega_0(n+\tfrac{1}{2}) + (\alpha_3 - \alpha_4^2/\alpha_2)\hbar^2 k_z^2/2m. \quad (3)$$

In these equations L is the length of an edge of the cubic sample which contains N electrons. The allowed values of the wave vectors k_y and k_z are obtained by imposing periodic boundary conditions, and μ_n is a normalized simple harmonic oscillator wave function for a particle of mass m/α_1, and characteristic frequency $\omega_0 = |e|B_0(\alpha_1\alpha_2)^{1/2}/mc$.

The calculation of the conductivity tensor is completely analogous to that presented in reference 2 for an isotropic electron gas. We shall only briefly describe the procedure here. If one introduces a disturbance into the system, it will set up a self-consistent field. We let $\vec{A}$ be the vector potential of the self-consistent field, and choose a gauge such that the scalar potential of the self-consistent field is zero. The Hamiltonian for an electron in the presence of the field $\vec{B}_0$ and the self-consistent field can be written

$$H = H_0 + H_1, \quad (4)$$

where H_0 is given by Eq. (1) and the linearized expression for H_1 is

$$H_1 = -(e/2c)(\vec{v}\cdot\vec{A} + \vec{A}\cdot\vec{v}). \quad (5)$$

The velocity operator $\vec{v}$ is given by

$$\vec{v} = (i/\hbar)[H_0, \vec{r}]. \quad (6)$$

As in reference 2, one calculates the current induced by the perturbation H_1 and uses Maxwell's equations to relate the induced current to the fields. The induced-current density is calculated using the single-particle density matrix, which is determined to first order in the self-consistent field.

The zero-temperature conductivity tensor which one obtains is

$$\sigma(\vec{q},\omega) = \frac{\omega_p^2}{4\pi i\omega}\left\{\alpha - \frac{m}{N}\sum_{n', nk_y k_z} f_0[E_n(k_z)] \times\left[\frac{\vec{F}_{n'n}\vec{F}^*_{n'n}}{E_{n'}(k_z+q_z) - E_n(k_z) - \hbar\omega} + \frac{\vec{F}^*_{n'n}\vec{F}_{n'n}}{E_{n'}(k_z+q_z) - E_n(k_z) + \hbar\omega}\right]\right\}. \quad (7)$$

In this equation $\omega_p^2 = 4\pi Ne^2/L^3$, and the tensor α has diagonal components α_1, α_2, α_3 and two off-diagonal components $\alpha_{yz} = \alpha_{zy} = \alpha_4$. The vector $\vec{F}_{n'n}$ is related to off-diagonal matrix elements of the operator $\frac{1}{2}(\vec{v}e^{i\vec{q}\cdot\vec{r}} + e^{i\vec{q}\cdot\vec{r}}\vec{v})$, and for propagation parallel to the magnetic field (i.e., $q_x = q_y = 0$) is given by

$$[F_{n'n}(q_z)]_x = i\left(\frac{\alpha_1\hbar\omega_0}{2m}\right)^{1/2}[(n+1)^{1/2}f_{n',n+1}(q_z) - n^{1/2}f_{n',n-1}(q_z)], \quad (8)$$

$$[F_{n'n}(q_z)]_y = \left(\frac{\alpha_2\hbar\omega_0}{2m}\right)^{1/2}[(n+1)^{1/2}f_{n',n+1}(q_z) + n^{1/2}f_{n',n-1}(q_z)] + \frac{\hbar}{2m}\alpha_4 q_z f_{n'n}(q_z), \quad (9)$$

$$[F_{n'n}(q_z)]_z = \left(\frac{\alpha_4^2}{\alpha_2}\frac{\hbar\omega_0}{2m}\right)^{1/2}[(n+1)^{1/2}f_{n',n+1}(q_z) + n^{1/2}f_{n',n-1}(q_z)] + \frac{\hbar}{2m}\left[\left(\alpha_3 - \frac{\alpha_4^2}{\alpha_2}\right)k_z + \tfrac{1}{2}\alpha_3 q_z\right]f_{n'n}(q_z), \quad (10)$$

where $f_{n'n}(q_z)$ is the following two-center integral of harmonic oscillator wave functions:

$$f_{n'n}(q_z) = \int_{-\infty}^{\infty} dx\,\mu_{n'}\left[x + \frac{\hbar}{m\omega_0}\left(\frac{\alpha_1}{\alpha_2}\right)^{1/2}\alpha_4 q_z\right]\mu_n(x). \quad (11)$$

The conductivity tensor is quite similar to that obtained in the usual perpendicular field magnetoacoustic effect in the case of a spherical Fermi surface. In fact, the functions $f_{n'n}(q_z)$ are exactly the same functions of the parameter ξ as those given in reference 2, but in this case $\xi = (\hbar/2m\omega_0)\alpha_4^2 q_z^2/\alpha_2$ instead of $\hbar q_y^2/2m\omega_0$. The

resonances in the normal magnetoacoustic effect arise from these matrix elements, which in the semiclassical limit reduce to Bessel functions[2] of order $(n'-n)$ whose argument is $(q_y v_0/\omega_0) \times (n/n_0)^{1/2}$. The parallel field magnetoacoustic resonances discussed in this work, like the normal magnetoacoustic resonances, are semiclassical in nature. The reduction of Eqs. (7)-(11) to the semiclassical limit[2] will again give Bessel functions in the conductivity tensor.

From the similarity in the conductivity tensor, we may assume that the resonances in the parallel field case arise in the same way as in the normal magnetoacoustic effect. The resulting condition for a maximum in attenuation is

$$2[(2\hbar c n_0/|e|B_0)\alpha_4^2/(\alpha_1\alpha_2^3)^{1/2}]^{1/2} = l\lambda, \qquad (12)$$

where l is any positive integer, λ the wavelength of the sound wave, and n_0 the quantum number of the last occupied Landau level.

It is quite simple to demonstrate that the quantity on the left-hand side of Eq. (12) is equal to the "oscillation distance," $\frac{1}{2}\oint |v_z|dt$, for the electrons on orbit A of Fig. 1. As we remarked previously, these electrons oscillate back and forth parallel to $\vec{B}_0$ as they go through one full cyclotron orbit. The condition for minimum attenuation would be that the "oscillation" distance equal a half-integral number of wavelengths. We should like to point out that these resonance conditions are quite different from those assumed by Daniel and Mackinnon,[6,7] although the calculations confirm the intuitive guess that anisotropy of the Fermi surface gives rise to this effect.

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[1]L. Mackinnon, M. T. Taylor, and M. R. Daniel, Phil. Mag. 7, 523 (1962).

[2]J. J. Quinn and S. Rodriguez, Phys. Rev. 128, 2487, 2494 (1962).

[3]M. H. Cohen, M. Harrison, and W. A. Harrison, Phys. Rev. 117, 937 (1960).

[4]V. L. Gurevich, V. G. Skobov, and Yu. A. Firsov, Zh. Eksperim. i Teor. Fiz. 40, 786 (1961) [translation: Soviet Phys. -JETP 13, 552 (1961)].

[5]J. J. Quinn (to be published).

[6]M. R. Daniel and L. Mackinnon, Phil. Mag. 8, 537 (1963). In this paper the authors assume that

$$\oint v_z\, dt = l\lambda$$

is the condition for maximum absorption. They show that

$$\oint v_z\, dt = -(c\hbar/|e|B_0)(\partial A/\partial k_z),$$

where A is the cross-sectional area of the orbit in k space. The electrons which we believe to be responsible for the resonances have $\partial A/\partial k_z = 0$.

[7]E. A. Kaner, V. G. Peschanskii, and I. S. Privorotskii {Zh. Eksperim. i Teor. Fiz. 40, 214 (1961) [translation: Soviet Phys. -JETP 13, 147 (1961)]} have made some consideration of the propagation of sound in anisotropic crystals. For the case of propagation parallel to the magnetic field, it seems that their resonance condition would be similar to that of reference 6 instead of the condition of the present paper.

OPTICAL MASER OSCILLATION FROM Ni^{2+} IN MgF_2 INVOLVING SIMULTANEOUS EMISSION OF PHONONS

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All of the solid state optical maser materials known at the present time take advantage of purely electronic transitions in solids. We wish to report the observation of fluorescence from Ni^{2+} in MgF_2 and optical maser oscillation in a line which involves a transition between electronic states of Ni^{2+} and simultaneous vibrational excitations of the MgF_2 lattice.

The polarization of infrared emission from Ni^{2+} in MgF_2 at 20°K is shown in Fig. 1. The sharp structure near 6500 cm^{-1} represents pure electronic transitions between the first excited state $^3T_2$ and the ground state $^3A_2$,[1] while the remainder of the spectrum arises from electronic transitions with simultaneous emission of phonons to the lattice. As can be seen from Fig. 1, the π spectrum coincides with the axial spectrum and both the electronic and phonon-accompanied transitions are, therefore, magnetic dipole. The magnetic-dipole character of the purely electronic transitions in MgF_2:Ni^{2+} has been reported earlier.[2] The intense narrow emission at 6500 cm^{-1} consists of three closely spaced lines representing transitions from the lowest $^3T_2$ level to the three components of the $^3A_2$ ground state at 0, 1, and 6 cm^{-1}. The 1-cm^{-1} separation is barely re-