[2]J. Berkowitz, K. O. Friedrichs, H. Goertzel, H. Grad, J. Killeen, and E. Rubin, Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1958 (United Nations, Geneva, 1958), Vol. 31, p. 171.

[3]M. N. Rosenbluth and C. L. Longmire, Ann. Phys. (N.Y.) 1, 120 (1957); M. N. Rosenbluth (private communication).

[4]J. B. Taylor, Culham Laboratory Report No. CLM-P22, June 1963 (unpublished).

[5]J. Andreoletti, Euratom Meeting on Mirror Confinement, Paris, France (Fontenay-aux-Roses), 15 July 1963 (unpublished).

[6]R. F. Post, Euratom Meeting on Mirror Confinement, Paris, France (Fontenay-aux-Roses), 15 July 1963 (unpublished).

rf PROPERTIES OF THE PLASMA SHEATH*

R. S. Harp, G. S. Kino, and J. Pavkovich†

Microwave Laboratory, W. W. Hansen Laboratories of Physics, Stanford University, Stanford, California

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We have been interested in determining if there are self-oscillating rf fields in the sheath of a plasma, and finding how an externally applied rf field penetrates through the sheath and into the plasma. We have carried out an experiment similar in basic concept to that of Ash, Gabor, and Dracott[1] and to a later one by Von Gierke, Ott, and Schwirzke[2] to measure both the rf and dc fields in the sheath. The rf results are in good agreement with our self-consistent small-signal calculation of rf-field penetration into the sheath. The theoretical attack used was to integrate the one-dimensional form of the collisionless Boltzmann equations by the orbital integration technique used by Rosenbluth and others.[3-5]

We have assumed a Maxwellian velocity distribution, and that the static potential variation in the sheath is of the form $\phi_0 = -Kx^2$. In the body of the plasma, the density is assumed to be uniform, i.e., $\phi_0 = 0$. This assumption of a parabolic potential variation implies that electrons, whatever their initial velocity, have constant transit time into the sheath and out again; this vastly simplifies the analysis.

Our results indicate that the sheath impedance always has a positive real part due to a type of Landau damping in the sheath. Consequently, at least for a perfect Maxwellian velocity distribution, we would not expect self-oscillations to be present.

The resistance term may well account for the anomalous loss which cannot be accounted for by collision processes, observed by Trivelpiece and Gould[6] in measuring propagation of waves along a plasma column at frequencies below the plasma frequency. Rough checks which we have already made indicate that this effect can, in typical cases, give rise to large values of loss of the same order as in the experiments. A schematic of the experimental apparatus which we constructed for checking the theory is shown in Fig. 1. A low-density plasma of plasma frequency 90 Mc/sec was formed in the central region of the system using a general concept for the design due to Takayama, Ikegama, and Miyazaki.[7] There were two cylindrical discharge diodes placed one at each end of the system. Plasma diffused out from these diodes into the central region. By this means, a uniform density could be obtained in the region of interest with a good Maxwellian velocity distribution as measured with a Langmuir probe. This result would appear to occur, because by careful shielding it is possible to stop the primaries from penetrating into the central region of the plasma. In addition, because there is no drift of electrons from one end to the other, there are no low-frequency oscillations present. Low-frequency oscillations were observed only when the system was unbalanced. Because of the low density, the sheath thickness obtained was typically of the order of 1 cm, thus making it possible to carry out detailed investigations of the sheath.

The sheath was formed on a movable metal plate inserted through one of the side arms shown in Fig. 1. An electron beam from a CRT gun, which was differentially pumped, was passed from another side arm parallel to the plate and collected on a detector placed in the third side arm. The system was filled with mercury vapor and pumped through a fourth side arm. Typically, measurements were made at a pressure 4×10^{-4} Torr, thus yielding a fairly low collision frequency compared with the working frequency. The deflection of the beam was detected by collecting it on a back-biased silicon diode. When the beam hit this diode, because of ionization in the diode, current twenty or thirty times that of the beam would be detected.

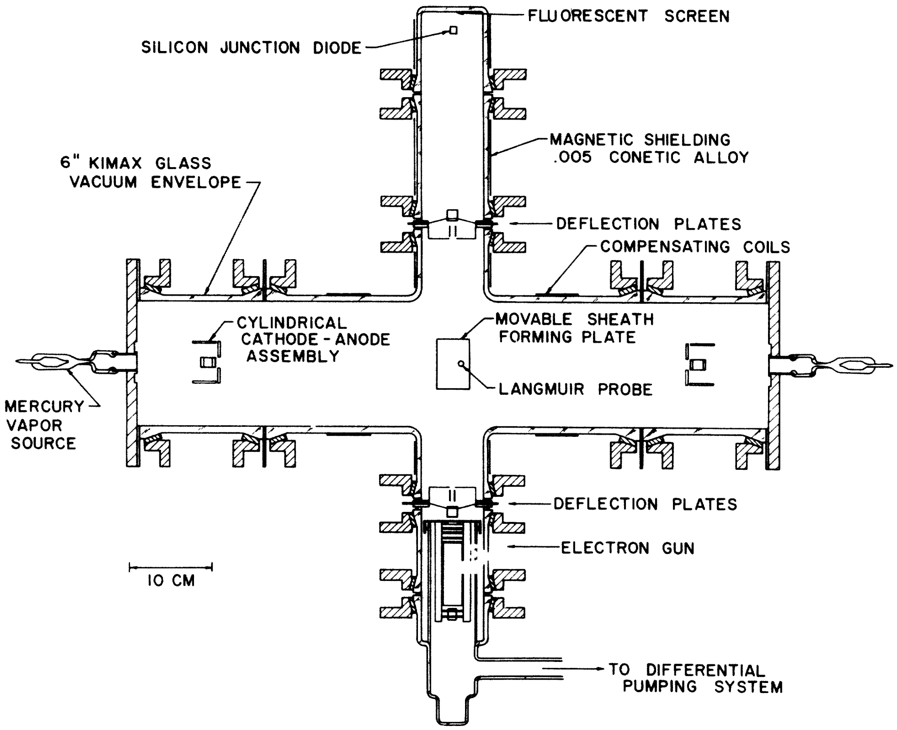

FIG. 1. A schematic of the experimental apparatus.

By using additional deflection circuitry, it was possible to display the position of the beam on a cathode ray oscilloscope, thus obviating the usual difficulties of measuring beam deflection by observing a luminous shot on a phosphorescent screen with a microscope. By this technique, deflection sensitivities of 1 V/cm could be accurately measured and crude measurements made with sensitivities as great as 0.1 V/cm. For static field measurements it was possible to use the pair of compensating coils shown in Fig. 1 to give a magnetic field which cancelled out the deflection of the beam. It was possible to calibrate directly in terms of the value of the required magnetic field, and hence measure the dc field directly.

A series of recordings of dc field as a function of distance from a metal plate are shown in Fig. 2. It will be seen that the field varies linearly with distance in a similar manner to that assumed in the rf theory, the results being in reasonable agreement with a theory for the plasma sheath given by Self.[8] On the same plot, it is shown how the theoretically assumed value of field was chosen to fit the experimental results for a current of 100 mA. This corresponds to a plasma frequency of 90 Mc/sec, a temperature of 3.6 eV, and a Debye length of 1.4 mm, as measured by a 7-mm-diameter probe inserted in and flush with the movable metal plate. At no time were self-oscillations at high frequency observed. This, we believe, was due to the fact that much care had been taken to obtain a good Maxwellian distribution of electron velocities in our plasma.

A series of measurements of rf field magnitude with respect to distance from the wall were taken

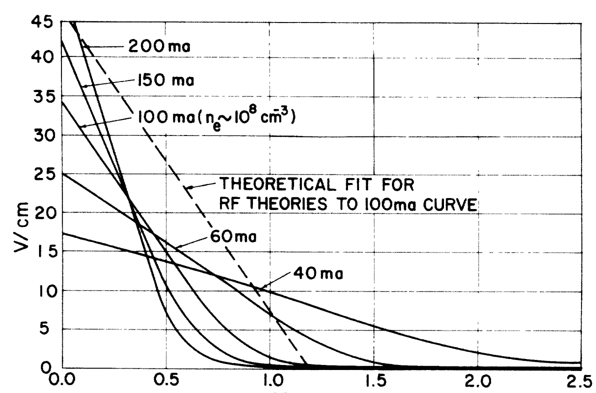

FIG. 2. A recorder plot of the measured static field in the sheath.

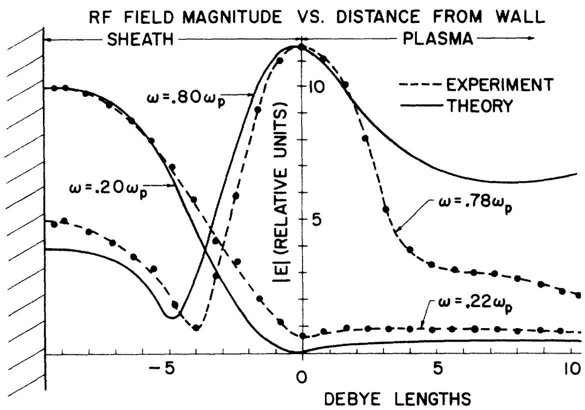

FIG. 3. A comparison of the experimentally measured and theoretically calculated rf fields in the sheath for two values of frequency. The experimental result for $\omega/\omega_p = 0.78$ is compared with the theory for $\omega/\omega_p = 0.8$. The experimental result for $\omega/\omega_p = 0.22$ is compared with the theory for $\omega/\omega_p = 0.2$.

with this apparatus and plotted against the theoretical results. Two plots of experimental and theoretical results are shown in Fig. 3. It will be seen that the agreement between experiment and theory is good. Several other similar plots have been taken at different frequencies below the plasma frequency and are uniformly in good agreement with theory. It will be noted that there is no evidence of a singularity in the field at the point where the working frequency is equal to the plasma frequency. There is some falling off, however, of the experimental field in the uniform plasma region, unlike the theoretical results. This, we believe, is due to the fact that the system was not truly one-dimensional, for without plasma present, the field value fell off approximately 50% over the range of distance from the plate used. In addition, we would expect that the plasma would not be as uniform as is assumed in the theory. Consequently, there should be differences in the experimental and theoretical results.

Further experimental results have also been taken at frequencies above the plasma frequency. But as no theoretical results are yet available, and we have not been able to extend the measurements over a region sufficiently long to measure a wavelength of a Bohm and Gross wave,[9] they are not given here.

It will be apparent that theories based on a solution of the collisionless Boltzmann equation in a region of nonuniform density can indeed yield results which are in reasonable agreement with experiment. We are intending to carry on this work further, and in particular measure the phase of the field. This should give information on the real and imaginary components of the field and hence the loss due to Landau damping in the plasma sheath.

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†Now at Varian Associates, Palo Alto, California.

[1]D. Gabor, E. A. Ash, and D. Dracott, Nature 176, 916 (1955).

[2]E. Von Gierke, W. Ott, and F. Schwirzke, Proceedings of the Fifth International Conference on Ionization Phenomena in Gases, Munich, 1961 (North-Holland Publishing Company, Amsterdam, 1962), Vol. 2, p. 1412.

[3]M. Rosenbluth, The Plasma in a Magnetic Field: A Symposium on Magnetohydrodynamics, edited by R. K. M. Landshoff (Stanford University Press, Stanford, California, 1963).

[4]J. Pavkovich, Microwave Laboratory, Stanford University, Report No. 1063, 1963 (unpublished).

[5]D. R. Dobrott, Electron Devices Laboratory, Stanford Electronics Laboratories, Stanford University Technical Report No. 104-8, August 1962 (unpublished).

[6]A. W. Trivelpiece and R. W. Gould, J. Appl. Phys. 30, 11, 1784 (1959).

[7]K. Takayama, H. Ikegami, and S. Miyazaki, Phys. Rev. Letters 5, 238 (1960).

[8]S. A. Self, Microwave Laboratory, Stanford University Report No. 1056, July 1963 (unpublished).

[9]D. Bohm and E. P. Gross, Phys. Rev. 75, 1851 (1949).