it would be desirable to have a direct measure of the K^- cascade time in liquid helium to further pin down the cascade mechanism and to give more direct evidence on the question of the angular momentum states from which K^- are absorbed.

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DISCUSSION OF A SUGGESTED BOUND ON COUPLING CONSTANTS

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In a recent Letter, Geshkenbein and Ioffe[1] have derived an upper bound on coupling constants, i.e., the mass-shell value of three-leg vertices. Their form of bound is remarkable in comparison to previous results[2] because it depends only on the masses of the three particles involved, and not on the nature and range of the forces between the particles or on the nonexistence of stable states in other (crossed) channels. In the present communication we conclude that an assumption on which the G-I bound is based (namely, that the proper vertex function has no pole) has no direct (i.e., phenomenological) physical significance, and hence that their bound on coupling constants likewise has no direct physical significance.

We recapitulate briefly their argument: The propagator of a spinless boson a has the representation

$$G^{-1} = (s - m_a^2)\left\{1 + (s - m_a^2)\left[\sum \frac{c_i}{s_i - s} + \int \frac{ds'\rho(s')}{(s' - m_a^2)^2(s' - s - i\epsilon)}\right]\right\}, \tag{1}$$

with $c_i \geq 0$. In the limit of infinite energy, $G^{-1} \xrightarrow{s\to\infty} Zs$, where Z, the propagator renormalization constant, should be nonnegative; this imposes the condition

$$\int \rho\, ds \leq 1 - \sum c_i \leq 1. \tag{2}$$

The spectral weight $\rho(s)$ is a linear combination of positive definite terms, each contributed by a state into which particle a can transform (conserving everything except energy). The contribution to ρ of a two-body state consisting of particles b and c is $\xi(s)g^2|\Gamma(s)|^2$, where $\xi(s)$ is a phase-space factor, g is the abc coupling constant, and Γ is the proper vertex part (as defined in renormalized field theory) of the abc vertex, normalized to unity on the mass shell, i.e., $\Gamma(m_a^2) = 1$. This implies the inequality $\rho > \xi(s)g^2|\Gamma|^2$, which, used in Eq. (2), leads to

$$g^2 < \Phi^{-1},$$

where

$$\Phi = \int \frac{ds\,\xi(s)|\Gamma|^2}{(s - m_a^2)}. \tag{3}$$

If a lower limit can be put on Φ, we have an upper limit on g^2.

G-I show that Φ does have a minimum value, if it is assumed that Γ has no singularities on the physical sheet besides its right-hand cut, $s \geq (m_b + m_c)^2$. G-I did consider that Γ might have a pole, noting that if G has a zero at the same point $G\Gamma$ will be nonsingular (for the significance of this, see below); but they rejected this possibility, on the grounds that such a pole would correspond to a bound state of particles b and c. They also remarked that this pole will appear in the scattering amplitude of particles b and c through the irreducible contribution of particle a as intermediate state: $g^2\Gamma G\Gamma$. Here we shall argue that such a pole of Γ has, in fact, no direct physical significance and does not appear in the scattering amplitude. [The pole of $g^2\Gamma G\Gamma$ due to the pole of Γ is actually a "ghost," i.e., its residue has the wrong sign for it to represent a bound state.] We shall first show that it is natural, when the coupling is strong, for G to have a zero, hence Γ to have a pole there, thus allowing the G-I bound to be violated. We shall then explain from the point of view of "dynamics" how the proper vertex part can have a pole, and how the consequent pole of $g^2\Gamma G\Gamma$ is canceled from the b-c scattering amplitude.

We start from the original Källén-Lehmann representation of the propagator of particle a:

$$G = \frac{1}{s - m_a^2} - \int^\infty ds' \frac{\sigma(s')}{s' - s}, \tag{4}$$

where the spectral weight is a sum over intermediate states,

$$\sigma \propto \sum_n \xi_n(s) |\langle n | \psi_a | 0\rangle|^2.$$

The weight σ of Eq. (4) is obviously related to the weight ρ of Eq. (1) by $\sigma = \rho |G|^2$. If G has a zero, at $s = s_0$, when σ does not, then ρ is singular: In particular, if the bc contribution to σ is not zero at $s = s_0$, then Γ has a pole there. Now if in a given model the Källén-Lehmann representation exists, the propagator exists, with limit $G \xrightarrow{s \to \infty} 1/Zs$ with $Z \geq 0$; no consideration of the representation of the reciprocal of the propagator, Eq. (1), is going to change this conclusion. Hence if, for a particular case, we assume that the Källén-Lehmann representation Eq. (4) exists, the G-I bound on the coupling may be violated, simply because G has a zero, and hence Γ a pole.

We exhibit explicitly this failure of the G-I argument for nonrelativistic elastic models. Writing k for the relative momentum of particles b and c, our notation for the abc coupling constant and for the binding energy of particle a below the b-c threshold is defined by saying that in the b-c scattering amplitude f, the pole term due to particle a is $f^B = -\beta/(k^2 + \gamma^2)$. The vertex function $\langle bc | \psi_a | 0\rangle$ is a multiple of $D^{-1}(k^2)$, where D is the Jost or "denominator" function[3]; D has no poles and we assume D to have just one zero, at $k^2 = -\gamma^2$. The representation Eq. (4) becomes

$$G = \frac{1}{k^2 + \gamma^2} - \frac{2}{\pi}\beta \int dk' \frac{k'^2}{(k'^2 - k^2 - i\epsilon)|D(k')|^2}, \tag{5}$$

where D is normalized so that $[\partial_{k^2} D]_{k^2 = -\gamma^2} = 1$. From (5) it follows that for the reciprocal of the propagator we can write the equivalent of Eq. (1):

$$G^{-1} = (k^2 + \gamma^2)\left\{1 + (k^2 + \gamma^2)\left[\sum_i \frac{c_i}{k_i^2 - k^2} + \frac{2}{\pi}\beta \int \frac{dk' k'^2}{(k'^2 + \gamma^2)^2 (k'^2 - k^2)}\right]\right\}, \tag{6}$$

where $\Gamma = 1/GD$. If it is assumed that Γ has no poles, the G-I bound follows: $\beta < 4\gamma$.

From (5) it follows that $G < 0$ for $k^2 < -\gamma^2$, and that for $-\gamma^2 < k^2 < 0$ G monotonically decreases from $+\infty$; hence the criterion for a zero of G is whether G is negative at threshold. For example, if we evaluate G at threshold using the zero-range approximation for D (for small γ this should give a good approximation for the integral), we find

$$G(0) \approx \frac{1}{\gamma^2} - \frac{2}{\pi}\beta \int \frac{dk}{|2\gamma(\gamma + ik)|^2} = \frac{1}{\gamma^2}\left(1 - \frac{\beta}{4\gamma}\right). \tag{7}$$

Thus in this approximation G acquires a zero on the physical sheet when $\beta > 4\gamma$, i.e., precisely when the G-I bound is violated.

We digress briefly to note that if the interaction between particles b and c is purely through a nonsingular potential, then $\delta(\infty) - \delta(0) = -\pi$ (Levinson's theorem) so $D \xrightarrow{k \to \infty}$ const and the Källén-Lehmann representation Eq. (5) does not exist, unless one makes a subtraction at infinity. The resulting "propagator" monotonically decreases from $+\infty$ to $-\infty$ as k^2 goes from $-\infty$ to 0, and thus has a zero in this region of k^2. The position of this zero depends on the choice of the subtraction and plays the same role of arbitrary constant in representation Eq. (6) of G^{-1} as does the subtraction constant in the representation of "G." Since "G" always has a zero in the case of potential scattering, the G-I argument never applies.[4]

If, compared to the potential case, the b-c scattering amplitude f has "extra" poles in the lower-half k plane (unphysical k^2 sheet), then D will have additional factors $(k - k_i)$, and the representation for G [Eq. (5)] will exist. As the simplest example, we can consider the case that f has just two poles, namely the bound-state pole and one extra pole, on the negative imaginary k axis; then $f = -(\varphi-\gamma)/(\gamma-ik)(\varphi-ik)$ and $D=(\gamma+ik)(\varphi-ik)/2\gamma(\varphi+\gamma)$, and we find $\beta=2\gamma(\varphi-\gamma)/(\varphi+\gamma)$ and $G=(\varphi+\gamma)/2\gamma(\varphi-ik)(\gamma+ik)$. This case is apparently too simple: β never exceeds 2γ and G never has a zero. The next most simple model has a "left-hand" (potential) pole in addition:

$$D=\frac{2\gamma(\mu+\gamma)}{\varphi+\gamma}\frac{(\gamma+ik)(\varphi-ik)}{(\mu-ik)};\quad \text{so } \beta=2\gamma\frac{\mu+\gamma}{\mu-\gamma}\frac{\varphi-\gamma}{\varphi+\gamma}$$

and

$$G=\frac{\varphi+\gamma}{2\gamma(\mu^2-\gamma^2)}\frac{\mu^2-\varphi\gamma-ik(\varphi-\gamma)}{(\gamma+ik)(\varphi-ik)}. \tag{8}$$

This G has a zero, which is in the upper-half k plane (physical k^2 sheet) when $\varphi>\mu^2\gamma^{-1}$, i.e., $\beta>\beta_c\equiv 2\gamma(\mu+\gamma)^2/(\mu^2+\gamma^2)$. Note that in accordance with G-I, $\beta_c\leq 4\gamma$.

Having seen that from a dispersion theory standpoint there is nothing surprising about a zero of G, and hence a pole of Γ, we would now like to discuss the significance and origin of such a pole from a dynamical point of view. The first point to make clear is that in order to define a unique[5] Γ and G, and Lagrangian of the theory must contain an elementary particle, a', in the a channel (one of the components of the physical particle a is a'). The proper vertex function Γ is then well defined: It is the (suitably renormalized) sum of all three-leg graphs <u>except</u> those in which a' occurs as an intermediate state in the a channel. It follows that Γ is proportional to D_P^{-1}, where D_P is the denominator function for b-c scattering in the case that a' is omitted as an intermediate state in the a channel (the subscript P stands for "potential," the sole interaction between b and c, where by "potential" interaction we mean exchange of particles: Even when omitted in the a channel, a' in the crossed channels may still contribute to the "potential" interaction). Hence Γ will have a pole if the "potential" interaction binds b and c into a stable state. This state, call it a_P, is, of course, not a physical state, because a', when reintroduced into the a channel, will mix with and shift a_P to the physical state a.

The b-c scattering amplitude can be written

$$f=f_P+g^2\Gamma G\Gamma, \tag{9}$$

where the second term on the right contains the terms in which a' occurs as an intermediate state one or more times. The pole of the second term at a_P, remarked by G-I, just cancels the pole of f_P there, thus removing this unphysical pole from the complete scattering amplitude.

The foregoing can be immediately exhibited in the nonrelativistic elastic models: For instance, in the model of Eq. (8), if we take the "extra pole" to infinity, $\varphi\to\infty$ (i.e., we remove the elementary particle), but keep the residue at $k=i\mu$ fixed (i.e., we keep the same potential interaction between b and c), then we find that the bound state moves to $\gamma_P=(\varphi\gamma-\mu^2)/(\varphi-\gamma)$, which is precisely the location of the zero of G. In fact, we can write generally

$$G=D_P/D, \tag{10}$$

where D_P is the denominator function of f_P, normalized to $D_P(k=i\gamma)=1$, where f_P has the same left-hand singularities as f, $[\mathrm{Im} f_P]_{\mathrm{lh}}=[\mathrm{Im} f]_{\mathrm{lh}}$, but has no extra poles, whereas f has a pair of extra poles.[6] Equation (10) exhibits the fact that where f_P has poles (dynamical bound states), G has zeros. Equation (10) is proved as follows[7]: From Eq. (10) we would have

$$\mathrm{Im}G=\frac{D\,\mathrm{Im}D_P-D_P\,\mathrm{Im}D}{|D|^2},$$

$$=-(k/|D|^2)(DN_P-D_PN). \tag{11}$$

The last factor of this, DN_P-D_PN, is an entire function, because it has no singularities; from the asymptotic behavior of the D's and N's it follows that it is a constant, and from the normalization of the D's it follows that it equals β. Comparing Eq. (11) with Eq. (5), we see that the form Eq. (10) for G has the correct imaginary part, and it has the correct poles according to the assumption made above that D has only one zero.

Our conclusion is that the only significance of the G-I bound on a coupling constant is the following: The bound is violated only if the "potential" interaction binds a stable state in the channel a in the absence of an elementary particle a' in that channel.

Of course, if the bound is not violated, we can say nothing.

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[4]In the case of potential scattering, a limit can be put on the coupling constant only if sufficient information about the left-hand cut of the scattering amplitude is given, and then one gets both upper and lower bounds. For instance, if it is given that the nearest left-hand singularity of the b-c scattering amplitude is at $k^2=-\mu^2$ and that $\mathrm{Im}f<0$ as the negative k^2 axis is approached from above, then

$$2\gamma<\beta<2\gamma(\mu+\gamma)/(\mu-\gamma).$$

[5]I.e., in contrast to the case of purely potential interaction, discussed above.

[6]That is, Eq. (10) applies to the case that there is one elementary particle in the a channel. If f had more than one pair of extra poles, i.e., there were more than one elementary particle in channel a, there would be more than one G, of a more complicated form than Eq. (10). It also might be remarked that the situation in the model of Eq. (8), where one of a pair of extra poles is at infinity, is the limiting case $Z=0$.

[7]One can prove the same formula in the relativistic case, and thus establish the comments in the preceding paragraph about Eq. (9). Of course f_p will, in general, not be a truly physical amplitude, because it will not satisfy crossing if a' can be exchanged between b and c.

IS G_2 OR SU_3 THE MORE SUITABLE SYMMETRY FOR STRONG INTERACTIONS?

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The very appealing idea that the elementary particles are a manifestation of the invariance of the strong intersections under the action of higher rank groups has had some success in the use of the simple second-rank Lie group SU_3. SU_3 has been applied to three different features of the elementary particles: (i) as a classification scheme, since by appropriate grouping the present knowledge can be fitted by representations having dimensions of 1, 8, and 10 with the appropriate spin and parity assignments[1-5]; (ii) to provide a reasonable interpretation of the mass splitting within a multiplet as the action of a symmetry-breaking interaction which transforms as a tensor operator under the action of the group[3,5-7]; and (iii) using the assignment of strongly interacting particles to representations of SU_3, to obtain partial widths for the two-body decays of the known resonances which are compatible with experiment.[8]

However, despite the success of SU_3 in the domain of the strong interactions, if we assume that the weak vector currents are conserved in the presence of the symmetric strong interactions,[3,9] we find that SU_3 forbids K_{e3} decays with $\Delta Q=-\Delta S$,[10] in disagreement with the results of Ely et al.[11] On the other hand, another simple Lie group, G_2, predicts the existence of K_{e3} decays with $\Delta Q=-\Delta S$. With this qualitative success as our motivation, we have explored the implications of G_2 for the strong interactions, with special emphasis on those points for which SU_3 has been considered successful. We find that except for two crucial experimental results, G_2 is compatible with the present experimental knowledge of the spectra of the strongly interacting particles. These two experimental results are the spin-parity assignment of the 1405-MeV $Y_0{}^*$ resonance which has been tentatively identified[12] as $1/2^-$ and which G_2 requires to be $3/2^+$, and the isotopic-spin assignment of the 1530-MeV Ξ^* resonance which has been identified[12] as $T=1/2$ and which G_2 requires to be $T=3/2$.[13]

The predictions of G_2 differ significantly from those of SU_3 only within the region of the resonances. In order to put G_2 to the experimental test, we list its predictions in Table I and, for comparison, those of SU_3 in Table II.

The dimensionalities of the representations of G_2 are $N=1,7,14,27,\cdots$. Only the 1, 7, and 14 dimensional representations are needed to fit the present experimental data while for SU_3 those needed are 1, 8, and 10. In the first column of each of the Tables I and II, we have listed, for the lowest lying levels,[12] the baryon multiplet assignments $B_{JP}{}^N$ where the superscript is the dimensionality of the representation, N, and the subscripts, the spin-parity assignment. In the second column is the usual notation for these states. These multiplet assignments are, of course, predictions of the model: They predict the number of states in the multiplet, their isotopic spin and hyper-