[3]M. J. Longo and B. J. Moyer, University of California Lawrence Radiation Report No. UCRL-10174, 1962 (unpublished); Phys. Rev. Letters 9, 466 (1962).
[4]A. N. Diddens, E. W. Jenkins, T. F. Kycia, and K. F. Riley, Phys. Rev. Letters 10, 262 (1963).
[5]T. J. Devlin, B. J. Moyer, and V. Perez-Mendez, Phys. Rev. 125, 690 (1962); J. C. Brisson et al., Nuovo Cimento 19, 210 (1961).
[6]J. Alitti et al., Nuovo Cimento 22, 1310 (1961); F. Shively (private communication).
[7]M. Chretien et al., Phys. Rev. 108, 383 (1957).
[8]K. W. Lai, L. W. Jones, and M. L. Perl, Phys. Rev. Letters 7, 125 (1961).
[9]J. A. Helland et al., Phys. Rev. Letters 10, 27 (1963); J. A. Helland, University of California Lawrence Radiation Laboratory Report No. UCRL-10378, 1962 (unpublished).
[10]L. Bertanza et al., Nuovo Cimento 19, 467 (1961).
[11]C. D. Wood et al., Phys Rev. Letters 6, 481 (1961).
[12]L. P. Kotenko, E. P. Kusnetsov, G. I. Merzon, and Yu. B. Sharov, Zh. Eksperim. i Teor. Fiz. 42, 1158 (1962) [translation: Soviet Phys.—JETP 15, 800 (1962)].
[13]W. D. Walker (private communication).
[14]V. Cook, B. Cork, W. R. Holley, and M. L. Perl, Phys. Rev. 130, 762 (1963).

EXPERIMENTAL STUDY OF THE CASCADE TIME OF NEGATIVE MESONS IN A LIQUID HELIUM BUBBLE CHAMBER*

J. G. Fetkovich and E. G. Pewitt
Carnegie Institute of Technology, Pittsburgh, Pennsylvania
(Received 29 July 1963)

We wish to report preliminary results of a measurement of the cascade time of π^- mesons in liquid helium, and to discuss some possible implications of the result. We mean by cascade time the period between initial atomic capture of the meson and its absorption by the nucleus. It is important to understand the mechanism of the cascade since it determines not only the cascade time, but other important effects such as the angular momentum states from which mesons are absorbed by the nucleus.

If channel k is one of r channels through which the meson cascades in average time T_k from initial atomic capture to nuclear absorption, and if N_k is the number of mesons cascading through channel k, then we define the average cascade time to be

$$T = \left(\sum_{k=1}^{r} N_k T_k\right)\Big/\left(\sum_{k=1}^{r} N_k\right).$$

This is equivalent to

$$T = \tau_d (N_d/N_s),$$

where τ_d is the mean life against decay of the meson, N_d is the number of mesons observed to decay at rest, from a total sample, N_s, of mesons observed to stop in liquid helium.

The technique used to determine the cascade time, T_π, is exactly the same as that used by Fields et al.[1] in liquid hydrogen. In the present case, π^- mesons were stopped in a 1.3-liter liquid helium bubble chamber,[2] and the range and angle of all backward π-μ decays were measured to determine the velocity of the pion at the instant of decay. We observe 11 decays at rest with $\theta_{\pi\mu} > \frac{1}{2}\pi$ for a sample of 2255 π^- stops in the chamber. Because of range straggling, we cannot distinguish pions slower than $\beta_\pi \approx 0.01$ from stopped pions; however, the time taken by the pion to go from $\beta_\pi = 0.01$ to atomic capture has been estimated[3] to be much shorter than the cascade time measured here. From ordinary stopping power theory we calculate that the number of events between $E_\pi = 0.175$ MeV and $E_\pi = 1$ MeV with $\theta_{\pi\mu} > \frac{1}{2}\pi$ should be 0.8. We observe none. From the observed number of backward decays at rest we calculate the cascade time to be

$$T_\pi = [2N_d(\theta_{\pi\mu} > \tfrac{1}{2}\pi)/N_s]\tau_d = (2.5 \pm 1.0) \times 10^{-10} \text{ sec.}$$

Day[4] has made theoretical estimates of the magnitudes of several effects which might be of importance in determining the history of a typical K^- meson during its cascade. Day assumes that the meson is initially captured from the continuum into a bound orbit with principal quantum number $n \approx 30$, ejecting one of the atom's electrons in the process. Because of the strong binding of electrons in helium, the $(K^- \text{He}^{++})$ ion cannot capture electrons from neighboring atoms, and therefore only one ordinary Auger transition of the K^- meson is possible. In addition to ordinary radiative transitions, Day considers three other mechanisms for depopulating any given mesonic atom state. The first, the external Auger effect, results in the meson making a transition to a lower level during a collision with an ordinary helium atom, ejecting an electron from the atom in the process. In polarization capture, a mesonic atom in an np state makes a transition to the ns state in a collision with a helium atom, while an electron in the atom makes a virtual transition to an excited state. Due to the strong nuclear interaction of K^- mesons, and the large amplitude of its wave function at the nucleus in s states, the meson is rapidly absorbed from s states. Finally, Day considers a molecular Stark effect which, at any given n level, mixes the substates of various l. The mixing is due to the production of an electric field at the mesonic atom through collisions with neighboring atoms, or through the actual formation of a molecule-ion with a neighboring atom. Mixing of angular momentum states allows for s-state capture from even high-n levels at a rapid rate. Some of Day's estimates[4] of the rates of these processes for K^- are given in Table I along with the rates (scaled from Day's K^- rates) appropriate for π^- mesons. In addition, the rates of direct nuclear absorption from ns and np states are given.

From Table I, it would appear that the Stark effect should result in the rapid s-state absorption of both K's and π's from states of large n. Thus the mesons should not have a chance to cascade down thru the lower p states from whence they might be absorbed, and thus, Day concludes that K^- absorption in He is almost entirely from zero angular momentum states. From the rates in Table I, we can deduce a pion cascade time, T_π, to compare with our experimental result.

The striking feature of the experimental result is that it is orders of magnitude longer than predicted.[5] This seems to indicate that for π mesons in helium, there is no Stark effect of the magnitude estimated by Day. Furthermore, polarization capture at the rate indicated in Table I yields $T_\pi \approx 10^{-11}$ sec, and seems ruled out. We may ask

Table I. The theoretical rates of various mechanisms for depopulating a given mesonic atom level in liquid helium. The K^- numbers are taken from Day (reference 4). The π^- numbers are Day's K^- recalculated for the π^- case.

Process	Kaon rate (sec^{-1})	Pion rate (sec^{-1})
Radiation from np:		
$\Gamma_{rad}(np)$	$6\times10^{13}/n^3$	$2\times10^{13}/n^3$
Nuclear capture:		
$\Gamma_{cap}(ns)$	$2\times10^{19}/n^3$	$(1\text{-}24)\times10^{17}/n^3$ [a,b]
$\Gamma_{cap}(np)$	$4\times10^{15}/n^3$	$5\times10^{12}/n^3$ [a,c]
Polarization capture:		
$\Gamma_{pol}(np\to ns)$	$2\times10^{11}n$	$10^{10}n$
External Auger transitions:		
$\Gamma_{aug}(n)$		
$n=5$	9×10^5	1×10^9
$n=6$	3×10^7	2×10^{10}
$n=7$	5×10^8	2×10^{11}
$n=10$	1×10^{11}	1×10^{13}
$n=20$	1×10^{12}	2×10^{12}
Stark capture:		
$\Gamma_{Stark}(ns)$	$2\times10^5n^6$	$8\times10^7n^6$
$\Gamma_{Stark}(np)$	$2\times10^{15}/n^4$	$3\times10^{11}/n^4$

[a]A. G. Petschek, Phys. Rev. 90, 959 (1953); D. West, Reports on Progress in Physics (The Physical Society, London, 1958), Vol. XXI, p. 271.
[b]S. G. Eckstein, Phys. Rev. 129, 413 (1963).
[c]M. Stearns and M. B. Stearns, Phys. Rev. 107, 1709 (1957).

Table II. The expected cascade time for K^- and π^- in liquid helium under various conditions. Case (A) assumes that radiative and Day's external Auger transitions both take place. Here the results are rather independent of $n_{\text{init.}}$. Case (B) assumes that only radiative transitions are important. The results here depend on $n_{\text{init.}}$. The last column gives the fraction of stopped K^- which will be expected to decay at rest in liquid helium instead of undergoing nuclear absorption. Note that under case (B) for K^- we must have $n_{\text{init.}} \lesssim 18$ or else most K^- at rest would decay rather than undergo nuclear interaction.

Conditions	T_π (sec)	T_K (sec)	K decays at rest
Radiation + Auger (A)	1×10^{-10}	3×10^{-10}	2.5%
Radiation only (B)			
$n_{\text{init.}} = 6$	5×10^{-11}	3×10^{-11}	0.25%
= 7	1×10^{-10}		
= 8	3×10^{-10}		
= 9	5×10^{-10}	3×10^{-10}	2.5%
= 11	1×10^{-9}		
= 14	7×10^{-9}	3×10^{-9}	25%
= 16	1×10^{-8}		

what cascade time would be expected if we assume no Stark effect or polarization capture at all. In this case we need to consider only radiative and external Auger transitions. In Table II is given purely radiative cascade times from a given initial state of principal quantum number $n_{\text{init.}}$ to the 1S state, vs $n_{\text{init.}}$. These numbers are estimated from tables in Bethe and Salpeter[6] and Burhop,[7] assuming that the initial state has a population distribution among angular momentum substates which is not too different from statistical.[8] When we include external Auger transitions, we see (Tables I and II) that for $n \gtrsim 6$, Auger transitions dominate, while for $n \lesssim 6$, radiative transitions are most important. Using an argument parallel to Day's[4,8] for K^-, we would assume that the initial states of π^--mesonic helium atoms are typically at $n \approx 16$. However, the resultant cascade time is essentially independent of the assumed value of $n_{\text{init.}}$, since the π^- are brought very rapidly to $n \approx 5$ or 6 through the Auger effect and the cascade time is primarily determined by the rate of subsequent radiative transitions. The expected result is, for this case (radiative and Day's external Auger transitions only),

$$T_\pi \approx 10^{-10} \text{ sec.}$$

It is seen from Table II that the experimental value of T_π favors the assumption that only radiative and external Auger transitions occur. If this is so, the radiative and Auger selection rules insure that almost all K^- stopped in helium must cascade down through the lower p states. The rate of direct K^- nuclear absorption from p states is given by (Table I)

$$\Gamma_{\text{abs}}(np) = 4\times10^{15}/n^3,$$

which dominates radiation everywhere, and dominates external Auger effect up to $n \approx 11$. A very rough estimate indicates that the fraction of K^- undergoing nuclear absorption from p states in He is $\gtrsim 95\%$ under these conditions.

We may note that even should it be that all K^- absorptions in liquid helium go via p-state capture, this would not affect the conclusion of odd K-Λ relative parity deduced from the reaction[9]

$$K^- + He^4 \rightarrow {}_\Lambda H^4 + \pi^0,$$

provided the hypernucleus can be shown to be produced with zero spin. However, if the hypernucleus is produced (presumably in an excited state) with spin 1, then s-state K^- absorption implies even K-Λ parity, while p-state absorption would allow no conclusion to be drawn.

In conclusion, the experimental result for π^- seems to rule out significant amounts of Stark effect and polarization capture in liquid helium. It appears that the measured cascade time is consistent with the assumption that radiation and external Auger effect are the primary cascade mechanisms. In addition to more precise calculations, it would be desirable to have a direct measure of the K^- cascade time in liquid helium to further pin down the cascade mechanism and to give more direct evidence on the question of the angular momentum states from which K^- are absorbed.

We are grateful for many helpful discussions with Professor Lincoln Wolfenstein. In addition, we are happy to acknowledge the untiring aid of Mr. Steve Stasak and Mr. John Griffith in reducing the data.

---

*Work supported by the U. S. Atomic Energy Commission.

[1]T. H. Fields, G. B. Yodh, M. Derrick, and J. G. Fetkovich, Phys. Rev. Letters 5, 69 (1960).

[2]E. G. Pewitt, J. G. Fetkovich, and R. H. Bicker (to be published).

[3]E. Fermi and E. Teller, Phys. Rev. 72, 399 (1947).

[4]T. B. Day, Nuovo Cimento 18, 381 (1960).

[5]The expected value of T_π depends on the assumed value of $n_{\text{init.}}$. Thus [using $\Gamma_{\text{Stark}}(ns)$ from Table I], $T_\pi \approx 10^{-15}$ sec if $n_{\text{init.}} = 16$, and for $n_{\text{init.}} = 8$ we have $T_\pi \approx 6 \times 10^{-14}$ sec. See reference 8.

[6]H. A. Bethe and E. E. Salpeter, Quantum Mechanics of One- and Two-Electron Atoms (Academic Press, Inc., New York, 1957), p. 266.

[7]E. H. S. Burhop, The Auger Effect (Cambridge University Press, New York, 1952), p. 168.

[8]The question of the atomic state into which the meson is initially captured is treated in some detail by G. A. Baker, Jr., Phys. Rev. 117, 1130 (1960); Y. Eisenberg and D. Kessler, Phys. Rev. 130, 2349 (1963); R. A. Mann and M. E. Rose, Phys. Rev. 121, 293 (1961). The validity of the results of Mann and Rose has been questioned by T. B. Day, L. S. Rodberg, G. A. Snow, and J. Sucher, Phys. Rev. 123, 1051 (1961).

[9]M. M. Block, E. B. Brucker, I. S. Hughes, T. Kikuchi, C. Meltzer, F. Anderson, A. Pevsner, E. M. Harth, J. Leitner, and H. O. Cohn, Phys. Rev. Letters 3, 291 (1959).

DISCUSSION OF A SUGGESTED BOUND ON COUPLING CONSTANTS

C. J. Goebel* and B. Sakita†
University of Wisconsin, Madison, Wisconsin
(Received 12 August 1963)

In a recent Letter, Geshkenbein and Ioffe[1] have derived an upper bound on coupling constants, i.e., the mass-shell value of three-leg vertices. Their form of bound is remarkable in comparison to previous results[2] because it depends only on the masses of the three particles involved, and not on the nature and range of the forces between the particles or on the nonexistence of stable states in other (crossed) channels. In the present communication we conclude that an assumption on which the G-I bound is based (namely, that the proper vertex function has no pole) has no direct (i.e., phenomenological) physical significance, and hence that their bound on coupling constants likewise has no direct physical significance.

We recapitulate briefly their argument: The propagator of a spinless boson a has the representation

$$G^{-1} = (s - m_a^2)\left\{1 + (s - m_a^2)\left[\sum \frac{c_i}{s_i - s} + \int \frac{ds' \rho(s')}{(s' - m_a^2)^2 (s' - s - i\epsilon)}\right]\right\}, \quad (1)$$

with $c_i \geq 0$. In the limit of infinite energy, $G^{-1} \xrightarrow{s\to\infty} Zs$, where Z, the propagator renormalization constant, should be nonnegative; this imposes the condition

$$\int \rho\, ds \leq 1 - \sum c_i \leq 1. \quad (2)$$

The spectral weight $\rho(s)$ is a linear combination of positive definite terms, each contributed by a state into which particle a can transform (conserving everything except energy). The contribution to ρ of a two-body state consisting of particles b and c is $\xi(s) g^2 |\Gamma(s)|^2$, where $\xi(s)$ is a phase-space factor, g is the abc coupling constant, and Γ is the proper vertex part (as defined in renormalized field theory) of the abc vertex, normalized to unity on the mass shell, i.e., $\Gamma(m_a^2) = 1$. This implies the inequality $\rho > \xi(s) g^2 |\Gamma|^2$, which, used in Eq. (2), leads to

$$g^2 < \Phi^{-1},$$

where

$$\Phi = \int \frac{ds\, \xi(s) |\Gamma|^2}{(s - m_a^2)}. \quad (3)$$

If a lower limit can be put on Φ, we have an upper limit on g^2.

G-I show that Φ does have a minimum value, if it is assumed that Γ has no singularities on the