coefficient, which in turn is equal to  $\exp(-2\chi_l)$ . Thus we have that

$$\chi_{l} = \frac{1}{2} \int_{-\infty}^{\infty} \rho [(x^{2} + l^{2}/k^{2})^{1/2}] dx.$$
 (14)

Thus for an actual scattering  $\chi_{l=kr}$  is a measure of the "perpendicular interaction probability density" for strong interactions which we call  $\rho_{\perp}(r)$ . This seems to be fairly independent of k if the shrinkage of the diffraction peak is ignored. Note that  $\rho_{\perp}(r)$  has units  $1/cm^2$  rather than  $1/cm^3$  as a true density. From the graph of  $\chi_l$  vs *l*, the perpendicular interaction probability density is seen to be strongly peaked near the center. In this region,  $\chi_l$  is very sensitive to slight changes in  $X(s, \tau)$  so that the details of the peaking are not reliable. Nevertheless  $\chi_1$  $=\rho_{\perp}(r)$  seems to be large near the center. This core region, which is associated with large- $\tau$ scattering, has thus far been seen only in p-pscattering. It would be interesting to see if the core region were still present in  $\pi$ -*p* scattering.

The concept and usefulness of the perpendicular interaction probability density is rather general and is independent of the model given here to illustrate its importance. It may be that the strong interaction distribution in the perpendicular direction is of more fundamental importance than was previously suspected. In any case the perpendicular interaction probability density is all that one can obtain from experiment without further assumptions about the nature of the strong interactions.

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<sup>2</sup>For 1 GeV see, for example, J. D. Dowell, W. R. Frisken, G. Martelli, B. Musgrave, H. B. van der Raay, and R. Rubinstein, Nuovo Cimento <u>18</u>, 818 (1960). For 2-6 GeV see, for example, B. Cork, W. A. Wenzel, and C. W. Causey, Phys. Rev. <u>107</u>, 859 (1957).

<sup>3</sup>R. Serber [Phys. Rev. Letters <u>10</u>, 357 (1963)] proposed a Yukawa-shaped, purely absorptive optical model.
<sup>4</sup>G. Molière, Z. Naturforsch. <u>2A</u>, 133 (1947); R. J. Glauber, <u>Lectures in Theoretical Physics</u> (Interscience Publishers, Inc., New York, 1958), p. 315.

## COHERENT PRODUCTION AS A MEANS OF DETERMINING THE SPIN AND PARITY OF BOSONS

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Presently available machines allow the possibility of studying the coherent reaction  $\pi(K) + A \rightarrow B_{\pi}(B_K) + A$ , where B is an integer-spin particle or resonance and A is a nucleus of mass number A.

For coherence, i.e., for the nucleus to remain in its ground state, the minimum momentum transfer at forward scattering angles must not exceed the reciprocal of the nuclear radius. This puts a lower limit on the incident beam energy  $\omega$ :

$$\omega > \frac{1}{2}m_B^2 A^{1/3}r_0 \sim m_B^2 A^{1/3}/2m_{\pi}.$$

This means that for production of bosons of mass  $m_B \lesssim 1.7$  BeV the process is a kinematically possible reaction for CERN and AGS machines. An

example of a coherent nuclear reaction is provided by the process  $\gamma + A \rightarrow \pi^0 + A$  which has been observed (at lower energies) by Tollestrup <u>et al</u>. and others.<sup>1</sup>

Our reason for considering the coherent process is that in the allowable angular range a single one of the 2J+1 states of the produced boson is produced with much higher probability than the other states. In many cases this leads to unique statements about the angular distribution of the decay products of B. Even in the cases when unique statements cannot be made, certain useful information can be established which can be used to determine both the spin and parity of B. Presumably the coherent process is identifiable by its dependence on the mass number A and by its characteristic cutoff at an angle  $\theta_c \approx m_\pi/\omega A^{1/3}$ .

The results of this analysis may be stated as follows: Consider an axis in the <u>rest system of</u> <u>B</u> which is along the incident projectile direction (seen from this system). Barring a special form of cancellation, we show for the case of B having the same intrinsic parity<sup>2</sup> as the projectile that one may apply an Adair<sup>3</sup> analysis to the decay distributions of B with respect to the above axis over the whole angular range in which the coherent process occurs. The case of opposite intrinsic parity has a zero in cross section for the forward direction, but nevertheless essentially only one state of B is excited (which is different from the same-parity case) and a similar analysis can be performed.

In order to become acquainted with the method, we consider first the case of B having spin one.

A vector particle is described by a set of three orthogonal polarization vectors,  $\epsilon^i$  (*i* = 1, 2, 3), each orthogonal to the momentum,  $\epsilon^i \cdot p = 0$ , and spacelike,  $\epsilon^i \cdot \epsilon^i = -1$ . In the rest system of the vector particle we define the three directions to be along the incident-beam direction (as seen from this system), along the normal to the production plane, and along the third direction perpendicular to the above two directions. The polarization four-vectors satisfying the above conditions are given, in the laboratory system, by Eqs. (1a) through (1c) in the order stated above. (The order of components is the following: time; incident-beam direction; normal to production plane; and in production plane and orthogonal to beam direction.)

$$\epsilon_{\mu}^{1} = \frac{1}{Am_{B}} [\omega p^{2} - pk\epsilon \cos\theta; \omega\epsilon p \cos\theta - k(\epsilon^{2} - p^{2}\sin^{2}\theta);$$

0; 
$$(\omega \epsilon - pk \cos \theta)p \sin \theta$$
], (1a)

$$\epsilon_{\mu}^{2} = [0; 0; 1; 0],$$
 (1b)

$$\epsilon_{\mu}^{3} = \frac{1}{A} [-kp\sin\theta; -p\omega\sin\theta; 0; p\omega\cos\theta - \epsilon k], \quad (1c)$$

where  $\epsilon$  and p are the energy and momentum of the boson,  $\omega$  and k are the energy and momentum of the incident particle,  $\theta$  is the angle between the produced boson and the incident-beam direction, and  $A^2 = (p \cdot k)^2 - m_{\pi} K^2 m_B^2$ .

The matrix element for the production process will be of the form  $\epsilon_{\mu}{}^{i} \cdot V_{\mu}{}^{\pm}$ , where  $V_{\mu}{}^{\pm}$  is a fourvector formed out of the various vectors available in the process and where the  $\pm$  refers to *B* having the same parity or opposite parity to the beam, respectively. In the coherent processes suggested here, only spin-zero targets are considered. Thus the general form of  $V_{\mu}^{\pm}$  is

$$V_{\mu}^{+} = a(s, t, m_{B}^{2})k_{\mu} + b(s, t, m_{B}^{2})(P \cdot k/M^{2})P_{\mu},$$
  
$$V_{\mu}^{-} = c(s, t, m_{B}^{2})\epsilon_{\mu\nu\sigma\tau}k_{\nu}p_{\sigma}P_{\tau},$$

where  $P_{\mu}$  is the target momentum four-vector, *M* is the target mass, and *a*, *b*, and *c* are arbitrary scalar functions of the energy *s*, momentum transfer *t*, and boson mass. These functions although, in general, arbitrary may not be singular for  $\theta \rightarrow 0$ , since this would yield a singularity in the physical region for the crossed  $\mu$  channel, i.e., for the inverse process  $B + A \rightarrow \pi$  (K) + A.

From the expression for  $V_{\mu}^{-}$  above it is easy to see that only  $\epsilon_{\mu}^{2}$  is excited in the case of a parity change. Furthermore, because  $c(s,t,m_{B}^{2})$ cannot be singular at  $\theta = 0$ , the cross section will vanish at least as fast as  $\theta^{2}$  for small  $\theta$ . In the case of the same parity no  $\epsilon_{\mu}^{2}$  will be excited, but both  $\epsilon_{\mu}^{-1}$  and  $\epsilon_{\mu}^{-3}$  will be excited. However, for small angles we find the relation

$$\frac{V^{\top} \cdot \epsilon^{3}}{V^{+} \cdot \epsilon^{1}} = \frac{(\epsilon/m_{B})\theta}{[1 - (a/b)m_{B}^{2}/2\omega^{2}]}.$$
 (2)

From Eq. (2) we can see the following three situations:

(i) The denominator of Eq. (2) is of order unity or larger, and for angles  $\theta \ll m/\epsilon$ ,  $\epsilon_{\mu}^{1}$  is excited with much larger probability than  $\epsilon_{\mu}^{3}$ . Since the coherent process is confined to angles  $\theta < m\pi/A^{1/3}\omega$ , the angles are indeed small enough to amply satisfy the above angular condition. A nonvanishing cross section as  $\theta \to 0$  would indicate that the denominator in Eq. (2) satisfies this criterion.

(ii) The denominator of Eq. (2) is zero or very small over the allowed coherent angular range. Then  $\epsilon_{\mu}^{3}$  will be excited with much larger probability than  $\epsilon_{\mu}^{1}$  and the cross section will decrease with  $\theta^{2}$  as  $\theta \rightarrow 0$ .

(iii) The denominator of Eq. (2) is such that in a certain portion of the allowed angular range Eq. (2) is of order unity. These special circumstances would mean that both  $\epsilon_{\mu}{}^{3}$  and  $\epsilon_{\mu}{}^{1}$  are excited comparably. However, by dividing the allowed angular region into several subdivisions and considering the data in each subdivision separately, this particular situation can be discovered. By comparing the data in a region with small  $\theta$  with the data in a region with larger  $\theta$ , one will see a change in the distribution of the decay products of *B* from a pattern characteristic of  $\epsilon_{\mu}{}^{1}$  alone to a pattern characteristic of both  $\epsilon_{\mu}{}^{1}$  and  $\epsilon_{\mu}{}^{3}$  or perhaps  $\epsilon_{\mu}{}^{3}$  alone. This last analysis, of course, requires rather good statistics.

Although one cannot, <u>a priori</u>, rule out situations (ii) and (iii), they are rather special circumstances and we expect that situation (i) will generally prevail. This is even more so for the higher spin cases where the analogs of situations (ii) and (iii) require increasingly subtler cancellations.

As an explicit example consider the case of production of a vector or axial-vector particle by an incident pseudoscalar  $\pi$  or K, with its subsequent decay into two spin-zero particles (of opposite relative parities for the axial-vector case). The vector case (opposite intrinsic parity relative to the  $\pi$  or K) has a decay amplitude  $\epsilon^{2}$ , where l is the momentum of one of the two spin-zero decay products and leads to a decay distribution of  $\sin^2 \alpha$  $\times \cos^2 \varphi$  in the *B* rest system. For the axial-vector case the decay amplitude is  $\epsilon^1 \cdot l$  for situation (i),  $\epsilon^{3} \cdot l$  for situation (ii), and  $\sqrt{x} \epsilon^{1} \cdot l + (1-x)^{1/2} \epsilon^{3} \cdot l$ for situation (iii), leading to decay distributions of  $\cos^2 \alpha$ ,  $\sin^2 \alpha \sin^2 \varphi$ , and  $x \cos^2 \alpha + (1 - x) \sin^2 \alpha$  $\times \sin^2 \varphi$  in the three cases; x increases and 1 - x $\rightarrow$  0 as the data is confined to smaller production angles  $\theta$ . The angle  $\alpha$  is measured in the rest system of the resonance with respect to the incident-beam direction, and the azimuthal angle  $\varphi$  is measured with respect to the normal to the production plane. Similarly for the decay of the vector to a spin-one vector of momentum p and a  $\pi$ of momentum l, the matrix element is proportional to  $\epsilon_{\mu\nu\sigma\tau}\epsilon_{\mu}{}^{2}p_{\nu}\tilde{\epsilon}_{\sigma}l_{\tau}$ , where  $\tilde{\epsilon}$  is the polarization vector of the decay particle and the decay distribution is  $1 - \sin^2 \alpha \cos^2 \varphi$ . For the decay of the axial vector to a vector and a  $\pi$ , the amplitude is

## $\epsilon \cdot \tilde{\epsilon} - r \epsilon \cdot l \tilde{\epsilon} \cdot p$ ,

where  $\epsilon \rightarrow \epsilon^1$  for situation (i) and  $\epsilon \rightarrow \epsilon^3$  for situation (ii). In general, r is arbitrary; if the vector particle couples with a conserved current  $r = 1/l \cdot p$ . The decay distributions are  $1 - \lambda \cos^2 \alpha$ ,  $1 - \lambda \sin^2 \alpha \sin^2 \varphi$ , and  $1 - \lambda [x \cos^2 \alpha + (1 - x) \sin^2 \alpha \sin^2 \varphi]$  for cases (i), (ii), (iii), respectively, with  $\lambda$ , in general, arbitrary and  $\lambda = \beta^2$  for the conserved current coupling, where  $\beta$  is the velocity of the vector decay particle in the *B* rest system.

From the polarization vectors (1a)-(1c), the general set of polarization tensors  $\epsilon_{11} \ldots {}_{\nu}^{l}$  for

a particle of spin J can be constructed  $(l = 1, 2, \dots, 2J + 1)$ . These tensors must be orthogonal and symmetric in the J lower indices and satisfy the additional condition that the trace on any two indices vanish (i.e., they are orthogonal to spinzero particles). We show these tensors explicitly for spin 2 and 3 for which there are 5 and 7 polarization tensors, respectively. For spin 2, they are

$$(\epsilon_{\mu}^{2}\epsilon_{\nu}^{3}+\epsilon_{\mu}^{3}\epsilon_{\nu}^{2})/\sqrt{2},$$
 (3a)

$$(\epsilon_{\mu}^{1}\epsilon_{\nu}^{2}+\epsilon_{\mu}^{2}\epsilon_{\nu}^{1})/\sqrt{2},$$
 (3b)

$$(\epsilon_{\mu}^{2}\epsilon_{\nu}^{2}-\epsilon_{\mu}^{3}\epsilon_{\nu}^{3})/\sqrt{2}, \qquad (3c)$$

$$(\epsilon_{\mu}^{1}\epsilon_{\nu}^{3}+\epsilon_{\mu}^{3}\epsilon_{\nu}^{1})/\sqrt{2}, \qquad (3d)$$

$$(\frac{2}{3})^{1/2} [\epsilon_{\mu}^{1} \epsilon_{\nu}^{1} - (\epsilon_{\mu}^{2} \epsilon_{\nu}^{2} + \epsilon_{\mu}^{3} \epsilon_{\nu}^{3})/\sqrt{2}]; \qquad (3e)$$

for spin 3,

$$\left[\epsilon_{\mu}^{1}\epsilon_{\nu}^{1}\epsilon_{\alpha}^{1} - \left\{\frac{1}{2}\epsilon_{\alpha}^{1}(\epsilon_{\mu}^{2}\epsilon_{\nu}^{2} + \epsilon_{\mu}^{3}\epsilon_{\nu}^{3})\right\}_{\text{sym}}\right]\left(\frac{2}{5}\right)^{1/2}, (4a)$$

$$\left\{\left(\epsilon_{\alpha}^{2}\epsilon_{\mu}^{2}-\epsilon_{\alpha}^{3}\epsilon_{\mu}^{3}\right)\epsilon_{\nu}^{1}\right\}_{\text{sym}}\left(\epsilon_{6}^{1}\right)^{1/2},\qquad(4b)$$

$$[\{(\epsilon_{\alpha}^{2}\epsilon_{\mu}^{3}+\epsilon_{\alpha}^{3}\epsilon_{\mu}^{2})\epsilon_{\nu}^{3}\}_{\text{sym}}-2\epsilon_{\mu}^{2}\epsilon_{\nu}^{2}\epsilon_{\alpha}^{2}](\frac{1}{10})^{1/2},(4c)$$

$$[\{(\epsilon_{\alpha}^{2}\epsilon_{\mu}^{3}+\epsilon_{\alpha}^{3}\epsilon_{\mu}^{2})\epsilon_{\nu}^{2}\}_{\text{sym}}-2\epsilon_{\mu}^{3}\epsilon_{\nu}^{3}\epsilon_{\alpha}^{3}](\frac{1}{10})^{1/2},(4d)$$

$$\left\{\epsilon_{\mu}^{1}\epsilon_{\nu}^{2}\epsilon_{\alpha}^{3}\right\}_{\text{sym}}^{\left(\frac{1}{3}\right)^{1/2}},\qquad(4e)$$

$$[\{\epsilon_{\alpha}^{1}\epsilon_{\mu}^{1}\epsilon_{\nu}^{2}\}_{\text{sym}}^{-}\{\frac{1}{8}\epsilon_{\nu}^{3}(\epsilon_{\mu}^{3}\epsilon_{\alpha}^{2}+\epsilon_{\alpha}^{3}\epsilon_{\mu}^{2})\}_{\text{sym}}^{-\frac{3}{4}}\epsilon_{\mu}^{2}\epsilon_{\nu}^{2}\epsilon_{\alpha}^{2}](\frac{4}{15})^{1/2}, \qquad (4f)$$

$$[\{\epsilon_{\alpha}^{1}\epsilon_{\mu}^{1}\epsilon_{\nu}^{3}\}_{\text{sym}} - \{\frac{1}{8}\epsilon_{\nu}^{2}(\epsilon_{\mu}^{2}\epsilon_{\alpha}^{3} + \epsilon_{\alpha}^{3}\epsilon_{\mu}^{2})\}_{\text{sym}} - \frac{3}{4}\epsilon_{\mu}^{3}\epsilon_{\nu}^{3}\epsilon_{\alpha}^{3}](\frac{4}{15})^{1/2}.$$
(4g)

By the symbol  $\{ \}_{sym}$  we mean to add the two additional terms that make each tensor symmetric on the indices  $\alpha, \mu, \nu$ .

To see which of these tensors is excited in the coherent production process, we consider, as in the spin-one case, the general production matrix element. For spin two this will be of the form

$$\epsilon^{i}_{\mu\nu}T^{\pm}_{\mu\nu},$$

where 
$$T_{\mu\nu}^{\dagger}$$
 is a tensor of the form  
 $T_{\mu\nu}^{\dagger} = [d(s,t)k_{\mu}k_{\nu} + e(s,t)P_{\mu}P_{\nu} + f(s,t)(k_{\mu}P_{\nu} + P_{\mu}k_{\nu})],$   
 $T_{\mu\nu}^{\dagger} = \{\epsilon_{\mu\rho\sigma\tau}k_{\rho}p_{\sigma}P_{\tau}[h(s,t)k_{\nu} + i(s,t)P_{\nu}] + (\mu - \nu)\},$ 

where  $d(s,t), \dots, i(s,t)$  are arbitrary functions except that they are not singular in the physical region.

Following the arguments in the spin-one case, we have again three similar situations. Analogous to situation (i) where the functions  $d(s,t), \dots, i(s,t)$  do not conspire to occasion a special cancellation, one has tensor (3b) excited in the opposite-parity case and tensor (3e) for the sameparity case.

For the conditions analogous to situations (ii) and (iii), we proceed in the same manner as for the spin-one case, i.e., in the case of no-parity change, situation (ii) gives tensor (3d) and situation (iii) gives tensors (3d) and (3e), tensor (3d) being less important the smaller the production angle. For a change in parity, situation (ii) gives tensors (3a), and (iii) tensors (3a) and (3b), with (3a) being less important than (3b) the smaller the production angle. A cross section which vanishes as  $\theta^2$  and not higher powers of  $\theta$  is sufficient evidence that tensor (3a) is absent in the paritychanging case. Similarly in the case of spin 3, tensor (4a) is excited for no intrinsic parity change, and tensor (4f) for parity change with situation (i). For situation (ii), one has tensor (4g) in the no-parity-change case and tensor (4e) for parity change. For situation (iii), one has both tensors (4a) and (4g) in the no-change-parity case, with tensor (4g) becoming less important than tensor (4a) as smaller production angles are considered. With a change in parity, one has both tensors (4e) and (4f), with (4e) becoming less important for smaller production angles.

In Tables I and II we list the angular distribution of the decay products<sup>4</sup> in the rest system of *B* for several decay channels and for the conditions of situation (i). It should be noticed that there is, in general, a distinct observable difference in the azimuthal dependence of the decay angular distribution depending upon whether there has or has not been a parity change in the production. For example, in the decay into two spin-zero particles, the decay distribution in the case of a parity change is always  $\cos^2 \varphi$ , while it is in the most general case  $a + b \sin^2 \varphi$  for no parity change with Table I. Decay angular distributions from a boson of spin 1, 2, or 3 into bosons of spin zero and unity (with no change in parity). In the three-body decays  $\theta_1 \varphi_1$  and  $\theta_2 \varphi_2$  refer to any two of the three particles and  $\vec{n}$  is a unit vector normal to the decay plane. When the distribution is not unique, we indicate the relative amount of one kind of angular distribution with another by arbitrary coefficients A, B, and C.

Decay	Angular distribution
$1^+ \rightarrow 0^- + 0^+$	$\cos^2 \alpha$
$\rightarrow$ 1 <sup>-</sup> + 0 <sup>-</sup>	$A+B \cos^2 \alpha$ $1-\beta_1^2 \cos^2 \alpha$ (for decay vector meson coupled to a conserved current)
$\rightarrow 0^{-} + 0^{-} + 0^{-}$	$ A\cos\alpha_1 + B\cos\alpha_2 ^2$
$2^{-} \rightarrow 0^{-} + 0^{+}$	$(3\cos^2\alpha-1)^2$
→ 1 <sup>-</sup> + 0 <sup>-</sup>	$\begin{array}{l} A[3\cos^{2}\alpha - 1]^{2} + B[3\cos^{2}\alpha + 1] \\ (3\cos^{2}\alpha + 1) - \beta_{1}^{2}(3\cos^{2}\alpha - 1)^{2} \\ (\text{for conserved current interaction}) \end{array}$
→ 0 <sup>-</sup> + 0 <sup>-</sup> + 0 <sup>-</sup>	$ \begin{array}{l} [A(3\cos^2\alpha_1-1)+B(3\cos^2\alpha_2-1)\\ +C \vec{\mathbf{p_1}}\times\vec{\mathbf{p_2}} ^2(3\cos^2\alpha_n-1)]^2 \end{array} $
	$= \{A(3\cos^{2}\alpha_{1}-1)+B(3\cos^{2}\alpha_{2}-1) \\ + C[\sin^{2}(\alpha_{1}-\alpha_{2})-2\sin^{2}\alpha_{1}\sin^{2}\alpha_{2} \\ \times \sin^{2}(\varphi_{1}-\varphi_{2})+2\sin^{2}\alpha_{1}\sin^{2}\alpha_{2} \\ \times \sin^{2}\frac{1}{2}(\varphi_{1}-\varphi_{2})]\}^{2}$
$3^+ \rightarrow 0^- + 0^+$	$\cos^2 \alpha [5\cos^2 \alpha - 1]^2$
$\rightarrow 1^{-} + 0^{-}$	$A[\cos\alpha(5\cos^2\alpha - 3)]^2 + B[4\cos^4\alpha + \sin^4\alpha]$
	$4\cos^4\alpha + \sin^4\alpha - \beta_1^2 \cos^2\alpha [5\cos^2\alpha - 3]^2 (for conserved current interaction)$

a and b both positive. This distinction is lost in noncoherent processes which allow the nuclear target to flip spin.

Furthermore, if the coherent production is purely an isospin-conserving reaction, then the isotopic spin of the produced boson must be equal to that of the beam particle. If the reaction is induced through the Coulomb field then, of course, it is possible to have a change in isospin of one unit. In this latter case the production is supposed to occur via the exchange of a photon with the nucleus. Because of current conservation there will be additional relations among the production coefficients a, b, etc. The effect of current conservation of this type is to depress the cross section in the very forward direction even for the case of no parity change. This same effect could be produced by the exchange of any Table II. Decay angular distributions from a boson of spin 1, 2, or 3 into bosons of spin zero and unity (with change in parity). In the three-body decays  $\theta_1 \varphi_1$  and  $\theta_2 \varphi_2$  refer to any two of the three particles and  $\vec{n}$  is a unit vector normal to the decay plane. When the distribution is not unique, we indicate the relative amount of one kind of angular distribution with another by arbitrary coefficients A, B, and C.

Decay	Angular distribution
$1^- \rightarrow 0^- + 0^-$	$\sin^2 lpha \cos^2 \!\! \varphi$
$1^{-} \rightarrow 1^{-} + 0^{-}$	$\cos^2lpha + \sin^2lpha \sin^2\!\! arphi$
$1^{-} \rightarrow 0^{-} + 0^{-} + 0^{-}$	$\frac{\sin^2 \alpha_n \cos^2 \varphi_n}{= \sin^2 \varphi_1 \sin^2 (\alpha_1 + \alpha_2)}$
$2^+ \rightarrow 0^- + 0^-$	$\cos^2lpha\sin^2lpha\cos^2\!\! arphi$
$2^+ \rightarrow 1^- + 0^-$	$\frac{[\cos^2\alpha + \sin^2\alpha \cos^2\varphi}{-4 \sin^2\alpha \cos^2\alpha \cos^2\varphi}]$
$2^+ \rightarrow 0^- + 0^- + 0^-$ $3^- \rightarrow 0^- + 0^-$	$[n_{x}V_{z}+n_{z}V_{x}]^{2}  \vec{\nabla} = A\vec{p}_{1}+B\vec{p}_{2}$ $[5\cos^{2}\alpha-1]^{2}\sin^{2}\alpha\cos^{2}\varphi$
$3^- \rightarrow 1^- + 0^-$	$\frac{9}{16}\sin^2\alpha\cos^2\varphi \left[5\cos^2\alpha - 1\right]^2 \\ -\frac{1}{2}\cos^2\alpha\sin^2\alpha \left[5\cos^2\varphi - \sin^2\varphi\right] \\ -\sin^2\varphi - \frac{1}{16}\sin^4\alpha \left[9\cos^4\varphi + \sin^4\varphi + 10\sin^2\varphi\cos^2\varphi\right] \\ -\cos^4\alpha$

spin-one particle which is coupled through a conserved current.

Finally we note that with slight modification

these results can all be repeated for incident photons and yield similar and slightly more complicated but richer results<sup>5</sup> in view of the photon's polarization vector (plane polarized photons dominate in electron production of resonances).

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<sup>1</sup>R. Barringer, R. Meunier, and L. S. Osborne, <u>Proceedings of the CERN Symposium on High-Energy</u> <u>Accelerators and Pion Physics, 1956</u> (European Organization of Nuclear Research, Geneva, 1956), p. 282; A. V. Tollestrup <u>et al.</u>, <u>Proceedings of the Tenth</u> <u>Annual International Rochester Conference on High-Energy Physics, 1960</u> (Interscience Publishers, Inc., New York, 1960), p. 27. H. Ruderman, thesis, Calfornia Institute of Technology, 1962 (unpublished).

<sup>2</sup>We exclude a factor  $(-1)^{J}$  in our definition of intrinsic parity. For example, we refer to the case of incident pion and emergent  $\rho$  meson as a change in parity, and incident pion and emergent axial-vector meson as no change in parity.

<sup>3</sup>R. K. Adair, Phys. Rev. <u>100</u>, 1540 (1955).
<sup>4</sup>C. Zemach (to be published) has performed some

independent studies of the three-body decays.

<sup>5</sup>S. M. Berman and S. D. Drell (to be published).

## PHOTOPAIR PRODUCTION OF CHARGED, SEMIWEAKLY INTERACTING VECTOR BOSONS\*

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The total cross section for the coherent photopair production of charged, semiweakly interacting vector bosons,  $W^{\pm}$ , in the Coulomb field of a nucleus (Fe<sup>56</sup>) has been calculated in lowest order perturbation theory. This calculation differs from previous ones<sup>1</sup> in three respects: (1) It should be valid even for cases in which both the  $W^+$  and  $W^-$  are produced at nonrelativistic energies, that is, it is not an extreme relativistic limit. (2) The matrix element is exact to order  $e^2$ , i.e., the Weizsäcker-Williams approximation is not used. (3) It includes as parameters not only an arbitrary magnetic-dipole moment of the vector boson but also an arbitrary electric-dipole moment. This latter term, if different from zero, would lead to violation of parity (P) and timereversal (T) invariance in lowest order for the photopair production process, and also for the process in which a high-energy neutrino produces a single W and a lepton in the Coulomb field of a nucleus.<sup>2</sup> It would also imply P and T violation in the electromagnetic radiative corrections to the weak interactions, if they are, in fact, mediated by such vector bosons.

For the process considered,

$$\gamma + \mathbf{F}\mathbf{e}^{\mathbf{56}} \rightarrow W^+ + W^- + \mathbf{F}\mathbf{e}^{\mathbf{56}}$$

we included the contribution to the matrix element that corresponds to the three Feynman graphs of Fig. 1, in which the nucleus is not ex-