PROTON-PROTON SCATTERING AND STRONG INTERACTIONS*

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There has recently been considerable interest in the high-energy differential cross section of strongly interacting particles. We will show that for proton-proton scattering the existing data can be fit by a simple function which can be understood in terms of a simple model.

We consider the differential cross section for elastic proton-proton scattering, which is normalized according to

$$X(s,t) = \frac{d\sigma/dt}{d\sigma/dt|_{t=0}^{\rm opt}} = \frac{d\sigma/d\Omega}{d\sigma/d\Omega|_{\theta=0}^{\rm opt}}, \qquad (1)$$

where $d\sigma/d\Omega_{\theta=0}^{\rm opt} = k^2\sigma_{\rm tot}^2/16\pi^2$. It is known experimentally that X depends on both the center-of-mass energy squared, s, and the four-momentum transfer squared, t. We would like to find a single parameter which contains most of this dependence. Now notice that since the two protons are identical, this cross section must be invariant under a 180° rotation in the center-of-mass system. Thus we look for a parameter which is invariant under this rotation. Such a parameter is the transverse momentum, $p_\perp = p\sin\theta$. Consequently we define the variable

$$\tau = p^2 \sin^2\theta. \qquad (2)$$

Note that τ can be written in terms of the Mandelstam variables as

$$\tau = tu/(s-4m^2) = -t[1+t/(s-4m^2)]. \qquad (3)$$

Notice that τ is invariant under a transformation from the laboratory to center-of-mass system. The transverse momentum is also thought to be an important variable in the production of secondary particles by high-energy cosmic rays. For small $-t$, τ is essentially equal to $-t$. However, for large $-t$, it is smaller than $-t$, especially for small values of $s-4m^2$. τ attains its maximum value, $\tau_{\rm max} = \frac{1}{4}(s-4m^2) = p^2$, for 90° scattering, when $t = u - \frac{1}{2}(s-4m^2)$.

One now proceeds by considering all available experimental data for proton-proton scattering above 10 GeV,[1] and determining the value of τ for each point from Eq. (3). These data are then plotted in Fig. 1. It is interesting to note that the shrinkage with increasing energy, seen in graphs of $X(s,t)$ vs $-t$, now seems to be reduced, roughly by a factor of two. There is also less curvature in the diffraction peaks than is seen in plots of X against t. These are consequences of the fact that for fixed t, $\tau(t,s)$ increases with increasing s. If we ignore the remaining s dependence, Fig. 1 can be fit by a rather simple sum of two exponentials in τ,

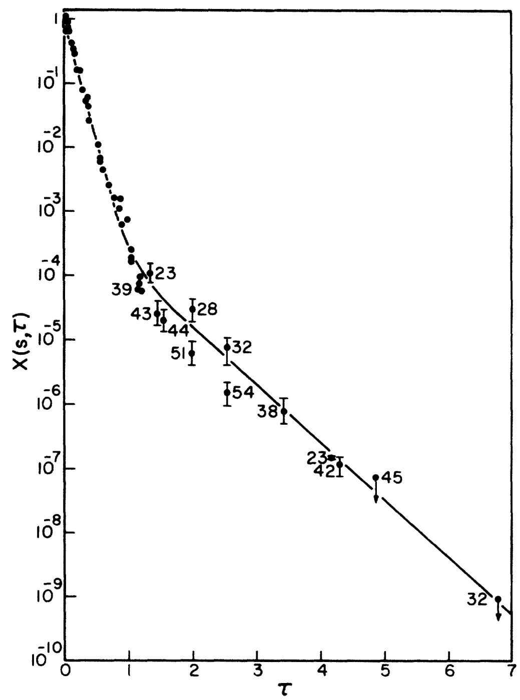

FIG. 1. Normalized differential cross section for elastic proton-proton scattering $X(s,\tau)$ vs τ, the transverse momentum squared in units of $({\rm GeV}/c)^2$. Some s values are shown in units of $({\rm GeV})^2$. Equation (4) is plotted.

$$X(s,\tau) = 0.999\,e^{-8.63\,\tau} + 0.001\,e^{-2.07\,\tau}. \qquad (4)$$

Lower energy proton-proton scattering data[2] down to 1 GeV can also be fit rather well by exponentials in τ. The coefficients, however, are s dependent. There may also be some interference between the two terms where they cross at $\tau = 1.25$ $({\rm GeV}/c)^2$. However, the existence or nature of any interference is of little consequence outside the region $1.0 < \tau < 1.5$, and the present experimental data in this region are not sufficient to determine the nature of any interference that may exist. Thus it is possible that (4) should be replaced by the interference equation which has the same behavior for large and small τ,

$$X(s,\tau)=0.94[e^{-4.315\tau}+10^{-1.5}e^{-1.035\tau}]^2. \quad (5)$$

It is interesting to see what can be learned about the structure of the proton from the differential scattering cross section. We will make a phase-shift analysis which involves a Legendre transformation of the scattering amplitude to extract this information.

The scattering amplitude for identical spinless particles may be expanded in terms of partial waves to obtain

$$f(z)=\frac{1}{ik}\sum_{n=0}^{\infty}(2n+1)P_n(z)[b_n\exp(2i\delta_n)-1], \quad (6)$$

where $z=\cos\theta$. The absorption coefficient is defined to be $b_n=\exp(-\chi_n)$, where χ_n is twice the imaginary part of the phase shift. Now we multiply both sides of (6) by $\int_{-1}^{1}dzP_l(z)$ to obtain

$$\int_{-1}^{1}dzP_l(z)f(z)=\frac{1}{ik}\sum_{n=0}^{\infty}(2n+1)[b_n\exp(2i\delta_n)-1]\times\int_{-1}^{1}P_l(z)P_n(z)dz. \quad (7)$$

Using the orthogonality relation

$$\int_{-1}^{1}P_l(z)P_n(z)dz=\frac{2}{2l+1}\delta_{nl},$$

we have that

$$b_l\exp(2i\delta_l)-1=\tfrac{1}{2}ik\int_{-1}^{1}P_l(z)f(z)dz. \quad (8)$$

The scattering amplitude is related to the normalized differential cross section by the equation

$$f(z)=(ik\sigma_{\text{tot}}/4\pi)[X(s,\tau)]^{1/2}. \quad (9)$$

We have here assumed that the optical theorem holds so that $f(z)$ is purely imaginary. Putting this into Eq. (8) we have a direct equation for calculating the absorption coefficients, b_l, from the differential cross section.

$$b_l\exp(2i\delta_l)-1=-\frac{k^2\sigma_{\text{tot}}}{8\pi}\int_{-1}^{1}P_l(z)[X(s,p^2(1-z^2))dz]^{1/2}. \quad (10)$$

One now proceeds by using either Eq. (4) or (5) for $X(s,\tau)$. The choice depends on the possible existence of interference between the two exponentials. Then we obtain integrals of the forms

$$I_a^{\,l}=\int_{-1}^{1}dzP_l(z)\exp(az^2),$$

$$I_{ab}^{\,l}=\int_{-1}^{1}dzP_l(z)[\exp(az^2)+C\exp(bz^2)]^{1/2}. \quad (11)$$

These are not elementary integrals but they can easily be done numerically by a computer. We now normalize by noting that $\sigma_{\text{tot}}=39.3$ mb and that (4) and (5) seem to be good fits to the data for $k^2\hbar^2=7$ (GeV/c)^2. It is then found that the absorption coefficients are distributed as shown in Fig. 2 where $b_l\exp(2i\delta_l)-1$ is plotted against l for the following four cases: (a) when only the large narrow diffraction peak, which we call the π peak, is present [$X(s,\tau)=e^{-8.63\tau}$]; (b) when only the small broad peak, which we call the core peak is present [$X(s,\tau)=10^{-3}e^{-2.07\tau}$]; (c) when both terms are present but do not in-

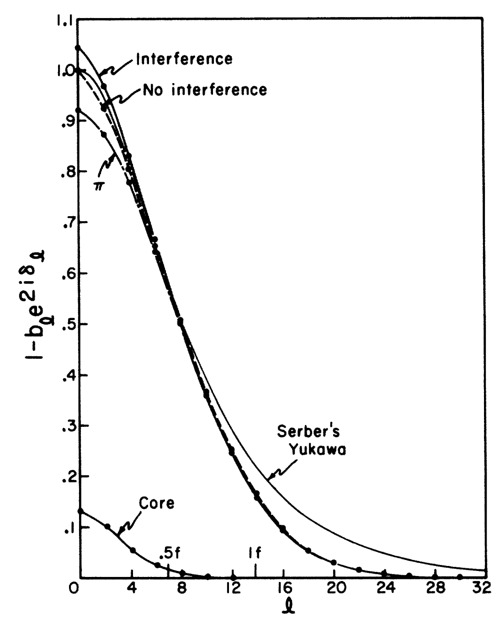

FIG. 2. Distribution of absorption coefficient $b_l\exp(2i\delta_l)-1$, as a function of angular momentum, l. Four cases are shown: (a) the distribution due to the core peak alone; (b) the distribution due to the π peak alone; (c) the distribution due to the core peak and π peak together assuming that there is no interference (4); and (d) that there is interference (5). The 1-fermi mark is shown. The prediction of Serber's Yukawa potential[3] is also plotted.

terfere [Eq. (4)]; (d) when there is interference between the π term and the core term (5). Notice that there is no absorption for odd integers. This is a consequence of the identity of the particles. The prediction of Serber's Yukawa potential[3] is also plotted. We can associate a spatial size with these distributions by employing $l = kr$. The corresponding 1-fermi mark is shown in Fig. 2. Such a spatial distribution is useful in that it is reasonably energy independent. In fact, changing the laboratory energy by a factor of four gives less than a 10% change in the distribution.

Using the distributions in Fig. 2 and assuming that $\delta_l = 0$, we can obtain the various cross sections associated with the different distributions. These are given in Table I.

Perhaps the most interesting parameter which can be obtained from this analysis is χ_l, which is twice the imaginary part of the phase shift. It is easily obtained from the distribution of the b_l's by employing the relation $\chi_l = -\ln b_l$ and assuming that all the δ_l are equal to zero. The resulting distribution of χ_l vs l is plotted in Fig. 3. Notice that the interference and no-interference distributions are both much larger near the center than a simple sum of the separate π and core distributions. This is a consequence of the fact that a great deal of absorbing material is needed to increase the absorption from 0.90 to 0.999, while there is only a 10% increase in cross section. This effect results in some masking of the inner region by the outer region, which leads to the core having a very small cross section of 1.2 mb. The importance of χ_l can be understood in terms of the following simple model for strong interactions, which is somewhat similar to one recently proposed by Serber.[3]

Table I. Cross sections calculated from the distribution of b_l's given in Fig. 2. The core cross sections are very small because of masking by the outer region. Note also that the elastic core cross section is only 0.03 of the inelastic core cross section.

	σ_{total} (millibarns)	$\sigma_{\text{inelastic}}$ (millibarns)	σ_{elastic} (millibarns)
Core	1.2	1.2	0.04
π	38.1	29.4	8.7
Interference	39.3	29.7	9.6
No interference	39.3	30.0	9.3

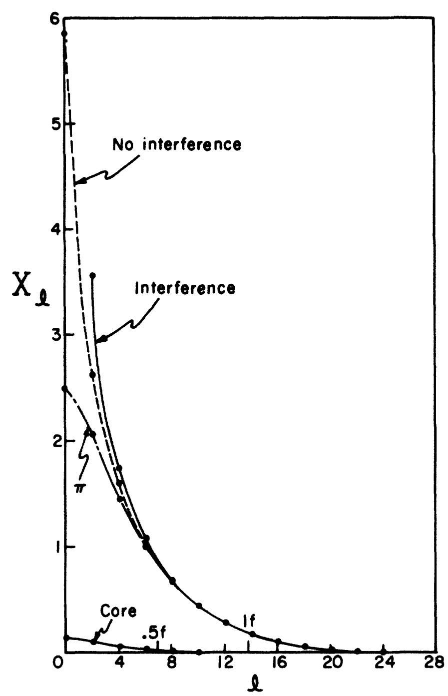

FIG. 3. Distribution of the imaginary part of the phase shift, χ_l, as a function of angular momentum, l. Four cases are shown: (a) core peak alone; (b) π peak alone; (c) interference; and (d) no interference. For the interference and no-interference cases χ_l is proportional to the perpendicular interaction probability density.

Assume that in strong interactions all scattering is caused by the inelastic or absorptive channels, and that the strong elastic scattering is solely the diffraction scattering associated with these channels. Next suppose that the strong interaction can be characterized by an "interaction probability density," $\rho(r)$, which depends on the distance, r, between the centers of the interacting particles and is mathematically similar to potentials which have been used before.[3,4] Then the probability of an interaction during the time that the distance between the particles changes by dx is given by

$$dP = P(\vec{r})\rho(r)dx, \qquad (12)$$

where x is the coordinate parallel to the motion and $P(\vec{r})$ is the probability that the particles do not interact before reaching $\vec{r}$. Then, for classical particles of impact parameter l/k, the probability that there is no interaction at all is given by

$$P(+\infty) = \exp\{-\int_{-\infty}^{\infty} \rho[(x^2 + l^2/k^2)^{1/2}]dx\}. \qquad (13)$$

Now notice that $P(+\infty)$ is just b_l^2, the absorption coefficient, which in turn is equal to $\exp(-2\chi_l)$. Thus we have that

$$\chi_l = \tfrac{1}{2}\int_{-\infty}^{\infty}\rho[(x^2+l^2/k^2)^{1/2}]dx. \qquad (14)$$

Thus for an actual scattering $\chi_{l=kr}$ is a measure of the "perpendicular interaction probability density" for strong interactions which we call $\rho_\perp(r)$. This seems to be fairly independent of k if the shrinkage of the diffraction peak is ignored. Note that $\rho_\perp(r)$ has units $1/\mathrm{cm}^2$ rather than $1/\mathrm{cm}^3$ as a true density. From the graph of χ_l vs l, the perpendicular interaction probability density is seen to be strongly peaked near the center. In this region, χ_l is very sensitive to slight changes in $X(s,\tau)$ so that the details of the peaking are not reliable. Nevertheless $\chi_l = \rho_\perp(r)$ seems to be large near the center. This core region, which is associated with large-τ scattering, has thus far been seen only in p-p scattering. It would be interesting to see if the core region were still present in π-p scattering.

The concept and usefulness of the perpendicular interaction probability density is rather general and is independent of the model given here to illustrate its importance. It may be that the strong interaction distribution in the perpendicular direction is of more fundamental importance than was previously suspected. In any case the perpendicular interaction probability density is all that one can obtain from experiment without further assumptions about the nature of the strong interactions.

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COHERENT PRODUCTION AS A MEANS OF DETERMINING THE SPIN AND PARITY OF BOSONS

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Presently available machines allow the possibility of studying the coherent reaction π $(K)+A \to B_\pi$ $(B_K)+A$, where B is an integer-spin particle or resonance and A is a nucleus of mass number A.

For coherence, i.e., for the nucleus to remain in its ground state, the minimum momentum transfer at forward scattering angles must not exceed the reciprocal of the nuclear radius. This puts a lower limit on the incident beam energy ω:

$$\omega > \tfrac{1}{2}m_B^{\,2}A^{1/3}r_0 \sim m_B^{\,2}A^{1/3}/2m_\pi.$$

This means that for production of bosons of mass $m_B \lesssim 1.7$ BeV the process is a kinematically possible reaction for CERN and AGS machines. An example of a coherent nuclear reaction is provided by the process $\gamma+A\to\pi^0+A$ which has been observed (at lower energies) by Tollestrup et al. and others.[1]

Our reason for considering the coherent process is that in the allowable angular range a single one of the $2J+1$ states of the produced boson is produced with much higher probability than the other states. In many cases this leads to unique statements about the angular distribution of the decay products of B. Even in the cases when unique statements cannot be made, certain useful information can be established which can be used to determine both the spin and parity of B. Presumably the coherent process is identifiable by its dependence on the mass number A