DISLOCATIONS IN SUPERCONDUCTORS

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(Received 18 April 1963)

Bizarre properties of superconductors have often been ascribed to dislocations although a mechanism for their potency has not been identified. They are implicated in magnetic hysteresis,[1,2] high-field, high-current superconduction,[3,4] giant anisotropy of critical currents,[5] and resistance minima and critical-current peaks at high fields,[6,7] particularly in negative surface energy or type-II superconductors.[8,9]

This Letter proposes a simple mechanism for some of these dislocation effects in superconductors, and provides estimates of their magnitudes. The pinning of mixed-state structures accounts for some magnetic properties of high-field superconductors.[10-12] The pinning of flux vortices or fluxoids[9] by cavities has recently been estimated.[13,14] Pinning by dislocations is estimated here for two limiting mixed-state forms, flux vortices or fluxoids[9] and superconducting filaments.[15]

A macroscopic model in which the superconducting state is locally stabilized by the nonuniform stress field of a dislocation provides the pinning mechanism. The relevant second-order stress effects are primarily changes in ground-state or cohesive energy. Therefore local changes in the microscopic parameters, energy gap and coherence length, with stress are neglected in this first approximation in the local limit.

Seraphim and Marcus[16] have given general formulas for critical-field changes with stress from which we have deduced formulas for the changes with the tensor stress components σ_i, $i=1,\cdots,6$, of the free-energy difference per unit volume $\delta(g_n-g_s)$. Specializing to cubic crystal symmetry with Cartesian coordinates parallel to cube edges, we find, to second order,[17]

$$\begin{aligned}-\delta(g_n-g_s) &= \tfrac{1}{2}\Delta\epsilon_1(\sigma_1+\sigma_2+\sigma_3)\\ &+(\tfrac{1}{6})\Delta(S_{11}+2S_{12})^{(2)}(\sigma_1+\sigma_2+\sigma_3)^2\\ &+(\tfrac{1}{6})\Delta(S_{11}-S_{12})[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2]\\ &+(\tfrac{1}{2})\Delta S_{44}(\sigma_4{}^2+\sigma_5{}^2+\sigma_6{}^2), \qquad (1)\end{aligned}$$

where the change of strain at the superconducting to normal transition is $\Delta\epsilon_1$ and the changes of the elastic compliances are ΔS_{ij}. The second term is the second-order dilatational term with the superscript indicating the omission of a negligible part, $12\pi\Delta\epsilon_1{}^2/H_c{}^2$. The last two terms are second-order shear terms that have no first-order counterpart. The second-order terms give the change of strain energy due to changes of the elastic compliances ΔS_{ij} at the phase transition. They are of primary interest here. The coefficients $\Delta\epsilon_i$ and ΔS_{ij} can be directly determined or deduced from δH_c and data have been reported on Nb, V, Pb, and Ta.[18,19] The ΔS_{ij} vary with temperature approximately as the square of the BCS energy-gap parameter, that is, proportional to the superconducting component in a two-fluid model.

To estimate pinning by the nonuniform stress field of a dislocation the interaction energy density, $\delta(g_n-g_s)$ of Eq. (1), is integrated over a volume selected to approximate a pinned configuration such as a fluxoid or a superconducting filament in a mixed-state structure. Variational minimization for the interaction energy would yield a more exact result.

The stress components due to dislocations are given by isotropic elastic continuum theory.[20] Core effects are neglected within a cutoff radius $R_0\sim 3\times10^{-8}$ cm. Anisotropies arise which depend on dislocation type, orientation with respect to the pinned structure, and dislocation orientation with respect to crystallographic axes. However, the anisotropies tend to wash out and permit a simple calculation to yield a good estimate of the magnitude of all the interactions. For the particular case of a [100] screw dislocation, the term in ΔS_{44} is the only nonzero term. This one term provides an illustrative solution exact for this case and correct to a factor ~2 for others.

The interaction energy for this case is

$$U=-\int_v\delta(g_n-g_s)dv=\Delta S_{44}\frac{1}{2}\left(\frac{\mu b}{2\pi}\right)^2\iiint\frac{1}{r}dr\,d\theta\,dz, \qquad (2)$$

where μ is the shear modulus and b is the dislocation Burgers vector. Other cases have more terms of similar form except for angular factors and constants. Because second-order terms dominate in the neighborhoods of dislocations, the first-order terms are negligible for all known

ΔS_{ij} and $\Delta\epsilon_i$ for type-II superconductors except near $T \simeq T_c$ where the second-order coefficients vanish. Since all known values of ΔS_{ij} are negative, one expects that all stresses stabilize the superconducting state for $T \ll T_c$.

A superconducting filament, with thickness near the coherence length, lying along a dislocation is of interest because such a configuration should be stabilized in high fields by the dislocation. We compute the interaction energy U' per unit length of a filament of effective radius R lying parallel with a [001] screw dislocation at a distance ω as

$$U_{\max}' = \Delta S_{44}(\mu b/2\pi)^2 \pi \ln R/R_0 \quad (\omega = 0), \tag{3a}$$

$$U' = \Delta S_{44}(\mu b/2\pi)^2 \tfrac{1}{2}\pi \ln[(R^2 - \omega^2)/R_0^2] \quad (\omega < R - R_0), \tag{3b}$$

$$U' = \Delta S_{44}(\mu b/2\pi)^2 \tfrac{1}{2}\pi \ln[\omega^2/(\omega^2 - R^2) \quad (\omega > R + R_0). \tag{3c}$$

The pinning force, $F_c' \equiv (\partial U'/\partial\omega)_{\max}$, for a unit length of filament is obtained by taking a limiting difference $[U'(\omega = R) - U'(\omega = 0)]/R$ to avoid a nonphysical singularity due to the dislocation core cutoff at R_0:

$$F_c' \cong \Delta S_{44}(\mu b/2\pi)^2 \tfrac{1}{2}\pi (1/R) \ln R/R_0.$$

The magnitude of the interaction can be estimated using, for niobium, $R_0 = 3\times10^{-8}$ cm, $\mu = 3\times10^{11}$ dyn cm^{-2}, $b = 3\times10^{-8}$ cm, $\Delta S_{44} = 4\times10^{-16}$ dyn^{-1}-cm^2 at 4.2°K. For R the largest likely value, 10^{-5} cm, is assumed for the smallest value of F_c. This yields U'(maximum) $\simeq 2\times10^{-8}$ erg cm^{-1}; $F_c \simeq 10^{-3}$ dyn/cm. Equating F_c to the Lorentz force in a transverse field, $B = 10^4$ G yields a critical current $i_c = 10^{-6}$ A. To account for the experimental current density of 10^4 A cm^{-2} requires $\sim10^{10}$ filaments per cm^2 or an effective dislocation density $\rho \sim 10^{10}$ cm^{-2} which is obtainable by severe cold working. Since $F_c \propto 1/R$, thinner filaments would be more strongly pinned. To account for the ubiquitous Nb-25% Zr alloy by dislocation pinning alone requires $R \sim 10^{-6}$ cm. The appropriate values of R, $10^{-7} < R < 10^{-5}$ cm, are near the superconducting coherence length.

Dislocation pinning of fluxoids in the Abrikosov mixed state[9] at $H \ll H_{c2}$ differs geometrically from filament pinning in that it involves a repulsive interaction. A "forest" of dislocations transverse to the fluxoids provides arrays of energy barriers between which the fluxoids can be pinned.

The total interaction energy U'' between one screw dislocation and an infinitely long perpendicular fluxoid of effective radius R at a least distance ω is, for $R \gg R_0$, approximately minus the value of Eq. (2) integrated over the fluxoid volume; that is,

$$U_{\max}'' = -\Delta S_{44}(\mu b/2\pi)^2 2\pi R \ln R/R_0 \quad (\omega = 0); \tag{4a}$$

$$U'' = -\Delta S_{44}(\mu b/2\pi)^2 \tfrac{1}{2}\pi^2 R^2/\omega \quad (\omega \gg R). \tag{4b}$$

The pinning force per unit length of fluxoid is approximately

$$F_c'' \cong -\Delta S_{44}(\mu b/2\pi)^2 \pi^2/d, \tag{5}$$

where d is the spacing of pinning dislocations. This result is rather insensitive to the fluxoid model because of its independence of R. However, at a dislocation density $\rho \sim d^{-2}$ such that $d \sim 1/R$, fluxoid pinning is maximum and is about as strong as pinning of a filament by a coaxial dislocation. For $R = 10^{-5}$ cm this occurs at $\rho = 10^{10}$ cm^{-2}. For smaller R, higher densities and stronger pinning may be effective. The nature of Lorentz forces in a fluxoid lattice is obscure, but formally equating the pinning force to the reaction to a Lorentz force yields an expression for the critical-current density (A cm^{-2})

$$J_c'' = 10 F_c''/\varphi^*, \tag{6}$$

where $\varphi^* = 2\times10^{-7}$ G cm^2 is the fluxoid quantum. In niobium with $\rho = 10^{10}$ cm^{-2} Eq. (6) gives $J_c'' \sim 10^5$ A cm^{-2}. This result neglects any periodicity of the fluxoid lattice, so may overestimate J_c''. A similar result appears for fluxoid pinning by small cavities.[13,14]

These pinning effects can account for some of the behavior of a type-II superconductor such as niobium. The large flux jumps at magnetic fields near H_{c1} in massive samples may occur as dislocation unpinning occurs where the fluxoid lattice pushes into the interior. The nearly field-independent critical current often observed in wires at fields above H_{c1} would be expected from Eq. (6). The fluxoid pinning energy is $U'' \sim 0.1$ eV or $U''/\sqrt{\rho} \sim 10^4$ eV/cm. To compare with Anderson's[12] flux creep result, we write $U = p(H_c^2/8\pi)\pi R^3$ at fixed T, where $p \approx 10^{-3}$ is compared with Anderson's[12] value given as $p = 7\times10^{-3}$ for cavity pinning in another material. At fields near H_{c2} the observed peaks in critical current[7] and dips in resistance[6] may be consequences of a shift from a dense fluxoid network to an impurity-stabilized filamentary structure which could have a higher current limit. If a transition from fluxoid network to some filamentary structure occurs near H_{c2}, then an anisotropy of the critical current may show a pronounced reorientation. Superconduction persists along dislocations a little above H_{c2} as pointed out by Fleischer.[21] Near T_c where ΔS_{ij} vanish, the terms proportional to $\Delta\epsilon_i$ and $\sigma_1, \sigma_2, \sigma_3$ remain, so that some pinning by edge components of dislocations persists to T_c in spite of the incomplete continuity of the effective dislocation segments.

In soft (type-I) superconductors, pinning of the relatively coarse and flexible intermediate-state structures[22-24] is weak. Thus solid solution alloying that reduces the coherence length enough to convert an alloy to type II enormously enhances the magnetic hysteresis produced by plastic deformation.[25]

It appears that the stress fields of dislocations can account for some of the effects attributed to dislocations in superconductors. The stress fields of the dislocations provide a mechanism of appropriate magnitude for pinning of mixed-state structures. However, improved theory should ultimately replace these calculations.

At many stages J. Silcox contributed helpful ideas and comments that are gratefully acknowledged, as are conversations with M. R. Beasley, W. A. Fietz, R. W. Rollins, and J. M. Conway, support of the U. S. Atomic Energy Commission, and the facilities of the Advanced Research Projects Agency.

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