the potential whereas the others have 1, 2, 3,··· nodes. Experimentally, the lowest mode is least damped, but our theory omits any consideration of dissipative effects.

The excellent agreement between theory and experiment at all densities $(r_w^2/\langle\lambda_D^2\rangle)$ supports the simple model used in calculating electron-density profiles. In addition, measurements of the quadrupole resonant frequencies are in good agreement with calculations for that case.

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TRANSVERSE FIELD INTERACTIONS OF A BEAM AND PLASMA*

G. S. Kino and R. Gerchberg
Microwave Laboratory, Stanford University, Stanford, California
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It is well known that strong rf interactions can take place when a cylindrical beam is passed through a plasma. The main interaction of this type, which has been considered in the past, is one in which there is no azimuthal variation of rf field and in which the dominant ac motion of the electrons is in the direction of the motion of the beam. Recently, we have been carrying out an extension of the Gabor, Ash, and Dracott[1] experiment in which they measured rf fields in a plasma sheath by determining the deflection of an electron beam through the sheath parallel to the sheath-plasma interface. It has, therefore, been of interest to know if there is another type of beam-plasma interaction in which an initial rf deflection of the beam increases in amplitude as the beam passes through the plasma. In the presence of such an interaction, not only would the field measurements be unreliable, but there might indeed be the possibility of generation of oscillations because of the introduction of the beam itself.

We came to the conclusion that such an rf hose instability exists. This is equivalent to a mode of interaction with azimuthally varying fields, and had been predicted by Budker[2] several years ago. However, our theory predicts, in addition, that there is a whole class of new interactions with azimuthally varying fields in a magnetic field for which growing waves would occur, not only near the electron plasma frequency, but also near the ion plasma frequency. Experimentally we have demonstrated deflection amplification at frequencies near the electron plasma frequency of the plasma.

Physically, the amplification process may be understood by considering the motion of an ion-neutralized electron beam. With no plasma present, an electron given an initial transverse deflection by an rf field experiences a restoring force toward its unperturbed dc trajectory. This effect is due to the attractive force of the relatively heavy beam-neutralizing ions which suffer virtually no deflection by the rf fields. Thus, the electron would oscillate about its original path, in simple harmonic motion, giving rise to a type of transverse space-charge wave. If, on the other hand, the same beam were passed through a plasma with a frequency of deflection below the plasma frequency, the plasma would behave as a medium with a dielectric constant $\epsilon = \epsilon_0(1 - \omega_p^2/\omega^2)$ which is negative. Consequently, electrons would be repelled from their initial path, and an initial deflection of the electron beam would increase with distance.

In our analysis, we consider a very thin beam of radius a such that $ka \ll 1$, where k is the propagation constant of the wave of interest. It may then be shown that under these conditions, the fields that are present are basically TEM; i.e., the transverse field components are much larger than the longitudinal field components, and that the electric field can be written in terms of a scalar potential, $E = \nabla\phi$, which obeys Laplace's equation

$$\nabla^2\phi = 0 \tag{1}$$

both in the beam and in the plasma. Consequently, if the potential varies as $\cos n\theta$, then within the beam $\varphi \propto r^{|n|}$ and outside the beam we may take $\phi \propto r^{-|n|}$. We assume that all fields vary as $\exp[i(kz - \omega t)]$, and by relating the ac motion of the charged particles to the electric fields present, we are able to determine the surface charge density at the beam-plasma interface[3,4]:

$$\sigma = -\epsilon_0\left\{\frac{\omega_{pb}^2 E_r(a^-)}{(kv_{0b} - \omega)^2} + \frac{\omega_{pe}^2}{\omega^2}[E_r(a^-) - E_r(a^+)]\right\}. \quad (2)$$

Then by matching the potential across the beam boundary and by constraining the discontinuity in E_r there, to be consistent with the surface charge density, we obtain the following dispersion relation for the system:

$$k = (\omega/v_0) \pm (\omega_{pb}/v_0)[2(1 - \omega_{pe}^2/\omega^2)]^{-1/2}, \quad (3)$$

where ω_{pe} and ω_{pb} are the electron plasma frequencies of the plasma and beam, respectively. It will be seen that for $\omega < \omega_{pe}$, the propagation constant k becomes complex and growing waves can occur. This simple theory predicts that the growth rate becomes infinite at the plasma frequency.

When there is a magnetic field present in the direction of motion of the electron beam, we consider the fields which vary as $\exp(in\theta)$. In this case, with ion motion taken into account, the dispersion relation becomes

$$k = \frac{\omega}{v_0} - \frac{n}{|n|}\frac{\omega_{ce}}{2v_0} \pm \frac{1}{v_0}\left\{\tfrac{1}{4}\omega_{ce}^2 + \frac{\omega_{pb}^2}{2[1 - \omega_{pe}^2/(\omega^2 - \omega_{ce}^2) - \omega_{pi}^2/(\omega^2 - \omega_{ci}^2)]}\right\}^{1/2}, \quad (4)$$

where ω_{ce} and ω_{ci} are the electron and ion cyclotron frequencies, respectively, and ω_{pi} is the ion plasma frequency of the plasma. For frequencies well above the ion plasma frequency, this dispersion relation yields growing waves with a growth rate which is a maximum at the frequency $\omega = (\omega_{pe}^2 + \omega_{ce}^2)^{1/2}$.

However, there is an additional set of growing wave modes at very low frequencies, the growth rate being maximum where $\omega^2 = \omega_{ci}^2 + \omega_{pi}^2/(1 + \omega_{pe}^2/\omega_{ce}^2)$.

Thus, for $\omega_{ce}/\omega_{pe} \gg 1$, the growth rate is maximum where $\omega \approx \omega_{pi}$. This corresponds physically to the radial motion of the electron being inhibited by the magnetic field, while the ions are free to move so that the beam can interact with the ions in the plasma. On the other hand, if $\omega_{pe} \gg \omega_{ce}$, the growth rate is a maximum at the electron-ion hybrid cyclotron resonance, i.e., at $\omega = (\omega_{ce}\omega_{ci})^{1/2}$.

In order to test this theory, we have constructed an experimental device similar to that of the original Boyd, Field, Gould experiment[5] shown in Fig. 1. A mercury vapor discharge is formed between an annular cathode and an annular anode four inches apart. An electron beam from an electron gun is passed through this plasma region. The beam is focused by the positive ions present in the plasma system, additional focusing being given by the three Einzel lens electrodes placed just after the gun. The beam is deflected before it enters the plasma by a pair of plates which are themselves connected to a balanced transmission line resonator. The rf deflection of the beam after it leaves the plasma is detected by a second pair of deflection plates also connected to a balanced transmission line resonator. The symmetry of the demodulating plates with respect to the beam precludes the possibility of coupling to any longitudinal beam bunching which might occur. A typical curve of gain at 450 Mc/sec as a function of discharge current is shown in Fig. 2. Amplification of the order of 25 dB is obtained when there is a plasma present.

In order to check this theory further, we varied the discharge current and observed the frequency for maximum gain. The results are shown in Table I.

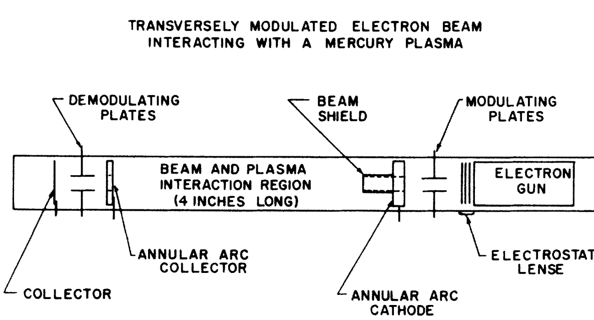

FIG. 1. A schematic of the experimental deflection amplifier.

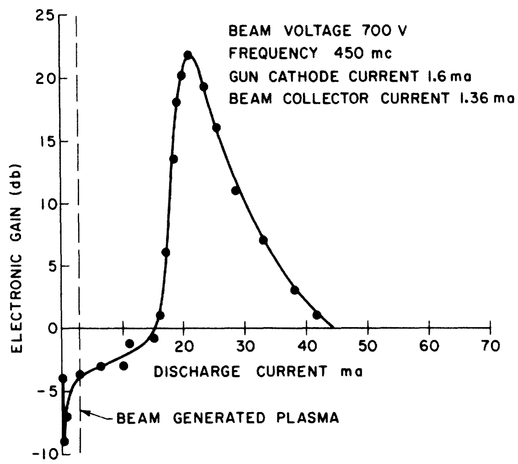

FIG. 2. A typical curve of gain as a function of discharge current taken at 450 Mc/sec.

It will be seen that the frequency for maximum gain is proportional to the square root of the discharge current. As the plasma frequency is also proportional to the square root of the discharge current, the results are in agreement with our theoretical expectations.

We did not include a probe in the system for measuring plasma density because of the possibility of interferences with the rf fields. We have made measurements of density with a dipole resonance technique. So far there is approximately a 50% error between the plasma-density measurements made by this technique and the frequency for maximum gain. This may be due to the relatively short length of the system and consequent errors in the diagnostic technique. We do not, as yet, fully understand the reason for this discrepancy. Further measurements are in progress to observe the same type of interaction in a magnetic field both at low and high frequencies. We also hope to clear up the discrepancy between our diagnostic measurements and our beam-plasma measurements.

Table I. The frequency for maximum gain as a function of discharge current.

Working frequency f (100 Mc/sec)	Arc current at maximum gain I (mA)	$f/I^{1/2}$
450	21	98.2
500	26	98.0
645	40	102.0
670	41	104.5

However, we feel that the results obtained already conclusively demonstrate the presence of a new type of beam-plasma interaction.

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OBSERVATION OF PARAMAGNETIC g-VALUE SHIFTS BY EXCHANGE INTERACTIONS*

M. T. Hutchings and W. P. Wolf†
Clarendon Laboratory, Oxford, England
(Received 22 July 1963)

In any ionic magnetic material in which the interactions between the neighboring magnetic ions are not negligible, the paramagnetic resonance condition for any one particular ion will depend explicitly on these interactions. In the simplest terms we may picture the neighbors polarized by the applied field producing an internal field which shifts the resonance from its "isolated" value. Unfortunately this effect is generally unobservable, because in addition to setting up a time-average internal field, the interactions also produce fluctuating fields which in most cases are much larger and broaden the one-ion absorption. The purpose of this note is to draw attention to cases where the broadening is absent, and the g-value shift, though small, is readily observable. Experimental results for Yb^{+3} in $Tm_3Al_5O_{12}$ and $Tm_3Ga_5O_{12}$