MESON-BARYON RESONANCES AND THE MASS FORMULA*

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In this note we wish to raise some problems connected with the analysis of the meson-baryon resonances in the framework of "higher symmetries." If such a symmetry does exist, it is clearly "broken" in the physical world. The problems we shall discuss concern the nature of this "brokenness," its effect on the resonances, and the possibility of formulating it in a physical theory. For concreteness we shall concentrate our discussion on the octet version[1] of the symmetry scheme based on the group SU_3; however, most of our remarks will be quite general and can be adapted to the discussion of any symmetry scheme.

Let us begin by considering the meson-baryon resonances in the octet model. It has been proposed[2] that the resonances $N_{3/2}^*$(1238 MeV), Y_1^*(1385 MeV), and $\Xi_{1/2}^*$(1530 MeV), together with a yet-to-be-found particle Ω^- belong to the 10-dimensional representation (3, 0) of SU_3.[3] This identification assumes first of all that they all have the same spin and parity $(3/2^+)$. The fact that all these states do not have the same mass is supposed to be the result of some symmetry-breaking interaction. It has been pointed out[4] that if one assumes that the symmetry-breaking interactions have certain transformation properties and that they can be treated as a first-order perturbation, then a "mass formula" can be derived, which gives equal spacing of masses for the 10-dimensional representation. For the representation of SU_3 denoted by (p, q), this mass formula is

$$M = a + bY - c\{2I(I+1) - \tfrac{1}{2}Y^2 + \tfrac{4}{3}(p-q)Y - \tfrac{1}{3}p(p+2) - \tfrac{1}{3}q(q+2) + \tfrac{1}{9}(p-q)^2\}, \qquad (1)$$

where a is the symmetric mass and b and c are proportional to the strength of the symmetry-breaking interaction. In fact, the proposed assignment of the resonances to the 10-dimensional representation was partly based on this mass formula, since the three observed levels $N_{3/2}^*$(1238 MeV), Y_1^*(1385 MeV), and $\Xi_{1/2}^*$(1530 MeV) are very nearly equally spaced.

To analyze the implications of the symmetry-breaking interactions we shall adopt the view that the meson-baryon resonances correspond to poles in the complex total energy plane of the meson-baryon scattering amplitude that has the appropriate quantum numbers. The observed mass splittings between the mesons and between the baryons cause large separations between the thresholds of various channels which are connected with each other by strong interactions, such as π^+p and $K^+\Sigma^+$. The positions of these channel thresholds in the meson-baryon scattering system are given in Fig. 1. In these diagrams all known two-particle channels consistent with the conservation of strangeness and isotopic spin quantum numbers are included without reference to the possible existence of higher symmetries. (By a particle we mean a system stable against decays due to strong interactions.)

The following questions now naturally arise:

(I) On which Riemann sheets are the poles that represent the resonances located? We emphasize that this question should be raised, independent of whether higher symmetries exist or not.

(II) If one gradually switches off the symmetry-breaking interactions, how do the thresholds and

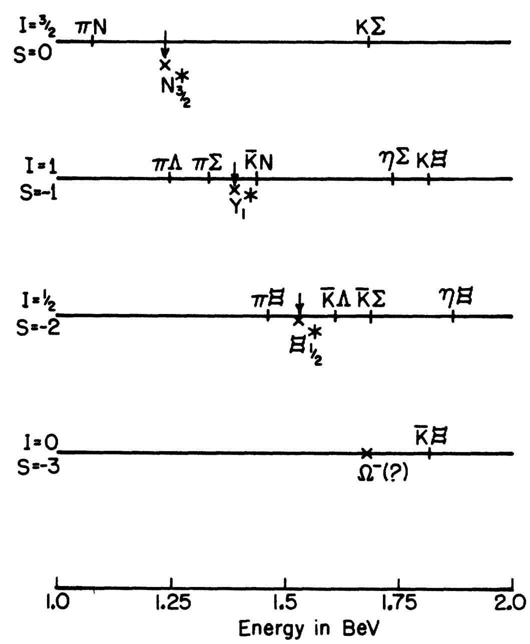

FIG. 1. Thresholds and positions of poles for some meson-baryon scattering systems. Three-particle channels are neglected. These diagrams are plotted independent of the existence or nonexistence of "higher symmetries"; however, isotopic-spin invariance is assumed. The paths by which the resonance poles are reached from the upper half complex energy plane are indicated by arrows.

the resonance poles move? What positions do they finally assume when the symmetry-breaking interactions are completely switched off? Can one assume a linear dependence between the mass of a resonance and the strength of the symmetry-breaking interactions?

(III) What is the criterion that a resonance pole will belong to a pure representation in the presence of a symmetry-breaking interaction?

Concerning question (I) it is clear that for the $N_{3/2}^*$ resonance the pole is reached from the upper half complex energy plane by a path passing between the π^+p and $K^+\Sigma^+$ thresholds. Any other path would not lead to the observed peaking of the π^+p scattering cross section at the $N_{3/2}^*$ resonance energy. The Y_1^* pole is reached from the upper half complex energy plane by passing between the $\pi\Sigma$ and $\bar{K}N$ thresholds. In this case the sheet reached by passing between the $\pi\Lambda$ and $\pi\Sigma$ thresholds is ruled out because experimentally the decay $Y_1^* \to \Lambda+\pi$ causes a well-defined peaking in the energy spectrum of the $\pi\Lambda$ system with practically all of the peak above the $\pi\Sigma$ threshold. The $\Xi_{1/2}^*$ is reached by passing between the $\pi\Xi$ and $\bar{K}\Lambda$ thresholds. These paths are indicated by arrows in Fig. 1. The conjectured particle Ω^- has been tentatively located so as to satisfy the mass formula (1), which requires it to be a bound state.

The arguments above that led to the location of the poles on definite sheets do not preclude the possible existence of additional poles in their neighborhood on different sheets. We shall only be concerned, however, with the poles identified with the resonances and shall not consider these additional poles.

To discuss question (II) let us imagine gradually turning off the symmetry-breaking interaction. We shall assume that the thresholds and the poles move continuously as the symmetry-breaking interaction is gradually switched off. In the octet scheme of SU_3 symmetry all the mesons belong to the same multiplet. So do all the baryons. Therefore all the meson-baryon thresholds will move toward a common energy E_t. If the resonances discussed above are in any approximate sense in the same multiplet, their corresponding poles must all move to the same (complex) energy E_p.

There are then the following possibilities:

(i) E_p is real $< E_t$, so that $N_{3/2}^*$, Y_1^*, $\Xi_{1/2}^*$, and Ω^- all become bound states.

(ii) E_p is complex, so that $N_{3/2}^*$, Y_1^*, $\Xi_{1/2}^*$, and Ω^- all become resonances.

(iii) $E_p = E_t$, so that $N_{3/2}^*$, Y_1^*, $\Xi_{1/2}^*$, and Ω^- all become zero-energy bound states.

Alternative (i) implies that in the case of the Y_1^*, the pole must change sheets. This can happen if the pole either passes through the $\pi\Sigma$ threshold or moves clockwise around the threshold, thus passing into the upper half of a Riemann sheet far removed from the physical one and returning to the lower half plane on another sheet. If the pole passes directly through the $\pi\Sigma$ threshold, it appears as a zero-width resonance in the $\pi\Lambda$ channel for some strength of the symmetry-breaking interaction. This is not possible, in general. Thus for some strength of the interaction the Y_1^* pole must move into a Riemann sheet far removed from the physical one. Its effect on the physical scattering amplitude would then not be easily observable. If one accepts this behavior one also must admit the possibility that for the actual strength of the symmetry-breaking interactions there are such difficult-to-observe poles which must be included to complete various multiplets.

It should be emphasized that if such complicated motion of the pole occurs, the validity of the linearity assumption leading to the mass formula remains to be investigated. [See discussion below concerning question (III).]

For alternative (ii) a similar discussion applies since the $N_{3/2}^*$, Y_1^*, and $\Xi_{1/2}^*$ poles must, respectively, change sheets one, three, and three times.

Alternative (iii) implies that each of the resonances become zero-energy bound states when the symmetry-breaking interaction is switched off. It would be interesting to investigate the compatibility of each of the above alternative (i), (ii), and (iii) with various dynamical models of the $N_{3/2}^*$ resonance.

We point out that if the mass formula (1) is to be taken seriously, then alternative (i) obtains. The values of the constants a, b, and c for the meson octet, baryon octet, and the presumed decuplet of meson-baryon resonances are given in Table I. Empirically for the meson octet, the squares of the masses and not the masses themselves more nearly satisfy the mass formula. Therefore the values of a, b, and c in Table I for the meson octet have been determined using Formula (1) with M, a, b, and c replaced, respectively, by their squares. (Here we are simply following a convention sometimes adopted.) In the case of the meson-baryon decuplet only the combination $b-7c$ appears, so we have put $c=0$

Table I. Mass formula parameters (in MeV).

	Meson octet	Baryon octet	Meson-baryon decuplet
a	412	1154	1385
b	0	- 195	- 146
c	276	- 20	0

for simplicity in Table I. As the strength of the symmetry-breaking interaction is switched off, each of the masses in a given multiplet must approach its mean value a, according to the mass formula. Thus the resonance pole is at $E_p = 1385$ MeV, which is 181 MeV below the meson-baryon threshold $E_t = (412+1154)$ MeV $= 1566$ MeV, in contradiction with alternatives (ii) and (iii).[5]

Next let us turn to question (III). We stress the importance of formulating such a criterion since it is essential to a deeper understanding of the meaning of multiplets under the influence of large symmetry-breaking interactions. We also stress that question (III) is quite different from, though related to, question (II). If one starts from a symmetry-respecting Hamiltonian, gradually switches on the symmetry-breaking interaction, and follows the motion of the various members of a given multiplet as poles in the complex energy plane, one is only tracing the "genetic" development of a multiplet. When the strength of the symmetry-breaking interaction is large enough, an individual member of a multiplet could have so much symmetry mixing in its wave function that it becomes meaningless to apply symmetry considerations to it regarding it as belonging to a pure representation.

To discuss this point we construct the following model in which the higher symmetry of the interactions is broken in a specific manner. Consider a many-channel two-body interaction for which the thresholds are different in the various channels. Assume that there exists a distance r_0 such that (i) for distances $r < r_0$, the interaction is strong and the system respects a certain symmetry, the symmetry-breaking interaction being then negligible; (ii) for distances $r > r_0$, the interaction is unimportant and the system exists as two spatially separated particles having various masses. The kinematic effects due to the mass differences and threshold differences then cause the wave numbers k_α to be different channels α in the region $r > r_0$. In this model the symmetry-respecting and symmetry-breaking parts of the system are spatially separated. In the R-matrix formulation of the scattering process of Wigner and Eisenbud,[6] this spatial separation means that the matrix R which describes the system for $r < r_0$ commutes with the symmetry group. But the S matrix, which is an explicit function[6] of R and the channel wave number matrix K, does not commute with the symmetry group because K does not.

In this model we can formulate the criterion that the mass splitting can be considered as a perturbation in determining the position of a pole of the S matrix. The criterion is that the wave numbers k_α in different channels, evaluated at the pole, must satisfy

$$|\Delta k| \ll |k_\alpha|, \tag{2}$$

where Δk represents the difference between the wave numbers of different channels.

For poles on the sheet reached by passing through the cut between thresholds, the different channel wave numbers k_α are complex numbers located in different quadrants and requirement (2) is never satisfied. Therefore, in this model the positions of the resonance poles cannot be determined by considering the symmetry-breaking interaction as a perturbation. We observe in this connection that in the case of isotopic spin symmetry the resonances, e.g., the singly charged component of the $N_{3/2}^*$ whose decays into $p+\pi^0$ and $n+\pi^+$ involve thresholds differing by 5.9 MeV, do satisfy criterion (2).

It seems to us that inasmuch as the internal structure of the resonances depend crucially on the channel wave number, criterion (2) should have general validity and is not restricted to the specific model discussed above. In Table II we list the wave numbers k_α for the various channels for the resonances $N_{3/2}^*$, Y_1^*, $\Xi_{1/2}^*$ and the hypothetical Ω^-. It is obvious that (2) is not satisfied.

Aside from the question of treating the symmetry-breaking interaction as a perturbation, one can ask whether a resonance could be considered as belonging to a pure representation even if (2) is not satisfied. Now in all cases at a pole of the S matrix, the residue at the pole defines a "decaying solution." In our model in the region $r < r_0$ the Hamiltonian exhibits exact symmetry. Therefore, in this interior region one can expand the "decaying solution" into solutions belonging to different representations of the symmetry group. The relative magnitudes of the various amplitudes determine the degree of mixing of the

Table II. Channel wave numbers at the resonance poles.

Channel	Resonance pole location (MeV)	Channel wave number at pole (MeV/c)
πN	$1238-50i$	$234-45i$
$K\Sigma$	$1238-50i$	$-13+477i$
$\pi\Lambda$	$1385-25i$	$209-25i$
$\pi\Sigma$	$1385-25i$	$125-32i$
$\bar{K}N$	$1385-25i$	$-42+182i$
$\eta\Sigma$	$1385-25i$	$-12+467i$
$K\Xi$	$1385-25i$	$-7+473i$
$\pi\Xi$	$1530-(<4)i$	$144-(<5)i$
$\bar{K}\Lambda$	$1530-(<4)i$	$-(<5)+231i$
$\bar{K}\Sigma$	$1530-(<4)i$	$-(<4)+318i$
$\eta\Xi$	$1530-(<4)i$	$-(<2)+463i$
$\bar{K}\Xi$	1676	$305i$

internal or intrinsic properties of the resonance. A minimum criterion, weaker than (2), that the resonance approximately belong to a pure multiplet, is that the interior part of the "decaying solution" in our model belongs to the pure multiplet with little mixing. We have attempted to see whether this weaker criterion for the pure multiplet assignment in the case of the $N_{3/2}^*$ possibly could be valid in this model. Assuming that $|kr_0| \gg 1$, the centrifugal barrier is negligible, and one finds that the equation defining the decaying solution ψ is

$$(R+iK^{-1})\psi=0, \tag{3}$$

where K is the wave-number matrix at the pole for the two channels, and R is the derivative matrix. Choosing the states $\varphi_{10}=(1/\sqrt{2})[\pi N-K\Sigma]_{I=\frac{3}{2}}$ and $\varphi_{27}=(1/\sqrt{2})[\pi N+K\Sigma]_{I=\frac{3}{2}}$ as the basis, these matrices assume the following form

$$K\simeq\begin{bmatrix}111+216i & 124-261i\\ 124-261i & 111+216i\end{bmatrix}(\mathrm{MeV}/c), \tag{4}$$

$$R=\begin{bmatrix}R_{10} & 0\\ 0 & R_{27}\end{bmatrix}. \tag{5}$$

The diagonal form of R is required by SU_3 symmetry in the interior region.

Writing

$$\psi=\begin{bmatrix}\psi_{10}\\ \psi_{27}\end{bmatrix},$$

one finds that the internal wave function for the decaying state has the values $R_{27}\psi_{27}$ and $R_{10}\psi_{10}$ at $r=r_0$, and the internal radial derivative has the values ψ_{27} and ψ_{10} at $r=r_0$. The requirement that the internal wave function approximately belongs to only the tenfold multiplet is thus equivalent to

$$|R_{27}\psi_{27}|^2 \ll |R_{10}\psi_{10}|^2 \tag{6}$$

and

$$|\psi_{27}|^2 \ll |\psi_{10}|^2. \tag{7}$$

It is easy to verify that (3), (6), and (7) cannot be simultaneously satisfied. This is a consequence of the off-diagonal elements of K^{-1} being non-negligible in this representation. [Of course, if criterion (2) were satisfied, these off-diagonal elements would be small.] Thus the $N_{3/2}^*$ resonance cannot be regarded as approximately belonging to only the tenfold multiplet in this model.

We have emphasized above some problems encountered in assigning the meson-baryon resonances to a pure multiplet in the octet symmetry scheme. In particular, we pointed out that the application of the mass formula to $N_{3/2}^*$, Y_1^*, $\Xi_{1/2}^*$ and Ω^-, regarded as forming a pure tenfold multiplet, is without theoretical justification. However, equally spaced energy levels are always empirically worthy of attention, and the search for the Ω^- should certainly be continued. We only emphasize that if the Ω^- is found and if it does satisfy the equal-spacing rule, it can hardly be interpreted as giving support to the octet symmetry model, at least not without the introduction of drastically new physical principles.

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[1]M. Gell-Mann, California Institute of Technology Synchrotron Laboratory Report, CTSL-20 (unpublished), and Phys. Rev. 125, 1067 (1962); Y. Ne'eman, Nucl. Phys. 26, 222 (1961).

[2]For a summary of the proposed multiplet assignments for the octet model and further references, see L. W. Alvarez et al., Phys. Rev. Letters 10, 184 (1963).

[3]For a discussion of the representations of the group

SU_3, see R. E. Behrends, J. Dreitlein, C. Fronsdal, and B. W. Lee, Rev. Mod. Phys. 34, 1 (1962).

[4]S. Okubo, Progr. Theoret. Phys. (Kyoto) 27, 949 (1962). See also M. Gell-Mann, reference 1. In the derivation of the mass formula, the Hamiltonian is separated into a term H_0 which is invariant under SU_3 and a term H_1 which transforms under SU_3 like Y, the hypercharge. This separation, if possible, is unique. We define H_1 to be the symmetry-breaking part of the interaction.

[5]Similar considerations could be applied to the proposed $3/2^-$ meson-baryon octet (see reference 2).

[6]E. P. Wigner and L. Eisenbud, Phys Rev. 72, 29 (1947).

NEW KINETIC EQUATION*

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The purpose of this Letter is to derive from first principles a new kinetic equation which applies to all the regimes for which Bogoliubov[1] has derived specialized equations. The new equation therefore constitutes a unified description of all kinetic regimes.

A kinetic equation is an equation for the single-particle distribution function, F, which has the form

$$\partial F/\partial t = A[F], \tag{1}$$

where A depends (functionally) on F only, and which should determine F in terms of its initial value alone.

Denote by ϕ and r the depth and the range of the repulsive two-body potential U, by $n = N/V$ the mean density of a gas of N particles enclosed in a volume V, and by kT the mean kinetic energy of a particle of the gas. Bogoliubov's theory yields the following three equations in the regimes indicated:

Boltzmann (short-range regime) if $\phi/kT \sim 1$, $nr^3 \sim \epsilon \ll 1$;

Landau (weak-coupling regime) if $\phi/kT \sim \epsilon \ll 1$, $nr^3 \sim 1$;

Bogoliubov (Debye regime) if $\phi/kT \sim \epsilon \ll 1$, $nr^3 \sim 1/\epsilon$. (2)

In each of these regimes the new equation coincides to lowest order in ϵ with the corresponding kinetic equation in (2).

A remarkable property of the new equation is that it is completely convergent for the Coulomb potential. The Landau equation, when applied to a plasma, requires two cutoffs: at small impact parameters in order to avoid close encounters with large momentum transfer, and at large impact parameters to simulate the effect of the Debye shielding. It will be seen below that the proper description of both the close encounters and of the dielectric screening are included in the new equation. The fact that convergence appears as a natural consequence of our theory is a strong argument in favor of the resulting equation.

Our derivation does not employ a local analysis in the $(\phi/kT,\ nr^3)$ plane as Bogoliubov's theory does, but relies directly on the existence of two well-separated time scales in the kinetic gas ($\epsilon = T^0/T^1 \ll 1$, where T^0 is the duration time of one collision and T^1 is the relaxation time into equilibrium).

Our theory is based on the following three assumptions: (I) The hierarchy of equations for the s-body distributions which follows from Liouville's theorem[1] is valid. As usual the limit $N \to \infty$, $V \to \infty$ with $n = N/V$ fixed is understood. (II) The two time scales T^0 and T^1 are well separated. (III) The correlation functions are "well ordered" in ϵ, that is, denoting by G and H the two- and three-particle correlation functions, we assume (subscripts denote particle variables)

$$G_{12}/F_1F_2 = O(\epsilon), \quad H_{123}/F_1F_2F_3 = O(\epsilon). \tag{3}$$

We write the equations for F and G in the form

$$\partial F/\partial t = LG, \tag{4}$$

$$\partial G/\partial t + \mathcal{H}G = IFF + \Gamma[F]G + L'H, \tag{5}$$

where $\mathcal{H}$ and I are the Poisson brackets for the total and interaction energies of two particles, Γ is the "polarization" operator

$$\Gamma G_{12} \equiv L_{13}F_1G_{23} + L_{23}F_2G_{13},$$

and L, L' are "phase-mixing" operators

$$L_{12} \equiv \int d\vec{x}_2 d\vec{p}_2 \vec{\nabla}_1 U_{12}\cdot\vec{\nabla}_{v_1},$$

$$L' \equiv L_{13} + L_{23}.$$