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SPIN-PARITY DETERMINATION OF THE $\Xi\pi$ RESONANCE (1.530 GeV)†

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We have continued a study of the $T=\frac{1}{2}$ $\Xi\pi$ resonance at 1.530 GeV.[1,2] In this Letter, we report a complete angular correlation study of the decay sequence $\Xi^{*0} \to \Xi^- + \pi^+$, $\Xi^- \to \Lambda + \pi^-$ which allows us to conclude that the simplest Ξ^* spin-parity assignment[3] is $(\frac{3}{2})^+$.

The resonance was produced by 1.80- and 1.95-GeV/c K^- in the Lawrence Radiation Laboratory 72-in. hydrogen bubble chamber by means of the reactions $K^- + p \to \Xi^{*0,-} + K^{0,+}$. Table I illustrates the production and decay systematics of Ξ^{*0}, where, as in reference 1, events in the interval $1.515 \leq M_{\Xi\pi} \leq 1.545$ GeV are defined as Ξ^* events.[4] The relative decay rates of Ξ^{*0} to Ξ^- and Ξ^0 confirm the $T=\frac{1}{2}$ assignment.[1,2] From a corrected number of 21 ± 6 events, $\Xi^{*-} \to \Xi^- + \pi^0$ (not shown in Table I), also produced in our film, we obtain a total Ξ^{*-} production cross section of 26 ± 7 μb, to be compared with 53 ± 8 μb for Ξ^{*0} production. Thus the Ξ^* production reaction proceeds through both the $T=0$ and $T=1$ channels[1] and the polarization states of Ξ^{*0} and Ξ^{*-} need not be the same. For these reasons, and also because $\Xi^{*0} \to \Xi^-$ is produced more copiously than $\Xi^{*-} \to \Xi^-$, the angular correlation analysis is confined to the 80 examples of the former reaction.

Figure 1 displays all 128 $\Xi^- + \pi^+ + K^0$ events on a Dalitz plot of $M_{K\pi}^2$ vs $M_{\Xi\pi}^2$. At 1.95 GeV/c there is some evidence for $K^*(885)$ production, but the Ξ^* and K^* bands do not overlap. There is no evidence for the $K^*(730)$[5] either outside the Ξ^* band or inside as a Ξ^*-K^* interference.[6] The $M_{\Xi\pi}$ histogram of the events in the vicinity of the Ξ^* region is shown in Fig. 2. A study of the error assignments and χ^2 distributions of the events yields an experimental resolution function with a width of 2 MeV. The best-fit Breit-Wigner distribution has $\Gamma = (7 \pm 2)$ MeV. The distribution folded with the resolution function is shown in Fig. 2. A value of 7 ± 2 MeV for Γ implies that the mean separation between Ξ^* and K^0 at the time of Ξ^* decay is ~30 fermis.[6] This distance, as well as the lack of evidence for K^*-Ξ^* interference, seems to justify the assumption that the Ξ^* decays as a free particle. The possibility of Ξ^* interference with the constant background is discussed below.

As discussed by several authors[7-9] and, more recently, by Byers and Fenster[10] (referred to hereafter as BF), the decay of a spin-J Y^* into a spin-$\frac{1}{2}$ Y and a spin-zero meson leads, in addition to the usual maximum-complexity conditions, to definite relations between the various moments of the Y polarization distribution. In principle, these relations permit a unique determination of

Table I. Production and decay systematics for Ξ^{*0}.

Production process	Decay mode	Topology scanned for	Detection efficiency	Number observed	Corrected number[a]	Predicted decay rate for T =3/2	=1/2	Production σ (μb)
$K^- + p \to K^0 + \Xi^{*0}$	$\Xi^-\pi^+$	2-prong (kink) with 1 or 2V's	$\frac{7}{9}$	80	93 ±11	1	2	53 ±8
	$\Xi^0\pi^0$	0-prong with 2V's	$\frac{2}{9}$	11[b]	36 ±15	2	1	

[a]Corrected for detection efficiency and backgrounds.
[b]See reference 4.

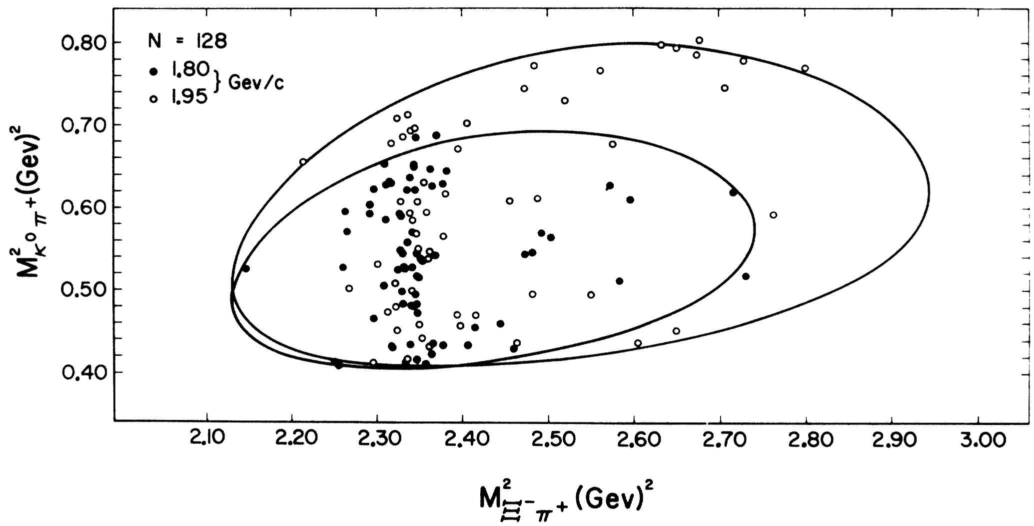

FIG. 1. Dalitz plot for $K^- + p \to \Xi^- + \pi^+ + K^0$. Beam spread at each momentum $\delta P/P \sim \pm 3\,\%$.

the spin J and the (Y^*-Y) relative parity. The usefulness of these tests depends crucially on the presence of some detectable spin alignment of the initial Y^*.

Following the treatment of BF, the spin state of a spin-J Ξ^* at production is described by a set of complex parameters t_L^M (spin-multipole moments) with $L \leq 2J$. These are defined in terms of the Ξ^* density matrix:

$$\rho_{\Xi^*} = \frac{1}{2J+1} \sum_{L=0}^{2J} \sum_{M=-L}^{+L} (2L+1) t_L^{M*} T_L^M, \qquad (1)$$

where the T_L^M are a complete orthonormal set of $(2J+1)^2$ complex matrices[11] formed from the components of the spin operator $\vec{S}$, and $t_L^{-M} = (-)^M t_L^{M*}$. T_0^0 is the unit matrix and $t_0^0 = 1$. For a parity-conserving production process, when the normal to the production plane, $\hat{n}$, is chosen as the polar axis, $t_L^M = 0$ for M odd.[7] Hereafter, the symbol M shall therefore refer only to even values. The intensity and polarization distributions, $I(\hat{\Xi})$ and $I\vec{P}(\hat{\Xi})$, respectively, are determined from the Ξ density matrix $\rho_\Xi = M_l \rho_{\Xi^*} M_l^+$, where M_l is the appropriate transi-

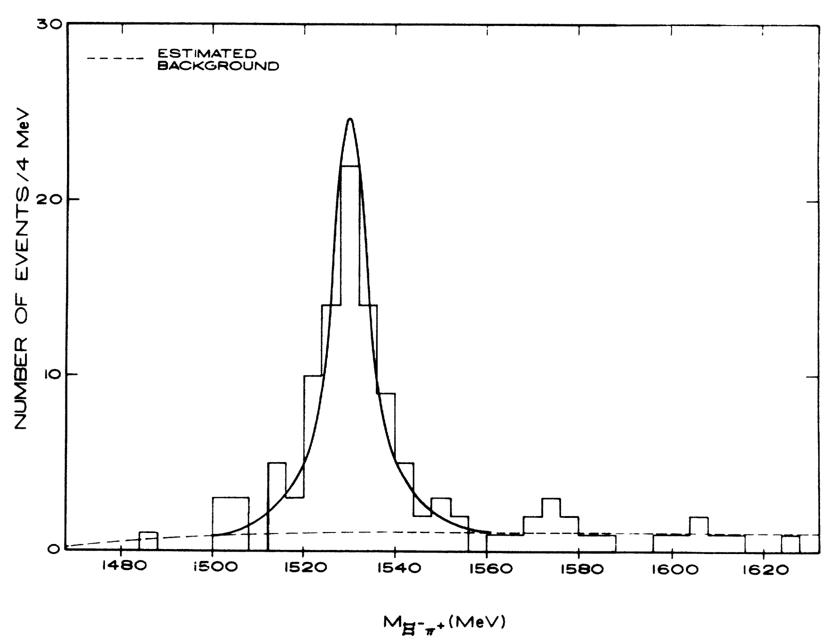

FIG. 2. $M_{\Xi\pi}$ histogram with best-fit Breit-Wigner curve ($\Gamma = 7 \pm 2$ MeV) folded with experimental resolution curve (width 2 MeV). Four events in the $K^*(880)$ region have been removed.

tion operator describing the parity-conserving decay $\Xi^* \to \Xi + \pi$ with orbital angular momentum l. The moments of these distributions are summarized by BF in the following set of relations:

Intensity distribution moments (L even only):

$$\langle Y_L^M \rangle = n_L^J t_L^M. \tag{2}$$

Longitudinal polarization moments (L odd only):

$$\langle \vec{P} \cdot \hat{\Xi} Y_L^M \rangle = n_L^J t_L^M. \tag{3}$$

Transverse polarization moments (L odd only):

$$\frac{L(L+1)}{(L+\frac{1}{2})^{1/2}} \sum_{L' \text{ even}} (L+L'+1)^{-1/2} \sum_{m=-1}^{+1} C(1,m,L',M-m\,|\,1,L',L,M) \langle P_1^m Y_{L'}^{M-m} \rangle = \gamma(2J+1) n_L^J t_L^M, \tag{4}$$

where

$$n_L^J = (-)^{J-1/2} \left(\frac{2J+1}{4\pi}\right)^{1/2} C(J, \tfrac{1}{2}, J, -\tfrac{1}{2}\,|\,J, J, L, 0), \tag{5}$$

and $\gamma = \pm 1$ for $l = J \mp \frac{1}{2}$ and thus is a measure of the $(\Xi^* \Xi \pi)$ relative parity; γ appears only in the transverse polarization distributions.[8] For the sake of brevity, the spherical tensor form of the polarization vector is retained: $IP_1^1 = -(IP_x + iIP_y)/\sqrt{2} = -IP_1^{-1*}$, and $IP_1^0 = IP_z$. In contrast to maximum complexity arguments, a comparison between the polarization moments calculated according to Eqs. (3) and (4) permits, with sufficient data, a unique spin-parity measurement.[10]

The normal to the production plane is defined in terms of the incident K and outgoing Ξ^* directions: $\hat{n} = (\hat{K} \times \hat{\Xi}^*)/|\hat{K} \times \hat{\Xi}^*|$. Each event is then described by the direction cosines along $(\hat{K}, \hat{n} \times \hat{K}, \hat{n}) \equiv (\hat{x}, \hat{y}, \hat{z})$ of the two vectors $\hat{\Xi}$ and $\hat{\Lambda}$ expressed in the Ξ^* and Ξ c.m. systems, respectively. The experimental moments were evaluated by means of relations such as

$$\langle Y_L^M \rangle = \int I(\hat{\Xi}) Y_L^M(\hat{\Xi}) d\Omega = (1/N) \sum_{i=1}^{N} Y_L^M(\hat{\Xi}_i), \tag{6}$$

$$\alpha_\Xi \langle P_x Y_L^M \rangle = \alpha_\Xi \int I \vec{P}(\hat{\Xi}) \cdot \hat{x} Y_L^M(\hat{\Xi}) d\Omega = (3/N) \sum_{i=1}^{N} (\hat{x} \cdot \hat{\Lambda}_i) Y_L^M(\hat{\Xi}_i), \tag{7}$$

where N is the number of events in the sample and, as an example, Eq. (7) shows the calculation of the Y_L^M moment of the polarization component along direction $\hat{x}$. Because of the size of the sample, we average over beam momentum and production angle in the analysis. The decay distribution for a spin-$\frac{1}{2}$ Ξ is given by $\frac{1}{2}(1 + \alpha_\Xi \vec{P} \cdot \hat{\Lambda})$ so that $\alpha_\Xi P_x = 3\hat{x} \cdot \hat{\Lambda}$. Consideration of Eqs. (3), (4), and (7) shows that J and γ may be determined without knowledge of the numerical value of α_Ξ.

The statistical correlations that exist between the various experimental moments were taken into account by means of the error matrix: $U_{XY} = (1/N^2)\sum_{i=1}^{N} (\bar{X} - X_i)(\bar{Y} - Y_i)$, where $\bar{X}$ and $\bar{Y}$ are two moments and X_i and Y_i are the values for the ith event. The chi squared (χ^2) for the hypothesis that a set of experimental moments $\bar{X}_K$ resulted from a population with moments X_K' is given the standard way by

$$\chi^2 = \sum_{K,L} (X_K' - \bar{X}_K) U_{KL}^{-1} (X_L' - \bar{X}_L).$$

The results of the analysis are shown in Tables II and III. Table II contains the experimental evaluations of the moments in Eqs. (2)-(4) for $L \leq 3$. Table III contains the χ^2, the associated probabilities that various sets of moments are zero, and the conclusions based on these probabilities.

The moments of the intensity distribution give strong evidence for the presence of t_2^M. The probability that a $\chi^2 = 16.5$ or larger (3 degrees of freedom) would result from an isotropic distribution is seen to be ~0.0003. We may reasonably conclude from this alone that $J \geq \frac{3}{2}$. However, the much larger χ^2 probabilities that t_4^M and t_5^M are zero show that the data do not require $J > \frac{3}{2}$.

The polarization moments given in Table II have been obtained by dividing the experimental

Table II. Multipole moments for $L=1,2,3$.

Intensity moments [Eq. (2)]:

$$n_2^J t_2^0 = 0.052 \pm 0.032$$
$$n_2^J \mathrm{Re}(t_2^2) = -0.053 \pm 0.017$$
$$n_2^J \mathrm{Im}(t_2^2) = 0.006 \pm 0.023$$

Polarization moments [Eqs. (3) and (4)]:

	$J=\frac{1}{2}$	$J=\frac{3}{2}$	$J=\frac{5}{2}$	$J=\frac{7}{2}$
$\alpha_\Xi t_1^0$	-0.08 ± 0.21	-0.18 ± 0.48	-0.26 ± 0.73	-0.36 ± 0.98
$\alpha_\Xi \gamma t_1^0$	-0.21 ± 0.13	-0.24 ± 0.14	-0.25 ± 0.14	-0.25 ± 0.14
$\alpha_\Xi t_3^0$	...	-0.01 ± 0.17	-0.02 ± 0.30	-0.03 ± 0.42
$\alpha_\Xi \gamma t_3^0$	...	-0.14 ± 0.12	-0.18 ± 0.15	-0.18 ± 0.15
$\alpha_\Xi \mathrm{Re}(t_3^2)$	...	-0.18 ± 0.08	-0.32 ± 0.15	-0.45 ± 0.21
$\alpha_\Xi \gamma \mathrm{Re}(t_3^2)$	...	-0.26 ± 0.09	-0.31 ± 0.12	-0.32 ± 0.12
$\alpha_\Xi \mathrm{Im}(t_3^2)$	...	-0.04 ± 0.11	-0.08 ± 0.21	-0.12 ± 0.29
$\alpha_\Xi \gamma \mathrm{Im}(t_3^2)$	...	$+0.09 \pm 0.09$	$+0.10 \pm 0.10$	$+0.11 \pm 0.11$

moments of Eqs. (3) and (4) by n_L^J and $(2J+1)n_L^J$, respectively, for the hypotheses $J=\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$. Using these values together with the complete error matrix, one can evaluate, for each assumed J, the set of linearly combined experimental moments: $(\alpha_\Xi t_L^M) \mp (\alpha_\Xi \gamma t_L^M)$, for t_1^0, t_3^0, and t_3^2. The χ^2 probability that a given set ($\mp$) is zero gives a measure of the compatibility of the data with the assumed J and $\gamma = \pm 1$. As indicated in Table III, the data are compatible with $P_{3/2}$, while the chance that they resulted from the decay of a $D_{3/2}$ particle is $\frac{1}{63}$. For $J=\frac{5}{2}$, $D_{5/2}$ is compatible, while $F_{5/2}$ is unlikely by $\frac{1}{44}$. The situation is similar for $J=\frac{7}{2}$. The paucity of data in this experiment makes it impossible to decide between the $J \geq \frac{3}{2}$ possibilities.

The simplest Ξ^* spin-parity assignment compatible with the data is $P_{3/2}$. Assuming $P_{3/2}$ and $\alpha_\Xi = -0.50$,[12] the best values for the t_L^M are

$$t_2^0 = -0.18 \pm 0.11, \quad \mathrm{Re}(t_2^2) = 0.19 \pm 0.06, \quad \mathrm{Im}(t_2^2) = -0.02 \pm 0.08,$$
$$t_1^0 = 0.47 \pm 0.28, \quad t_3^0 = 0.20 \pm 0.20, \quad \mathrm{Re}(t_3^2) = 0.42 \pm 0.13, \quad \mathrm{Im}(t_3^2) = -0.08 \pm 0.13.$$

Table III. χ^2 summary of Ξ^* spin-parity analysis.

Hypothesis	χ^2	Degrees of freedom	Probability	Conclusion
$t_2^M = 0$	16.5	3	0.0003	$J \geq \frac{3}{2}$
$t_3^M = 0$	12.0	6	0.036	
$t_{2,3}^M = 0$	29.2	9	0.0002	
$t_4^M = 0$	7.6	5	0.11	Data do not require $J > \frac{3}{2}$
$t_5^M = 0$	7.6	10	0.58	
$t_{4,5}^M = 0$	18.2	15	0.20	
$J = \frac{3}{2}$				
$(1-\gamma)t_{1,3}^M = 0$	1.5	4	0.65	Data compatible with $P_{3/2}$
$(1+\gamma)t_{1,3}^M = 0$	10.3	4	0.016	$D_{3/2}$ unlikely
$J = \frac{5}{2}$				
$(1-\gamma)t_{1,3}^M = 0$	0.9	4	0.84	Data compatible with $D_{5/2}$
$(1+\gamma)t_{1,3}^M = 0$	9.5	4	0.023	$F_{5/2}$ unlikely

These values as well as the corresponding values for $D_{5/2}$ are consistent with the allowed ranges of the t_L^M as discussed in BF. Therefore, additional potentially useful tests based on the theoretical limits of the t_L^M yield no further statements with the data.

As can be seen in Fig. 2, the background intensity is ~5% of the Ξ^* intensity at its peak. Therefore, one cannot disregard the possibility that the observed anisotropies may be due to interference between the resonance and nonresonant background. However, interference effects depend on the phase difference between the two production amplitudes and therefore are expected to vary strongly with the $\Xi\pi$ mass. For this reason, the Ξ^* events with $M_{\Xi\pi} > 1.530$ and < 1.530 GeV were examined separately. Within statistics, the results for these two sets of data are consistent with one another. Thus, there are no detectable interference effects of this type. Furthermore, no "impossible" moments were found in the analysis; that is, neither odd M moments nor odd (even) L moments were found in the intensity (polarization) distributions.

As a check on the analysis and on the IBM-7090 computer program MOMENT which was used to perform the statistical analysis, 100 experiments, each with 80 events, were generated by Monte-Carlo techniques, assuming isotropic distributions of $\hat{\Xi}$ and $\hat{\Lambda}$. Each of these experiments was analyzed by MOMENT. Histograms of the resultant χ^2 distributions for various hypotheses were then compared with the theoretical χ^2 distribution and found to agree within statistics. In addition, 100 experiments were Monte-Carlo generated, assuming, in turn, a $P_{3/2}$ and a $D_{3/2}$ Ξ^* spin state described by the values of t_L^M obtained in the real experiment. Histograms of the t_L^M obtained from the MOMENT analysis of these "fake" experiments were found to agree in both position and width with the real experiment. The results of these Monte-Carlo experiments also justified the χ^2 probabilities given in Table III.

The simplest spin-parity assignment,[3] $(\frac{3}{2})^+$, agrees with the prediction of SU(3) symmetry.[13,14] The existence of a second $\Xi\pi$ resonance[14] at 1600 MeV receives no support from this experiment. Based on Fig. 2, the upper limit on the production cross section of a $\Xi^*(1600)$ at a mean incident momentum of 1.87 GeV/c is ~1 μb.

We wish to express our gratitude to Professor Nina Byers and Stanley Fenster for numerous very helpful conversations on the theoretical questions in their analysis, and for prepublication information of their results. The support of Professor Luis Alvarez is gratefully acknowledged, as is the friendly cooperation of the entire Lawrence Radiation Laboratory hydrogen bubble chamber group.

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[3]Assuming that the relative Ξ-nucleon parity is even.

[4]There are 21 zero-prong-plus-2V events which do not fit $K^- + p \to \Xi^0 + K^0$. For these events, the missing mass above the K^0 was calculated. For 11 of the events, this mass is consistent with the Ξ^* mass. The remaining 10 events form a smooth background.

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