for Λ beta decay given by $0.016(g_\Lambda/g_\beta)^2(1+3\lambda^2)/[1+3(1.14)^2]$. With $\eta=+0.75$, the value appropriate to λ is -0.45, and this branching ratio becomes $0.016/27=0.6\times10^{-3}$, quite compatible with the value $(0.82\pm0.13)\times10^{-3}$ reported recently by R. Ely, G. Gidal, G. Kalmus, L. Oswald, W. Powell, W. Singleton, F. Bullock, C. Henderson, D. Miller, and F. Stannard, Phys. Rev. 131, 868 (1963).

[11]In general, the factor D will include form factors corresponding to the two vertices shown in Figs. 1(c) and 1(d). Since $q=400$ MeV/c, there could be quite appreciable uncertainty in the magnitude of the matrix element given for the Karplus-Ruderman terms by expression (15).

[12]With the Karplus-Ruderman terms alone, (Λp) deexcitation would be the dominant process. With $x=(pq/sq_\Lambda)^2\approx2.2$, the ratio $C({}_\Lambda\mathrm{He}^5)$ would be $(6+14x)/(3+x)\approx7.1$.

ELECTROMAGNETIC AND WEAK INTERACTIONS IN THE UNITARY SYMMETRY SCHEME*

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In the unitary symmetry scheme of Gell-Mann[1] and Ne'eman,[2] particles in a given unitary multiplet are usually classified by means of isotopic spin and hypercharge. It has, however, been observed by Levinson, Lipkin, and Meshkov[3,4] that other classifications can be obtained by considering U_2 subgroups of U_3 that are different from the isotopic-spin subgroup. Here we take advantage of these alternative classifications to derive general formulas for magnetic moments and electromagnetic mass differences of elementary particles, and to make some speculations about the weak interactions. As far as the metastable baryons are concerned, our formulas yield no relations other than those obtained by other authors[5-8]; they are, however, valid for all representations of SU_3, and, as an illustration, they are applied to the baryon-meson resonances of the "tenfold way."[9]

Following Okubo[7,8] we consider the generators $A_\nu^{\ \mu}$ $(\mu,\nu=1,2,3)$ of infinitesimal unitary transformations in U_3. Their commutation rules

$$[A_\beta^{\ \alpha},A_\nu^{\ \mu}]=\delta_\beta^{\ \mu}A_\nu^{\ \alpha}-\delta_\nu^{\ \alpha}A_\beta^{\ \mu} \tag{1}$$

and the unitary restriction

$$(A_\beta^{\ \alpha})^\dagger=A_\alpha^{\ \beta} \tag{2}$$

enable us to divide the generators into three sets, each containing an angular momentum type operator and a corresponding hypercharge operator. They are

$$T_+=-A_1^{\ 2},\quad T_-=-A_2^{\ 1},\quad T_3=\tfrac{1}{2}(A_2^{\ 2}-A_1^{\ 1}),\quad Y_T=A_3^{\ 3},$$

with

$$T_\pm=T_1\pm iT_2; \tag{3}$$

$$L_+=-A_1^{\ 3},\quad L_-=-A_3^{\ 1},\quad L_3=\tfrac{1}{2}(A_3^{\ 3}-A_1^{\ 1}),\quad Y_L=A_2^{\ 2},$$

with

$$L_\pm=L_1\pm iL_2; \tag{4}$$

and

$$K_+=-A_2^{\ 3},\quad K_-=-A_3^{\ 2},\quad K_3=\tfrac{1}{2}(A_3^{\ 3}-A_2^{\ 2}),\quad Y_K=A_1^{\ 1},$$

with

$$K_\pm=K_1\pm iK_2. \tag{5}$$

From each of these sets we can construct a set of mutually commuting operators

$$\vec{T}^2=T_1^{\ 2}+T_2^{\ 2}+T_3^{\ 2},\ T_3,\ Y_T; \tag{6}$$

$$\vec{L}^2=L_1^{\ 2}+L_2^{\ 2}+L_3^{\ 2},\ L_3,\ Y_L; \tag{7}$$

and

$$\vec{K}^2=K_1^{\ 2}+K_2^{\ 2}+K_3^{\ 2},\ K_3,\ Y_K. \tag{8}$$

Because of the commutation rules in (1), $\vec{T}^2$, $\vec{L}^2$, and $\vec{K}^2$ do not commute with one another; hence, only one of the three sets of operators (6), (7), (8) can be diagonalized in an arbitrary matrix representation of the $A_\nu^{\ \mu}$.

We identify $\vec{T}^2$, T_3 with the usual isotopic-spin operators, and Y_T with the usual hypercharge

$$Y_T=(B+S), \tag{9}$$

where B denotes baryon number and S strangeness. If we restrict ourselves to representations $U(f_1,f_2,f_3)$ of U_3, such that[9]

$$f_1+f_2+f_3=0, \tag{10}$$

then[8]

$$A_1^{\ 1}+A_2^{\ 2}+A_3^{\ 3}=0. \tag{11}$$

From Eqs. (3) and (11), we can identify the electric charge operator Q as

$$Q = T_3 + \tfrac{1}{2}Y_T = -A_1{}^1; \tag{12}$$

in addition, from (4), (5), (11), and (12), we find

$$L_3 = \tfrac{1}{2}(Q + Y_T), \quad Y_L = (Q - Y_T), \tag{13}$$

$$K_3 = \tfrac{1}{2}(2Y_T - Q), \quad Y_K = -Q. \tag{14}$$

It follows from (13) and (14) that the axes 2 and 3 in the weight diagrams of references 3 and 4 correspond to the Y_K and Y_L axes, respectively.

If we regard the baryons as members of the eightfold representation $U(1,0,-1)$ of U_3, then their classifications, based on (7) and (13), and on (8) and (14) are as shown in Tables I and II. The corresponding classifications for pseudoscalar and vector mesons can be obtained by means of the substitutions

$$(\Sigma, \Lambda, N, \Xi) \to (\pi, \eta, K, \bar{K}) \tag{15}$$

and

$$(\Sigma, \Lambda, N, \Xi) \to (\rho, \omega, K^*, \bar{K}^*). \tag{16}$$

Turning to the electromagnetic interactions, we assume that the electromagnetic current behaves like the $T_1{}^1$ component of a tensor $T_\nu{}^\mu$ under the transformations of U_3.[7] The magnetic moment of a particle α is then given by

$$\mu(\alpha) = \langle \alpha | T_1{}^1 | \alpha \rangle; \tag{17}$$

and the electromagnetically induced contribution to its rest mass is, in lowest order perturbation theory,

$$\delta m(\alpha) = \langle \alpha | T_{11}{}^{11} | \alpha \rangle, \tag{18}$$

where $T_{11}{}^{11}$ is a component of a tensor $T_{\nu\eta}{}^{\mu\lambda}$. The effects of strong interactions that violate unitary symmetry have been neglected in both of these formulas.

To evaluate the matrix element in (17), we make use of the most general form[7] of $T_\nu{}^\mu$,

$$T_\nu{}^\mu = a\delta_\nu{}^\mu + bA_\nu{}^\mu + c[A_\lambda{}^\mu A_\nu{}^\lambda - \tfrac{1}{3}\delta_\nu{}^\mu A{:}A] \tag{19}$$

and the identity

$$A_\lambda{}^1 A_1{}^\lambda = \tfrac{1}{2}A{:}A - \tfrac{3}{2}Q + (\tfrac{1}{4}Q^2 - \vec{K}^2), \tag{20}$$

where

$$A{:}A \equiv A_\lambda{}^\mu A_\mu{}^\lambda = f_1{}^2 + f_2{}^2 + f_3{}^2 + 2(f_1 - f_3) \tag{21}$$

for a representation $U(f_1, f_2, f_3)$ of U_3. The quantities a, b, c in (19) play a role analogous to that of the reduced matrix element in the quantum theory of angular momentum[10]; if $T_\nu{}^\mu$ is traceless, then

$$a = 0. \tag{22}$$

In the derivation of (20) and (22) we have made use of Eqs. (11) and (12).

From (17), (19), and (20), we obtain the formula

$$\mu(\alpha) = \langle \alpha | a' + b'Q + c(\tfrac{1}{4}Q^2 - K^2) | \alpha \rangle, \tag{23}$$

where

$$a' = a + \tfrac{1}{6}(A{:}A)c, \quad b' = -\tfrac{1}{2}(2b + 3c). \tag{24}$$

If α is an eigenstate of $\vec{K}^2$ (as is the case for all baryons, except Σ^0 and Λ^0; see Table II), then

$$\mu(\alpha) = a' + b'Q + c[\tfrac{1}{4}Q^2 - K(K+1)]. \tag{25}$$

To evaluate the matrix element in (18), we note that the most general form of $T_{11}{}^{11}$ is a linear combination of the six terms

$$\delta_1{}^1\delta_1{}^1,\ \delta_1{}^1A_1{}^1,\ \delta_1{}^1(A_\lambda{}^1A_1{}^\lambda),\ A_1{}^1A_1{}^1,\ A_1{}^1(A_\lambda{}^1A_1{}^\lambda),\ (A_\lambda{}^1A_1{}^\lambda)(A_\lambda{}^1A_1{}^\lambda).$$

It then follows from (12) and (20) that

$$\delta m(\alpha) = \langle \alpha | \{d + eQ + fQ^2 + g(\tfrac{1}{4}Q^2 - \vec{K}^2) + hQ(\tfrac{1}{4}Q^2 - \vec{K}^2) + j(\tfrac{1}{4}Q^2 - \vec{K}^2)^2\} | \alpha \rangle; \tag{26}$$

and if α is an eigenstate of $\vec{K}^2$,

$$\delta m(\alpha) = d + eQ + fQ^2 + g[\tfrac{1}{4}Q^2 - K(K+1)] + hQ[\tfrac{1}{4}Q^2 - K(K+1)] + j[\tfrac{1}{4}Q^2 - K(K+1)]^2. \tag{27}$$

Table I. Classification of baryons by means of $\vec{L}^2, L_3, Y_L$.

Y_L	L	Eigenstates
1	$\frac{1}{2}$	$\Sigma^+, -\Xi^0$
0	1	$p,\ \frac{1}{2}(\Sigma^0 + \sqrt{3}\Lambda^0),\ -\Xi^-$
0	0	$\frac{1}{2}(\sqrt{3}\Sigma^0 - \Lambda^0)$
-1	$\frac{1}{2}$	n, Σ^-

Table II. Classification of baryons by means of $\vec{K}^2, K_3, Y_K$.

Y_K	K	Eigenstates
1	$\frac{1}{2}$	$\Sigma^-, -\Xi^-$
0	1	$n,\ \frac{1}{2}(-\Sigma^0 + \sqrt{3}\Lambda^0),\ \Xi^0$
0	0	$\frac{1}{2}(\sqrt{3}\Sigma^0 + \Lambda^0)$
-1	$\frac{1}{2}$	$p, -\Sigma^+$

It is worth noting that the magnetic moment formula (25) and Formula (27) for $\delta m(\alpha)$ are the analogs of Okubo's first-order[7] and second-order[8] mass formulas, respectively, with Y replaced by Q, and T replaced by K.

Table II and Eqs. (23) and (25) yield the relations

$$\mu(\Sigma^-)=\mu(\Xi^-),\quad \mu(p)=\mu(\Sigma^+),\quad \mu(n)=\mu(\Xi^0),$$
$$2\mu(\Sigma^0)=\mu(\Sigma^+)+\mu(\Sigma^-),$$
$$6\mu(\Lambda^0)=\mu(p)+\mu(\Sigma^-)+4\mu(n),$$

and

$$2\sqrt{3}\,\mu(\Sigma^0\to\Lambda^0)=3\mu(\Lambda^0)+\mu(\Sigma^0)-2\mu(n)-2\mu(\Xi^0),\qquad(28)$$

immediately; in addition, the traceless condition (22) implies

$$\mu(\Lambda^0)=\tfrac{1}{2}\mu(n),$$
$$\mu(\Sigma^-)=-[\mu(p)+\mu(n)].\qquad(29)$$

Table II and Eqs. (26) and (27) yield

$$\delta m(\Xi^-)-\delta m(\Xi^0)=\delta m(\Sigma^-)-\delta m(\Sigma^+)+\delta m(p)-\delta m(n),$$
$$2\sqrt{3}\,\delta m(\Sigma^0\to\Lambda^0)=3\delta m(\Lambda^0)+\delta m(\Sigma^0)-2\delta m(n)-2\delta m(\Xi^0).\qquad(30)$$

The relations in (28), (29), and (30) have already been obtained by other authors,[5-8] who use methods different from ours. The particular advantage of our method is that it yields general formulas that can be applied to the particles of any unitary multiplet. As an example, let us consider the "tenfold way" of Glashow and Sakurai.[9]

We assume that the $\frac{3}{2}^+$ baryon-meson resonances are members of the tenfold representation[9] $U(2,-1,-1)$ of U_3. The classification of particles by means of $(\vec{K}^2, K_3, Y_K=-Q)$ is given in Table III (see reference 4 where the symbol U is used instead of K). As is well known for this representation,[9] the isotopic spin T and hypercharge Y_T are related by

$$T=1+\tfrac{1}{2}Y_T.$$

Similarly, for the K spin and Y_K, we find

$$K=1+\tfrac{1}{2}Y_K=1-\tfrac{1}{2}Q.\qquad(31)$$

This relation, together with Eq. (21), leads to a considerable simplification of the magnetic-moment formula (25) for members of the tenfold representation (all of which are eigenstates of $\vec{K}^2$), namely,

$$\mu(\alpha)=a-bQ,\qquad(32)$$

where a,b are the same as in Eq. (19). Similarly, the formula in (27) reduces to

$$\delta m(\alpha)=d'+e'Q+f'Q^2.\qquad(33)$$

Equation (33) is reminiscent of the "equal-spacing" mass rule[9] and leads to the following relations:

$$\mu(Z^-)=\mu(\Xi^{*-})=\mu(Y_1^{*-})=\mu(N^{*-}),$$
$$\mu(\Xi^{*0})=\mu(Y_1^{*0})=\mu(N^{*0}),$$
$$\mu(Y_1^{*+})=\mu(N^{*+}),\qquad(34)$$

$$\mu(N^{*-})+\mu(N^{*+})=2\mu(N^{*0}),$$
$$\mu(N^{*++})+2\mu(N^{*-})=3\mu(N^{*0}).\qquad(35)$$

If the traceless conditions (22) hold, then we have, in addition,

$$\mu(\Xi^{*0})=0,\ \text{etc.};$$
$$\mu(Z^-)=-\mu(Y_1^{*+})=-\tfrac{1}{2}\mu(N^{*++}).\qquad(36)$$

From (33) we obtain

$$\delta m(N^{*-})-\delta m(N^{*0})=\delta m(\Xi^{*-})-\delta m(\Xi^{*0})=\delta m(Y_1^{*-})-\delta m(Y_1^{*0})\qquad(37)$$

and

$$\delta m(N^{*+})-\delta m(N^{*0})=\delta m(Y_1^{*+})-\delta m(Y_1^{*0})=\tfrac{1}{3}[\delta m(N^{*++})-\delta m(N^{*-})].\qquad(38)$$

It is evident from the preceding discussion that the classification of particles based on the operators $(\vec{K}^2, K_3, Y_K)$ is of special significance for electromagnetic interactions. The reason is not hard to find, for, as Lipkin[11] has pointed out, the electromagnetic current conserves K spin. Hence K spin plays the same role for electromagnetic interactions as does isotopic spin for the strong interactions. In view of this close association

Table III. Classification of baryon resonances by means of $\vec{K}^2, K_3, Y_K$.

Y_K	K	Eigenstates
1	$\frac{3}{2}$	Z^-, Ξ^{*-}, Y_1^{*-}, N^{*-}
0	1	Ξ^{*0}, Y_1^{*0}, N^{*0}
-1	$\frac{1}{2}$	Y_1^{*+}, N^{*+}
-2	0	N^{*++}

between two of the three classification schemes and two of the three general classes of interaction, we would like to suggest that the third classification, namely that based on $(\vec{L}^2, L_3, Y_L)$ [see Eqs. (4) and (13)], may be closely associated with the weak interactions.

To explore the consequences of this suggestion, we note that for leptonic decays, the selection rules

$$\Delta S = \Delta Q = \pm 1, \tag{39a}$$

$$\Delta S = -\Delta Q = \pm 1, \tag{39b}$$

and

$$\Delta S = 0, \quad \Delta Q = \pm 1 \tag{39c}$$

are equivalent to [see Eq. (13)]

$$\Delta L_3 = \pm 1, \quad \Delta Y_L = 0, \tag{40a}$$

$$\Delta L_3 = 0, \quad \Delta Y_L = \pm 2, \tag{40b}$$

and

$$\Delta L_3 = \pm \tfrac{1}{2}, \quad \Delta Y_L = \pm 1, \tag{40c}$$

respectively. For nonleptonic decays,

$$\Delta S = \pm 1, \quad \Delta Q = 0 \tag{41}$$

is equivalent to

$$\Delta L_3 = \pm \tfrac{1}{2}, \quad \Delta Y_L = \mp 1. \tag{42}$$

From (39b) and (40b), we see that the only processes that may conserve L spin are those with $\Delta S = -\Delta Q$. If we assume

$$\Delta L = 0 \tag{43}$$

in this case, then it follows from Table I that

$$\langle \Sigma^+ | n e^+ \nu \rangle = -\langle \Xi^0 | \Sigma^- e^+ \nu \rangle. \tag{44}$$

If we assume that

$$\Delta L = 1 \tag{45}$$

for $\Delta S = \Delta Q$ leptonic decays [see (39a) and (40a)], then we find

$$\langle \Xi^- | \Lambda e^- \nu \rangle = -\langle \Lambda | p e^- \nu \rangle. \tag{46}$$

Similar relationships can be obtained for the other types of weak interactions, and we hope to examine these elsewhere. One final point worth noting is that if $|\Delta L_3| \geq \frac{3}{2}$, the corresponding values of ΔQ, ΔS are not consistent with the presently observed weak interactions.

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