Magnetic Energy Harvesting and Concentration at a Distance by Transformation Optics

Carles Navau, Jordi Prat-Camps, and Alvaro Sanchez*

Grup d'Electromagnetisme, Departament de Física, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain

(Received 21 August 2012; published 28 December 2012)

Based on transformation optics, we introduce a magnetic shell with which one can harvest magnetic energy and distribute it as desired in space with unprecedented efficiency at an arbitrary scale. It allows a very large concentration of magnetic energy in a free space region, which can be used for increasing the sensitivity of magnetic sensors, and the transfer of magnetic energy from a source to a given distant point separated by empty space, with possible applications in wireless transmission of energy.

DOI: 10.1103/PhysRevLett.109.263903 PACS numbers: 41.20.−q, 75.90.+w, 85.25.−j, 85.75.Ss

Transformation optics has pushed the possibilities of controlling light towards unexplored limits [1,2], including perfect lenses [3] and electromagnetic cloaks [4,5]. When applied to static magnetic [6,7] or electric fields [8,9], transformation optics ideas have allowed unique results such as the experimental realization of an exact cloak [10]. An important application of transformation optics is concentration of electromagnetic energy, which is attempted using plasmonics [11,12] or macroscopic concentrators [13]. The former only concentrates at subwavelength scales (typically, nanometers for visible or infrared light [14]) and the latter requires filling the concentration space with material [13,15]. Here we apply transformation optics ideas to shape and concentrate magnetic fields in an unprecedented way, by introducing a design that does not require material in its interior, so it represents an ideal collector for energy harvesting or for increasing the sensitivity of a magnetic sensor. The same device surrounding a magnetic source will enhance the field in the exterior. The combination of these two features will allow us to transfer magnetic energy from a source to a desired distant point through free space. This is an unusual effect because static magnetic fields naturally decay with the distance from the source (in the typical case of a magnetic dipole, as one over the third power of the distance). We will show that other unique properties can be achieved by combining several specimens of our magnetic shell with or without magnetic sources in their interior; for example, two magnetic sources each surrounded by our shells get an enhanced magnetic coupling, which may have applications in wireless transmission of energy.

Consider an infinitely long (along the z direction) cylindrical shell of interior and exterior radii R_1 and R_2, respectively, dividing the space in three domains: interior ($\rho < R_1$), exterior ($\rho > R_2$), and shell ($R_1 < \rho < R_2$) regions. To fulfill our goals, we will demonstrate that it is sufficient that (i) given a source of magnetic energy in the exterior, the shell concentrates all the magnetic energy that would be in the material transferring it to the hole, and (ii) if the source is in the interior, the shell energy is transferred to the exterior.

We start by considering the magnetic source in the exterior domain; we want our shell to transfer all the shell energy into its hole. The required shell material can be determined by the transformation optics technique [1,2], linearly compressing the region from $\rho = 0$ to $\rho = R_2 - \xi$ in the interior domain and expanding the annulus from $\rho = R_2 - \xi$ to $\rho = R_2$ in the shell region through a higher-order polynomial transformation (see Supplemental Material, Sec. I [16]) [17]. In the $\xi \to 0$ limit no space is left in the shell region which means that all the energy is concentrated inside. As the exterior space is not transformed, the exterior magnetic field (and energy) distribution is unaffected. This allows us to understand why magnetic energy is totally transferred from the shell to the hole: the exterior magnetic energy is unaltered, whereas in the shell volume the energy is zero (from the transformation it can be seen that within the shell $\mathbf{B}$ is radial and $\mathbf{H}$ angular so its product $\mathbf{B} \cdot \mathbf{H}$, the magnetic energy density, is zero), so it can only go to the hole. The material in the shell has to be homogeneous and anisotropic with radial and angular relative permeabilities $\mu_\rho \to \infty$ and $\mu_\theta \to 0$. Interestingly, even though the interior space is transformed, no material is needed in the hole (unlike the electromagnetic waves case [13]). For such a shell in a uniform applied field, the field in the hole $\mathbf{H}^{\text{HOL}}$ is uniform, has the direction of the applied field $\mathbf{H}_0$, and its magnitude is increased with respect to H_0 by a factor of R_2/R_1,

$$H^{\text{HOL}} = H_0 \frac{R_2}{R_1}. \qquad (1)$$

Similar results can be obtained for an actual 3D shape, like a spherical shell. The magnetic behavior of the shell can be seen in Fig. 1(a); a large field concentration is achieved inside while keeping the external field not distorted.

One can arrive at Eq. (1) by another path. From Maxwell equations, we have analytically solved the general case of a homogeneous and anisotropic shell in a uniform applied field (see Supplemental Material, Sec. III [16]). We notice especially two results. First, the condition $\mu_\theta = 1/\mu_\rho$ implies no distortion of the external field [7]. Second, for

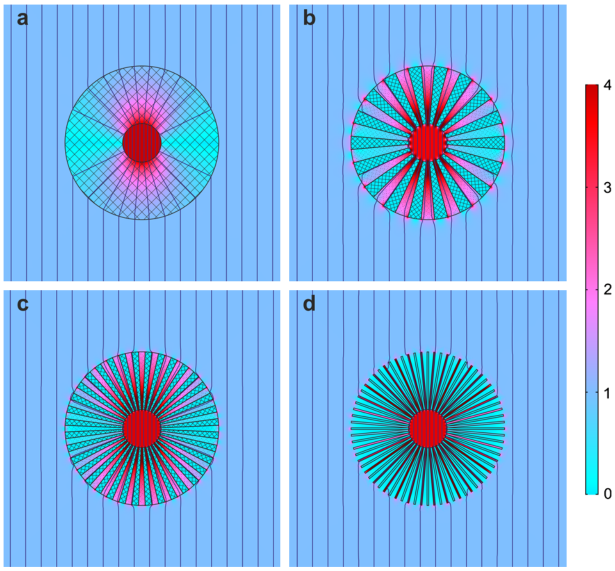

FIG. 1 (color online). Magnetic field lines and their density (in color; in units of applied magnetic field) for (a) the ideal cylindrical magnetic concentrator shell with homogenous anisotropic permeabilities $\mu_\rho \to \infty$ and $\mu_\theta \to 0$ and radii ratio $R_2/R_1 = 4$, and three discretized versions: (b) 36 wedges of alternating homogenous and isotropic ideal superconducting ($\mu = 0.0001$) and ideal soft ferromagnetic ($\mu = 10\,000$) wedges; (c) same with 72 wedges; and (d) same with 72 rectangular prisms, for which a very good behavior is obtained even though materials do not fill the whole shell volume.

a given R_2/R_1 ratio, the field in the hole increases when increasing μ_ρ (for fixed μ_θ) and also when decreasing μ_θ (for fixed μ_ρ). The absolute maximum for magnetic field concentration is achieved in the limit $\mu_\rho \to \infty$ and $\mu_\theta \to 0$, so it corresponds to a nondistortion case, and has the value given in Eq. (1).

Concerning the practical realization of our device, our concentrator requires $\mu_\rho \to \infty$ and $\mu_\theta \to 0$; actual materials with such anisotropy do not exist. But an approximation consisting of an alternation of N radially displaced ferromagnetic and superconductor wedges (or even rectangular prisms) constitutes a natural discretization of the required material as the ferromagnets give the large radial permeability and the alternated superconductors cancel the angular components of the **B** field, leading to an effective $\mu_\theta = 0$, as demonstrated in Figs. 1(b)–1(d) (all numerical calculations in this Letter use the magnetostatics module of COMSOL MULTIPHYSICS software). For large N—easily achieved in practice with thin sheets—the field in the hole is very homogeneous and approaches the exact limit. Such superconducting and ferromagnetic materials are commercially available. In Ref. [10] we fabricated a magnetic cloak using these two materials, and their ideal behavior was experimentally confirmed for fields as large as 40 mT and liquid nitrogen temperatures (in the present case the permeability of the magnetic layers should be larger, so permalloy or similar alloys could be used). Regarding an eventual application of our results to low frequency magnetic fields, we have experimentally confirmed that the cloaking effect obtained in Ref. [10] is still present when magnetic field oscillates at frequencies up to around 100 Hz at least [18].

For small applied field values as required for sensitive magnetic sensors, the behavior of the real materials will be very close to the ideal case. For large applied fields, superconductors have been used to concentrate magnetic energy with standard procedures for applied fields up to several teslas in magnetic lenses [19] but the saturation of the ferromagnets may decrease their permeability.

Our results could be applied to increasing the sensitivity of magnetic sensors, like superconducting quantum interference devices, magnetoresistance, or Hall sensors. Magnetic concentration is typically used to enhance their sensitivity [20–25], by either ferromagnetic materials, attracting magnetic flux, or diamagnetic ones, repelling it—superconductors being the optimum diamagnetic materials. The usual concentration strategy is based on two superconductors—or two ferromagnets—separated by a gap at which flux concentration is produced. We show in Figs. 2(a) and 2(b) a typical example of concentration of a

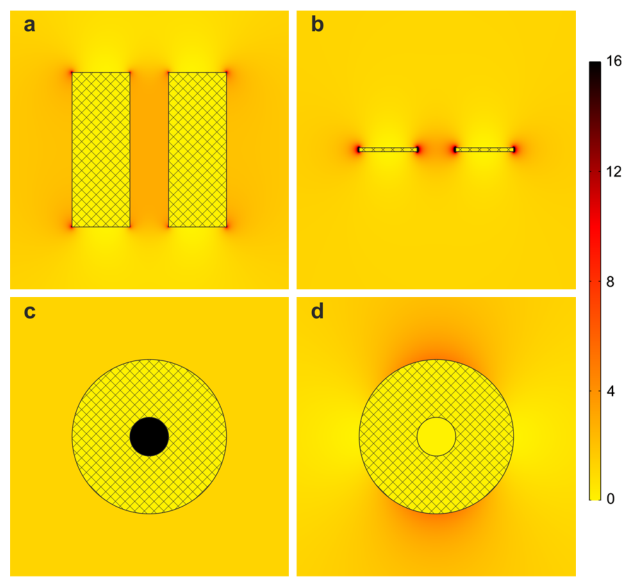

FIG. 2 (color online). Common strategies for field concentration use the gap between two superconductors. In (a) and (b) we show the magnetic energy density (normalized to that of applied field) for two ideal superconducting blocks ($\mu = 0.0001$) with a thickness w separated by a gap b and decreasing heights $h =$ (a) $4b$, and (b) $b/10$, in a vertical applied magnetic field. In (c), our optimum homogeneous anisotropic concentrating shell ($\mu_\rho \to \infty$ and $\mu_\theta \to 0$) with $R_2/R_1 = 4$ shows a large uniform energy density in its hole (for the comparison, notice that $2R_1 = b$ and $w = R_2 - R_1$). In (d), a homogeneous isotropic shell made of ideal soft ferromagnetic material with the same dimensions as (c) attracts more field lines from outside but cannot bring them to the hole, so the inner energy density is zero.

uniform applied magnetic field in the gap of two superconducting slabs (assumed ideal, with zero relative permeability μ [10]). The field gets enhanced at the edges of the slabs, not only on the gap region but also on the exterior ones. When reducing the slabs' thicknesses, the contribution from the edges adds up and a rather intense magnetic field develops in the gap [Fig. 2(b)], although the gap field is not homogeneous but increases towards the strip edges and decreases at the central region. For our shell with permeabilities $\mu_\rho \to \infty$ and $\mu_\theta \to 0$, an applied magnetic field will be enhanced homogeneously in the shell hole by a factor R_2/R_1 [Eq. (1)], so a magnetic sensor placed there would detect a much larger flux—increasing its sensitivity by a large known factor—at the price of an increase of the overall footprint of the sensor [24,26]. By comparing with the superconductors case, in our design not only is the average field in the hole always larger [Fig. 2(c)], but also the magnetic flux is always constant, whereas for the gap strategy the flux tends to zero (Supplemental Material, Fig. S7 [16]) [27] because field lines are mainly diverted toward the exterior edges when the gap is narrowed. Our concentration design is optimum because all the magnetic energy enclosed in the material region is totally transferred to the hole. Another case of interest is for sensing nonuniform fields from nearby magnetic sources, as in biosensors [28], in measuring human brain response in magnetoencephalography [29,30] or for detecting single magnetic microbeads [31]. Often in these situations the gradient is measured rather than the field, in order to separate the signal of the source from other distant noise sources. Transformation optics allows us to analytically obtain the magnetic field and its gradient at any point of the hole (Supplemental Material, Sec. IB [16]). The gradient becomes scaled by a higher power of the radii relation [$(R_2/R_1)^2$ for the gradient of a dipolar source], which makes our concentrator particularly useful for magnetic gradiometers.

We now study the case when the magnetic source is placed inside; we want all the magnetic energy in the shell to be expelled towards the exterior domain. The required material is designed by transforming the space so that the shell space is totally pushed out. Interestingly, applying analogous linear and higher-order polynomial transformations we find that the required shell is exactly the same homogeneous anisotropic shell with $\mu_\rho \to \infty$ and $\mu_\theta \to 0$ that we designed for energy concentration above (Supplemental Information, Sec. II [16]).

The shell property of transferring all the magnetic energy in the shell to the exterior when there is a source in the hole is illustrated in Figs. 3(a)–3(c). The magnetic energy of a magnetic dipole (e.g., a small magnet) in free space [Fig. 3(a)] is expelled outward by enclosing the dipole with our shell [Fig. 3(b)]. Interestingly, outside the shell the field is exactly the same as it would be if there were a centered dipole in empty space, with a magnetic moment enhanced

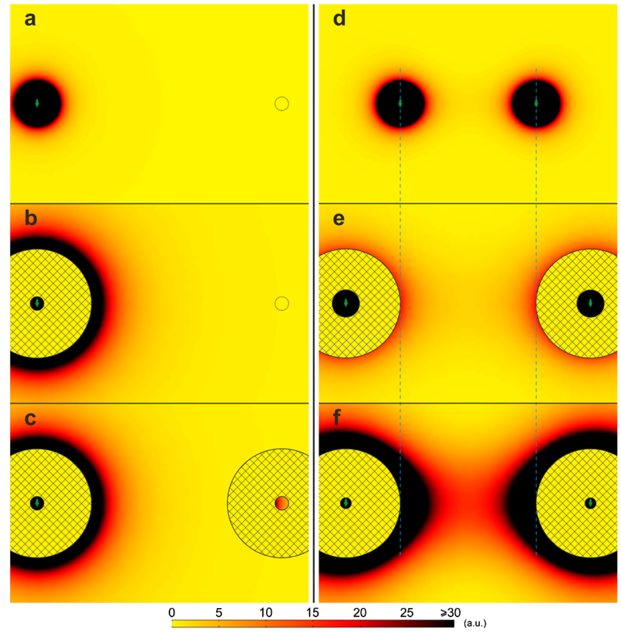

FIG. 3 (color online). Left panels: the magnetic energy density of a cylindrical dipole (e.g., a small magnet) (a) is spatially redistributed towards larger radial distances by the use of our shell with $R_2/R_1 = 8$ (b); in (c) a second concentrator harvests the enhanced magnetic energy in its volume and transfers it to its hole, where a large value of magnetic energy is achieved. Right panels: in (d) magnetic energy density of two identical cylindrical dipoles separated a given gap; when separating and enclosing them with two of our shells with $R_2/R_1 = 4$ [(e)] the magnetic energy density in the middle free space is similar to that in (d); when the inner radii of the shells are reduced to $R_2/R_1 = 10$ [(f)] the magnetic energy is concentrated in the free space between the enclosed dipoles, enhancing the magnetic coupling.

by a factor R_2/R_1. In particular, this enhanced magnetic moment can ideally be made arbitrarily large by making R_1 arbitrarily small. Intuitively, this can be understood if we take into account that the shell expels the magnetic energy that would be in the shell volume towards the exterior, and a small R_1 implies that the shell extends to points near the dipole, where the energy density arbitrarily increases. When adding a second shell at a given far distance, the enhanced field of the first dipole is harvested by the second shell and concentrated in its hole [Fig. 3(c)]: magnetic energy has been transferred from the dipole to a desired position through empty space. Analytical expressions for the field in all these cases are obtained by the corresponding space transformations (Supplemental Material, Eqs. S32 and S15 [16]), and also for any other configuration different from the vertical centered dipole (through the general transformations in Eqs. S8 and S26 [16]). This transfer of magnetic energy through free space is achieved in spite of the general property—also fulfilled in the region between the two concentrators in our case—that any static magnetic field naturally decays with distance in free space [32].

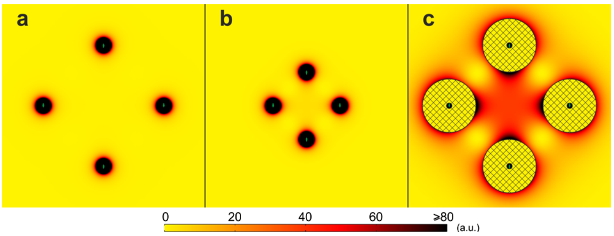

FIG. 4 (color online). Four magnetic dipoles show a weak interaction when they are separated by free space (a) and the field in the region between them is very small (light yellow color), even when they are closer (b). However, when all four dipoles are surrounded by our magnetic shells [$R_2/R_1 = 10$] (c) the field in their exterior becomes greatly enhanced yielding a larger energy density in the intermediate region between the dipoles.

Many combinations of shells with or without inner sources may be constructed based on these ideas. As an example with possible interest for applications, we show in Figs. 3(d)–3(f) how the magnetic coupling between two dipoles separated by a given free-space gap can be substantially enhanced by separating and surrounding them with two of our shells. This enhanced magnetic coupling may have relevance to wireless power transmission [33], where a key factor for achieving this goal is to increase the mutual inductive coupling between the source and the receiver resonators [34]. Although our ideas strictly apply to static fields, extension to other frequency ranges such as those used in power wireless transmission would be possible if materials with the required permeability values at such nonzero frequencies are available, possibly using metamaterials [35].

Another example of interest is shown in Fig. 4, where we present a strategy to increase the magnetic energy created by a collection of sources (four magnets in this example) in a region of free space. By surrounding each of the magnets with our magnetic shells, a large concentration of magnetic energy is obtained in the central region [red color in Fig. 4(c)], with a value much larger than not only that for the original "naked" magnets [Fig. 4(a)] but also that for the same magnets when they are brought to a closer distance [Fig. 4(b)]. These ideas could be applied to medical techniques like transcranial magnetic stimulation [36], for which large magnetic fields are required at a given position in the brain. As explained in Ref. [37], transcranial magnetic stimulation generally targets superficial cortical regions, and deeper targets, such as the basal ganglia, are beyond the range of current technology. Our ideas may help extend the reach of magnetic fields toward deeper depths in the body.

Finally, there are other applications that may result from our ideas. Magnetic energy is one the main agents powering our society: generating energy in power plants, saving information in magnetic memories, moving our devices with motors. All of these applications require a certain spatial distribution of magnetic energy—for example, concentrating it in a transformer core or in a magnetic sensor. Although our results have been strictly derived in the dc limit, early indications [18] show that our work may be also relevant for the low-frequency magnetic fields acting in these applications. Devising new ways of distributing magnetic energy in space becomes essential for developing new applications and improving existing ones. Superconductors and ferromagnetic materials have been traditionally used for shaping magnetic fields, separately. In terms of magnetic energy, both exclude the magnetic energy from their interior (ideal superconductors because $\mathbf{B} = 0$ and ideal soft ferromagnets because $\mathbf{H} = 0$, so that magnetic energy density $\mathbf{B} \cdot \mathbf{H}$ is zero in both cases). Our device, both in the ideal case of a homogeneous anisotropic material and in its discretized versions (Fig. 1), also expels the magnetic energy from its interior, because in this case $\mathbf{B}$ is always perpendicular to $\mathbf{H}$ in the material. The new property that we exploit in this Letter is that when the topology is such that the device divides the space into an exterior and an interior zone (e.g., a hollow cylinder or a sphere), then the energy of the magnetic field is totally transferred from one domain to the other (e.g., from an external source to the hole of the cylinder or sphere), contrary to the case of only superconductors and ferromagnets, where the energy is returned to the domain where the magnetic sources are (Supplemental Material, Fig. S8 [16]). This is the key physical fact that has allowed the main results of this Letter—unprecedented concentration of magnetic energy and its teletransportation to a distant point through free space—as well as future configurations that may be developed based on these ideas.

We are thankful for financial support from Spanish Projects No. NANOSELECT (CSD2007-00041) and No. MAT2012-35370. J. P. C. acknowledges a FPU grant from the Spanish government (No. AP2010-2556).

*alvar.sanchez@uab.cat

[1] A. J. Ward and J. B. Pendry, J. Mod. Opt. **43**, 773 (1996).
[2] H. Chen, C. T. Chan, and P. Sheng, Nat. Mater. **9**, 387 (2010).
[3] J. B. Pendry, Phys. Rev. Lett. **85**, 3966 (2000).
[4] J. B. Pendry, D. Schurig, and D. R. Smith, Science **312**, 1780 (2006).
[5] D. Schurig, J. J. Mock, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr, and D. R. Smith, Science **314**, 977 (2006).
[6] B. Wood and J. B. Pendry, J. Phys. Condens. Matter **19**, 076208 (2007).
[7] A. Sanchez, C. Navau, J. Prat-Camps, and D.-X. Chen, New J. Phys. **13**, 093034 (2011).
[8] F. Yang, Z. L. Mei, T. Y. Jin, and T. J. Cui, Phys. Rev. Lett. **109**, 053902 (2012).
[9] M. Liu, Z. L. Mei, X. Ma, and T. J. Cui, Appl. Phys. Lett. **101**, 051905 (2012).

[10] F. Gomory, M. Solovyov, J. Souc, C. Navau, J. Prat-Camps, and A. Sanchez, Science **335**, 1466 (2012).
[11] J. A. Schuller, E. S. Barnard, W. Cai, Y. C. Jun, J. S. White, and M. L. Brongersma, Nat. Mater. **9**, 193 (2010).
[12] A. Aubry, D. Y. Lei, A. I. Fernandez-Domínguez, Y. Sonnefraud, S. A. Maier, and J. B. Pendry, Nano Lett. **10**, 2574 (2010).
[13] M. Rahm, D. Schurig, D. A. Roberts, S. A. Cummer, D. R. Smith, and J. B. Pendry, Photon. Nanostr. Fundam. Appl. **6**, 87 (2008).
[14] D. K. Gramotnev and S. I. Bozhevolnyi, Nat. Photonics **4**, 83 (2010).
[15] A possible exception of this would be the particular case of a TEM wave propagating along the translational symmetry axis z. In this unique situation both electric and magnetic fields would be in the xy plane (having no z component) so that we could simultaneously ignore the equal nonzero values of permittivity ε_{zz} and permeability μ_{zz} required in the interior part of the concentrator. In this very special situation no material would be needed inside the concentrator even for electromagnetic waves.
[16] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.109.263903 for additional theoretical developments.
[17] S. A. Cummer, R. Liu, and T. J. Cui, J. Appl. Phys. **105**, 056102 (2009).
[18] J. Souc, F. Gomory, M. Solovyov, C. Navau, J. Prat-Camps, and A. Sanchez (unpublished).
[19] S. Choi, J.-H. Yoon, B.-S. Lee, M.-S. Won, J.-W. Ok, Z. Y. Zhang, T. Kiyoshi, S. Matsumoto, and S.-H. Lee, J. Appl. Phys. **111**, 07E728 (2012).
[20] D. Robbes, Sens. Actuators A **129**, 86 (2006).
[21] M. Pannetier, C. Fermon, G. Le Goff, J. Simola, and E. Kerr, Science **304**, 1648 (2004).
[22] P. Ripka and M. Janosek, IEEE Sens. J. **10**, 1108 (2010).
[23] J. Lenz and A. S. Edelstein, IEEE Sens. J. **6**, 631 (2006).
[24] W. C. Griffith, R. Jimenez-Martinez, V. Shah, S. Knappe, and J. Kitching, Appl. Phys. Lett. **94**, 023502 (2009).
[25] R. Kleiner, D. Koelle, L. Ludwig, and J. Clarke, Proc. IEEE **92**, 1534 (2004).
[26] A. Alu and N. Engheta, Phys. Rev. Lett. **102**, 233901 (2009).
[27] A. A. B. Brojeny, Y. Mawatari, M. Benkraouda, and J. R. Clem, Supercond. Sci. Technol. **15**, 1454 (2002).
[28] S. X. Wang and G. Li, IEEE Trans. Magn. **44**, 1687 (2008).
[29] V. Pizzella, S. Della Penna, C. Del Gratta, and G. L. Romani, Supercond. Sci. Technol. **14**, R79 (2001).
[30] R. L. Fagaly, Rev. Sci. Instrum. **77**, 101101 (2006).
[31] P.-A. Besse, G. Boero, M. Demierre, V. Pott, and R. Popovic, Appl. Phys. Lett. **80**, 4199 (2002).
[32] Throughout this Letter, we have used the concept "transfer of magnetic energy." In the strict dc case this concept may be not well defined, but when considering the dc case as actually the limit of very low-frequency ac fields the transfer has now full meaning since energy could be put in at the source end and extracted at the sink end to do useful work.
[33] A. Kurs, A. Karalis, R. Moffatt, J. D. Joannopoulos, P. Fisher, and M. Soljacic, Science **317**, 83 (2007).
[34] D. Huang, Y. Urzhumov, D. R. Smith, K. H. Teo, and J. Zhang, J. Appl. Phys. **111**, 064902 (2012).
[35] M. C. K. Wiltshire, J. B. Pendry, I. R. Young, D. J. Larkman, D. J. Gilderdale, and J. V. Hajnal, Science **291**, 849 (2001).
[36] M. Kobayashi and A. Pascual-Leone, Lancet Neurology **2**, 145 (2003).
[37] G. Bonmassar, S. W. Lee, D. K. Freeman, M. Polasek, S. I. Fried, and J. T. Gale, Nat. Commun. **3**, 921 (2012).