Determining the Density Content of Symmetry Energy and Neutron Skin: An Empirical Approach

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(Received 10 October 2012; published 26 December 2012)

The density dependence of nuclear symmetry energy remains poorly constrained. Starting from precise empirical values of the nuclear volume and surface symmetry energy coefficients and the nuclear saturation density, we show how in the ambit of microscopic calculations with different energy density functionals, the value of the symmetry energy slope parameter L along with that for neutron skin can be put in tighter bounds. The value of L is found to be $L = 64 \pm 5$ MeV. For ^{208}Pb, the neutron skin thickness comes out to be 0.188 ± 0.014 fm. Knowing L, the method can be applied to predict neutron skin thicknesses of other nuclei.

DOI: 10.1103/PhysRevLett.109.262501 PACS numbers: 21.65.Ef, 21.10.Gv, 21.65.Mn

In recent times, there has been a cultivated focus to achieve a better understanding of the density properties of the symmetry energy of nuclear matter. Particular attention has been given to constrain in a narrow window the value of the symmetry energy slope parameter L at the nuclear matter saturation density ρ_0. In terrestrial context, this parameter affects the nuclear binding energies [1] and the nuclear drip lines and has a crucial role in determining the neutron density distribution in neutron-rich nuclei. In astrophysical context, it is also of seminal importance. The pressure $P_n(= 3\rho_0 L)$ of neutron matter at ρ_0 influences the radii of cold neutron stars. The cooling of protoneutron stars through neutrino convection [2], the dynamical evolution of the core collapse of a massive star, and the associated explosive nucleosynthesis depend sensitively on the symmetry energy slope parameter [3,4]. In the droplet model [5,6] of the nucleus, the neutron skin is proportional to L, and a linear correlation between the neutron-skin thickness of the nucleus and neutron-star radius [7] could thus be envisaged.

The symmetry energy slope parameter is defined as

$$L = 3\rho_0 \frac{\partial C_v(\rho)}{\partial \rho}\bigg|_{\rho_0}, \tag{1}$$

where $C_v(\rho)$ is the volume symmetry energy per nucleon of homogeneous nuclear matter at density ρ. Estimates of L are fraught with much uncertainty. Isospin diffusion predicts $L = 88 \pm 25$ MeV [8,9], nucleon emission ratios [10] favor a value closer to $L \sim 55$ MeV, and isoscaling gives $L \sim 65$ MeV [11]. Analysis of the giant dipole resonance of ^{208}Pb [12] is suggestive of $L \sim 45$–59 MeV, whereas pygmy dipole resonance [13] in ^{68}Ni and ^{132}Sn would yield a weighted average in the range of $L = 64.8 \pm 15.7$ MeV. Of late, from a sensitive fit of the experimental nuclear masses to those obtained in the finite-range droplet model [1], the value of L could be fixed in the bound $L = 70 \pm 15$ MeV. Astrophysical observations of neutron star masses and radii reportedly provide tighter constraints to L to $43 < L < 52$ MeV [14] within 68% confidence limits.

Correlation systematics of nuclear isospin with the neutron skin thickness [15,16] for a series of nuclei in the framework of the nuclear droplet model has been undertaken by the Barcelona Group. This has yielded a value of $L = 75 \pm 25$ MeV. The neutron skins were measured from antiprotonic atom experiments [17,18]; systematic uncertainties involving model assumptions to deal with strong interaction are therefore unavoidable. The novel Pb-radius experiment (PREX) at the Jefferson Laboratory has now been attempted through parity violation in electron scattering as a model-independent probe of the neutron density in ^{208}Pb [19]. The neutron skin $R_{\mathrm{skin}} = R_n - R_p$ was found to be $0.33^{+0.16}_{-0.18}$ fm, where R_n and R_p are the point neutron and proton root-mean squared (rms) radii. A reanalysis yielded the value to be 0.302 ± 0.175 fm [20]. The droplet model as well as calculations with class of different interactions, Skyrme or relativistic mean-field (RMF), have now clearly established that the neutron skin thickness of ^{208}Pb is strongly correlated with the density dependence of symmetry energy around saturation [21–24]. In the backdrop of this information, the large uncertainty in the experimental neutron radius of ^{208}Pb seems to be of not much help in putting L in a tighter bound.

Nuclear dipole polarizibility α_D has been suggested [25,26] as an alternative observable constraining the neutron skin. The recent high-resolution (p, p') measurement [27] of α_D yields the neutron skin thickness of ^{208}Pb to be $0.156^{+0.025}_{-0.021}$ fm, but the model dependence [28] in the correlation between R_{skin} and α_D assessed in systematic calculations in the framework of nuclear density functional theory is seen to shift the value of R_{skin} to 0.168 ± 0.022 fm.

In this Letter, we suggest a new method for determining the symmetry energy slope parameter L by exploiting the empirical information on the volume and surface symmetry energy coefficients, $C_v(\rho_0)$ and C_s. The symmetry coefficient of a finite nucleus $a_{\rm sym}(A)$ can be parametrized as

$$a_{\rm sym}(A) = C_v(\rho_0) - C_s A^{-1/3}. \tag{2}$$

These coefficients have recently been meticulously studied [29] by using the double differences of "experimental" symmetry energies. This has the advantage that other effects (such as pairing and shell effects) in symmetry energy can be well canceled out from the double differences for neighboring nuclei. The correlation between the double differences and the mass number of nuclei is found to be very compact, yielding values of $C_v(\rho_0)$ and C_s as 32.10 ± 0.31 and 58.91 ± 1.08 MeV, respectively. The uncertainties in these symmetry components are much smaller than those found earlier. We show below that these experimental values of C_v and C_s along with empirical information of the proton rms radius in a heavy nucleus yield the value of L within narrower limits; precision information on the neutron skin of nuclei also follows from the analysis.

We start with the ansatz

$$C_v(\rho) = C_v(\rho_0)\left(\frac{\rho}{\rho_0}\right)^{\gamma}, \tag{3}$$

where γ measures the density dependence of the symmetry energy. In a considerable density range around ρ_0, this ansatz is found to be very consistent with the density dependence obtained from the nuclear equation of state (EOS) with different interactions [8,9,30] and also from experiments in intermediate-energy heavy-ion collisions [10,11]. At very low densities, however, there are some small deviations. From Eqs. (1) and (3), $L = 3\gamma C_v(\rho_0)$ and the symmetry incompressibility $K_{\rm sym} = 9\rho_0^2 \frac{\partial^2 C_v(\rho)}{\partial\rho^2}|_{\rho_0} = 3L(\gamma - 1)$. One can expand the volume symmetry coefficient around ρ_0 as

$$C_v(\rho) = C_v(\rho_0)\left[1 + \sum_{n=1}^{\infty}\frac{1}{n!}\left(\frac{\rho-\rho_0}{\rho_0}\right)^n \prod_{k=0}^{n-1}(\gamma - k)\right]. \tag{4}$$

In the above expansion, keeping terms up to second order has been found to be a reliable approximation in our subsequent calculations. In terms of the symmetry energy slope parameter and symmetry incompressibility, $C_v(\rho)$ is then given as

$$C_v(\rho) = C_v(\rho_0) - L\epsilon + \frac{K_{\rm sym}}{2}\epsilon^2, \tag{5}$$

where $\epsilon = (\rho_0 - \rho)/(3\rho_0)$.

For a finite nucleus of mass number A, the symmetry coefficient $a_{\rm sym}(A)$ is always less than $C_v(\rho_0)$. The coefficient $a_{\rm sym}(A)$ can be equated to $C_v(\rho_A)$ where ρ_A is an equivalent density, always less than ρ_0. Using relations (2) to (5), we show below how ρ_A and γ (hence L and $K_{\rm sym}$) can be calculated. From Eqs. (2) and (5), it follows that

$$C_s A^{-1/3} = L\epsilon_A - \frac{K_{\rm sym}}{2}\epsilon_A^2, \tag{6}$$

where $\epsilon_A = (\rho_0 - \rho_A)/(3\rho_0)$. From Eq. (6),

$$C_s = 3C_v(\rho_0)A^{1/3}\left[\gamma\epsilon_A - \frac{3}{2}\gamma(\gamma-1)\epsilon_A^2\right]. \tag{7}$$

The value of C_s is an empirically determined constant (58.91 ± 1.08 MeV).

In the local density approximation, the symmetry coefficient $a_{\rm sym}(A)$ can be calculated [30] as

$$a_{\rm sym}(A) = \frac{1}{AX_0^2}\int d^3r \rho(r) C_v[\rho(r)][X(r)]^2, \tag{8}$$

where X_0 is the isospin asymmetry $[(N-Z)/A]$ of the nucleus, $\rho(r)$ is the sum of the neutron and proton densities inside the nucleus, and $X(r)$ is the local isospin asymmetry. The left-hand side of Eq. (8) is $C_v(\rho_A)$; hence, Eq. (8) can be rewritten as

$$C_v(\rho_0)\left(\frac{\rho_A}{\rho_0}\right)^{\gamma} = \frac{1}{AX_0^2}\int d^3r \rho(r) C_v(\rho_0)\left(\frac{\rho(r)}{\rho_0}\right)^{\gamma}[X(r)]^2. \tag{9}$$

Given the neutron-proton density profiles in the nucleus, from Eq. (9), a chosen value of γ gives ρ_A and hence ϵ_A. The one that satisfies Eq. (7) is the desired solution for γ. Once γ is known, the equivalent density ρ_A, the symmetry energy slope parameter, and symmetry incompressibility are determined. From Eq. (7), there are two solutions for γ and hence for ρ_A. Calculations show that one of the solutions is unphysical, as this gives $\rho_A \sim 5\rho_0$. The procedure so described is expected to work best for heavy nuclei where volume effects predominate over those coming from surface. The heavy spherical nucleus ^{208}Pb usually serves as a benchmark for extracting nuclear bulk properties, so we choose this nucleus for our calculation.

A priori knowledge of $C_v(\rho_0)$, C_s, ρ_0, and the proton and neutron density distributions in the nucleus is required to extract values of γ and ρ_A. In different parametrizations of the nuclear masses [31–35], $C_v(\rho_0)$ is $\sim$31 MeV, C_s is $\sim$55 MeV, and ρ_0 hovers around the canonical value $\sim$0.16 fm^{-3} in different nuclear EOS models. As stated earlier, we take the values $C_v(\rho_0) = 32.1 \pm 0.31$ MeV and $C_s = 58.91 \pm 1.08$ MeV. This value of $C_v(\rho_0)$ matches very well with 32.51 MeV, the one obtained from the latest mass systematics by Möller *et al.* [1]; from their quoted value of 28.54 MeV for the surface stiffness parameter Q, the value of C_s from ^{208}Pb also comes very close, 58.16 MeV. For saturation density, we fix $\rho_0 = 0.155 \pm 0.008$ fm^{-3}; this encompasses the saturation densities that come out from the EOSs of different Skyrme and RMF models. The point proton distribution is known from experiments, but the neutron density distribution is laced with much uncertainty.

From a recent covariance analysis [25], a lack of correlation of the neutron skin with some of the fundamental properties of nuclei like the isoscalar incompressibility, saturation density and the nucleon effective mass is suggested. The binding energy is also seen to be a poor isovector indicator. This is in consonance with the suggestion of effective nucleon-nucleon interactions of different genres, nonrelativistic (Skyrme) and relativistic (RMF), that give in the framework of microscopic mean-field theory different values of the neutron skin in ^{208}Pb [21,36] without compromising the basic nuclear properties mentioned earlier. Aided by the further information that the neutron skin calculated with an effective interaction is strongly correlated with the corresponding symmetry energy slope parameter L [15] and with empirical knowledge of C_v, C_s, ρ_0, and the proton density distribution, we now show how using Eqs. (7) and (9) for a heavy nucleus both L and $R_{\rm skin}$ can be calculated.

The parameters of the interactions BSR8-BSR14 [36], FSUGOLD [24], NL3 [37], and TM1 [38] have been used to generate the proton and neutron density profiles of ^{208}Pb in the RMF model. Along with many experimentally observed properties of finite nuclei and nuclear matter, these interactions reproduce the proton rms radius in ^{208}Pb ($R_p = \langle r_p^2\rangle^{1/2} = 5.451$ fm) extremely well. The neutron rms radii vary considerably though, as the calculated neutron skin varies from 0.17 to 0.28 fm. The symmetry energy slope parameters evaluated with these interactions using Eq. (1) are displayed in Fig. 1 as a function of the corresponding calculated neutron skin $R_{\rm skin}$ (blue filled triangles). The magnitude of L increases with $R_{\rm skin}$, and its functional dependence $L(R_{\rm skin})$ shows the usual linear correlation. These different interactions yield, using Eq. (1), different values of $C_v(\rho_0)$ (31–38 MeV). We also use Eqs. (7) and (9) to calculate γ and hence L by employing the microscopic densities for the protons and neutrons obtained within the RMF models using the above mentioned parameter sets. These values of L are depicted in the same figure by the shaded region, the spread at a particular $R_{\rm skin}$ arising from the uncertainties in the values of C_v, C_s, and ρ_0.

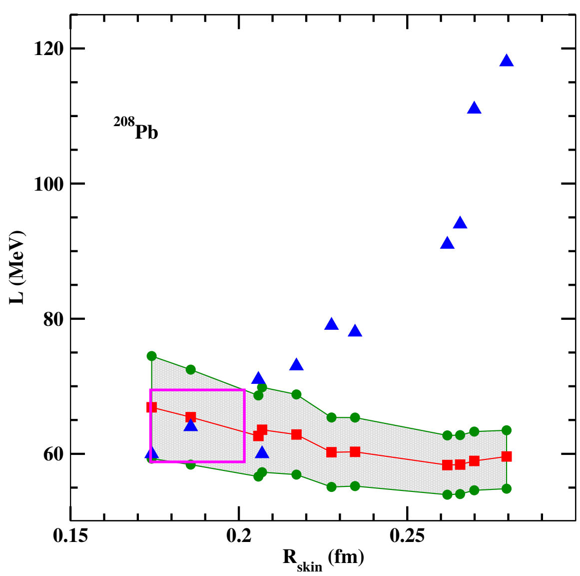

FIG. 1 (color online). Blue triangles representing L calculated using Eq. (1) with different RMF interactions. They are plotted as a function of corresponding $R_{\rm skin}$ for ^{208}Pb. The shaded region represents the envelope of possible L values with different RMF interactions obtained using Eqs. (7) and (9). The acceptable window for the values of L and $R_{\rm skin}$ for ^{208}Pb is represented by the magenta box.

The shaded region projects a strikingly different dependence of L on $R_{\rm skin}$. The filled red squares in the shaded area represent the median values for L, and the filled green circles represent its lower and upper bounds. The slope parameter L is seen to decrease here weakly with the neutron skin. This possibly originates from the fact that with given values of ρ_0, $C_v(\rho_0)$, and C_s, for a particular nucleus, $C_v(\rho_0)(\rho_A/\rho_0)^\gamma = a_{\rm sym}(A)$ [Eq. (2)] is a fixed quantity, and $(\rho_A)^\gamma$ is thus a constant for all the chosen interactions. Because ρ_A and γ have to satisfy Eq. (7), there is not much latitude in their values, resulting in the weak variation in L as shown. The intersection of the linear function $L(R_{\rm skin})$ with the shaded region projects out those values of neutron skin for ^{208}Pb that are commensurate with the given experimental windows for $C_v(\rho_0)$, C_s, and ρ_0. The corresponding calculated values of L are also accordingly projected out. The section depicting the bounds of L and the neutron skin of ^{208}Pb is shown by the box (magenta) in the figure. We find them to be $L = 64 \pm 5$ MeV and $R_{\rm skin}(^{208}\text{Pb}) = 0.188 \pm 0.014$ fm. Our scheme for finding L was found to be quite robust. This is tested by choosing Woods-Saxon density profiles, which are realistic but may not be very accurate. The proton density is adjusted to reproduce the experimental proton rms radius for ^{208}Pb, and the neutron density profiles are varied so that the entire range of 0.13 to 0.47 fm for $R_{\rm skin}$ as obtained from the PREX experiment is covered. Even in this large range of $R_{\rm skin}$, L is confined within 55 to 85 MeV; the median value (70 MeV) is not much different from the one obtained with microscopic densities. Knowledge of L helps in predicting neutron skins of nuclei. Estimates for the neutron skins for a few nuclei are displayed in Fig. 2; they are obtained from the intersection of the shaded region showing the calculated limits of $L = 64 \pm 5$ obtained from microscopic mean-field densities with the linear function $L(R_{\rm skin})$ calculated for those nuclei in the RMF model with different energy density functionals. The values of the neutron skins displayed for the nuclei ^{124}Sn, ^{90}Zr, and ^{48}Ca are seen to be 0.196 ± 0.014, 0.107 ± 0.007, and 0.182 ± 0.008 fm, respectively.

In summary, on the basis of empirical knowledge of the volume and surface symmetry coefficients and the nuclear saturation density, we have presented a model that yields

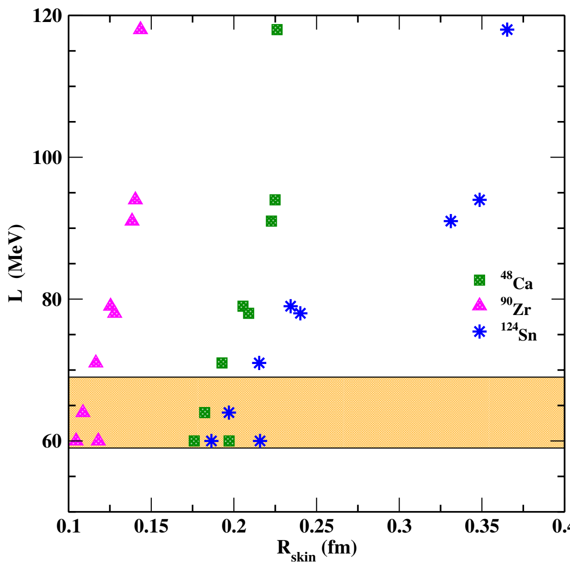

FIG. 2 (color online). Values of L shown as a function of $R_{\rm skin}$ for ^{124}Sn (blue stars), ^{90}Zr (magenta triangles), and ^{48}Ca (green squares) calculated with different RMF interactions. The shaded region corresponds to the projected-out range in the values of L as obtained from Fig. 1.

the density dependence of the symmetry energy in tighter bounds. This helps in making a precision prediction of the neutron skin thicknesses of different nuclei, including the currently experimentally studied nucleus ^{208}Pb, in PREX. We suggest that our determination on the limits on L and $R_{\rm skin}$ of experimentally studied nuclei be considered in properly evaluating nuclear energy density functionals.

J. N. D. acknowledges the support of DST, Government of India. The authors gratefully acknowledge the assistance of Tanuja Agrawal in the preparation of the manuscript.

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