Cosmic Magnetization: From Spontaneously Emitted Aperiodic Turbulent to Ordered Equipartition Fields

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It is shown that an unmagnetized nonrelativistic thermal electron-proton plasma spontaneously emits aperiodic turbulent magnetic field fluctuations of strength $|\delta B| = 3.5\beta_e g^{1/3} W_e^{1/2}$ G, where β_e is the normalized thermal electron temperature, W_e the thermal plasma energy density, and g the plasma parameter. For the unmagnetized intergalactic medium, immediately after the reionization onset, the field strengths from this mechanism are about 2×10^{-16} G in cosmic voids and 2×10^{-10} G in protogalaxies, both too weak to affect the dynamics of the plasma. Accounting for simultaneous viscous damping reduces these estimates to 2×10^{-21} G in cosmic voids and 2×10^{-12} G in protogalaxies. The shear and/or compression of the intergalactic and protogalactic medium exerted by the first supernova explosions locally amplify these seed fields and make them anisotropic, until the magnetic restoring forces affect the gas dynamics at ordered plasma betas near unity.

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The interstellar medium (ISM) is filled with (1) a dilute mixture of charged particles, atoms, molecules, and dust grains, referred to as interstellar gas and dust; (2) partially turbulent magnetic fields; (3) dilute photon radiation fields from stars, dust, and the universal microwave background radiation; and (4) cosmic ray particles with relativistic energies. It has been known for a long time [1–3], even before the discovery of the universal cosmic microwave background radiation, that these ISM components have comparable energy densities and pressures, each of the order of 10^{-12} erg cm^{-3}, commonly referred to as the equipartition condition in the ISM. Since today, this truly remarkable equipartition in the ISM has neither been understood nor explained theoretically. One refers to pressure partition if the ratio of any two individual pressures is a constant and to pressure equipartition if this ratio is near unity.

In other astrophysical objects, equipartition conditions and the closely related minimum-energy assumption for the total magnetic field energy density and the kinetic energy density of plasma particles are also often invoked for convenience [4] in order to analyze cosmic synchrotron intensities. The minimum-energy assumption was first proposed by Burbidge [5] and applied to the optical synchrotron emission of the jet in M87. Duric [6] argued that any major deviation from equipartition would be in conflict with radio observations of spiral galaxies. Observationally, for a variety of nonthermal sources, the equipartition concept is supported by magnetic field estimates as, e.g., in the Coma cluster of galaxies and radio-quiet active galactic nuclei [7]. Also, the solar wind plasma exhibits near equipartition conditions: 10 years of wind/SWE satellite data [8] near 1 AU showed that the proton and electron temperature anisotropies $A = T_\perp/T_\parallel$ are bounded by the mirror and fire-hose instabilities at large values of the parallel plasma beta $\beta_\parallel = 8\pi n k_B T_\parallel / B^2 \geq 1$ and by parallel propagating Alfvén waves [9] at small values of $\beta_\parallel < 1$, resulting in near magnetic field equipartition.

Because of their comparably low gas densities, all cosmic fully and partially ionized nonstellar plasmas are collision poor, as indicated by the very small values of the plasma parameter $g = \nu_{ee}/\omega_{p,e} \leq 10^{-10}$, given by the ratio of the electron-electron Coulomb collision frequency ν_{ee} to the electron plasma frequency $\omega_{p,e}$, characterizing interactions with electromagnetic fields, so that fully kinetic plasma descriptions are necessary. Because of the large sizes of astrophysical systems compared to the plasma Debye length, the fluctuations are described by real wave vectors ($\vec{k}$) and complex frequencies $\omega(\vec{k}) = \omega_R(\vec{k}) + \imath\gamma(\vec{k})$, implying for the space and time dependence of, e.g., magnetic fluctuations the superposition of $\delta\vec{B}(\vec{x}, t) \propto \exp[\imath(\vec{k} \cdot \vec{x} - \omega_R t) + \gamma t]$. One distinguishes between collective modes with a fixed frequency–wave-number dispersion relation, also referred to as normal modes, and noncollective (no frequency–wave-number relation) modes in the system. Regarding frequency, basically two fundamental types of fluctuations occur: (1) weakly amplified or damped solutions (e.g., Alfvén waves, electromagnetic waves) with $|\gamma| \ll \omega_R$ and (2) weakly propagating solutions (e.g., fire-hose and mirror fluctuations) with $\omega_R \ll \gamma$, including aperiodic solutions with $\omega_R = 0$ (e.g., Weibel fluctuations [10]). Aperiodic modes fluctuate only in space and do not propagate as $\omega_R = 0$ but permanently grow or decrease in time depending on the sign of γ. Past research [11] has concentrated predominantly on the fluctuations from collective weakly amplified modes in the plasma.

All plasmas, including unmagnetized plasmas and those in thermal equilibrium, have fluctuations so that their state variables such as density, pressure, and electromagnetic fields fluctuate in position and time. Unlike for weakly amplified or damped modes, however, for aperiodic fluctuations the expected fluctuation level has never been calculated quantitatively. Only recently have general expressions for the electromagnetic fluctuation spectra (electric and magnetic field, charge and current densities) from uncorrelated plasma particles in unmagnetized plasmas for arbitrary frequencies been derived [12] using the system of the Klimontovich and Maxwell equations, which are appropriate for fluctuation wavelengths longer than the mean distance between plasma particles, i.e., $k \leq k_{\max} = 2\pi n_e^{1/3}$. The electric [13] and magnetic field fluctuations in unmagnetized plasmas with the plasma frequency $\omega_{p,a}^2 = (4\pi e^2 n_a/m_a)^{1/2}$,

$$\begin{pmatrix} \langle \delta E_\parallel^2 \rangle_{k,\omega} \\ \langle \delta E_\perp^2 \rangle_{k,\omega} \\ \langle \delta B^2 \rangle_{k,\omega} \end{pmatrix} = \sum_a \frac{\omega_{p,a}^2 m_a}{4\pi^3 k^2} \begin{pmatrix} \frac{K_\parallel(k,\omega)}{|\omega \Lambda_L(\vec{k},\omega)|^2} \\ \frac{K_\perp(k,\omega)}{|\omega \Lambda_T(\vec{k},\omega)|^2} \\ \frac{c^2 k^2 K_\perp(k,\omega)}{|\omega^2 \Lambda_T(\vec{k},\omega)|^2} \end{pmatrix}, \tag{1}$$

are given in terms of the parallel and perpendicular form factors

$$\begin{pmatrix} K_\parallel(k,\omega) \\ K_\perp(k,\omega) \end{pmatrix} = k^2 \Re \int d^3p \frac{f_a(\vec{p})}{\gamma + \imath(\vec{k}\cdot\vec{v} - \omega_R)} \begin{pmatrix} v_\parallel^2 \\ v_\perp^2 \end{pmatrix} \tag{2}$$

and the general longitudinal and transverse dispersion functions $\Lambda_{L,T}(\vec{k},\omega)$ involving the respective parallel and perpendicular dielectric tensor elements. The form factors (2) are the generalizations of the standard expressions found in the literature in which the weak amplification limit of $\gamma \to 0^+$ is taken at the outset to approximate $\lim_{\gamma\to 0^+}(-\imath)[\gamma + \imath(\vec{k}\cdot\vec{v} - \omega_R)]^{-1} \to \pi\delta(\omega_R - \vec{k}\cdot\vec{v})$.

We now consider the unmagnetized intergalactic medium (IGM) immediately after the reionization onset, assuming that any earlier cosmological magnetization has vanished during the long recombination era with a fully neutral IGM. Modeling the photoionization by the first forming stars [14–16] indicates IGM temperatures of about $T_e = T_p = T = 10^4 T_4$ K at redshift $z = 4$ and ionized gas densities ranging from $n_e = 10^{-7} n_{-7}$ cm^{-3} in cosmic voids to $n_{-7} = 10^9$ (i.e., $n_e = 100$ cm^{-3}) in protogalaxies. For this isotropic thermal IGM proton-electron plasma, we follow recent work [12] to calculate from Eqs. (1) and (2) the energy in aperiodic ($\omega_R = 0$) magnetic fluctuations generated per unit volume at k and γ due to spontaneous emission as

$$U(k,\gamma) = \langle \delta B^2 \rangle_{k,\gamma} = \sum_a \frac{\omega_{p,a}^2 m_a u_a D(\frac{\gamma}{k u_a})}{4\pi^{5/2} k[\gamma^2 + c^2k^2][1 + \pi^{1/2}\sum_a \frac{\omega_{p,a}^2}{k^2c^2}\frac{|\gamma|}{k u_a} D(\frac{\gamma}{k u_a})]^2}, \tag{3}$$

with the thermal velocity $u_a = \sqrt{2k_B T_a/m_a}$ and $D(x) = e^{x^2}\mathrm{erfc}(|x|)$ denoting the complementary error function. The related collective Weibel mode [10] has a positive growth rate in anisotropic plasma distribution functions but is not excited in isotropic plasma distributions.

Integrating over all values of γ and k provides the energy density of spontaneously emitted fully random magnetic fluctuations

$$(\delta B)^2 = 4\pi \int_0^{k_{\max}} dk k^2 \langle \delta B^2 \rangle_k, \tag{4}$$

with

$$\langle \delta B^2 \rangle_k = 2\int_0^\infty d\gamma U(k,\gamma) = \frac{\omega_{p,e}^2 m_e \beta_e^2}{2\pi^{5/2}k^2} \times \int_0^\infty dx \frac{F(x,\mu)}{[1 + \beta_e^2 x^2][1 + \frac{\pi^{1/2}\omega_{p,e}^2}{k^2c^2}|x|F(x,\mu)]^2}, \tag{5}$$

where $F(x,\mu) = D(x) + \mu^{-1}D(x\mu)$, the mass ratio $\mu^2 = m_p/m_e = 1836$, and $\beta_e = u_e/c = 1.84 \times 10^{-3} T_4^{1/2}$. The factor 2 in Eq. (5) accounts for the proper analytical continuation for negative values of γ.

Because of the large value of $\mu = 43$, we can neglect the proton contribution, so that $F(x,\mu) \simeq D(x)$, implying in terms of the normalized wave vector $\kappa = kc/\omega_{p,e}$ that

$$\langle \delta B^2 \rangle_k = \frac{m_e c^2 \beta_e^2}{2\pi^{5/2}\kappa^2} J_0(\beta_e, \kappa), \tag{6}$$

where we define the integral

$$J_n(\beta,\kappa) = \int_0^\infty \frac{dx}{1 + \beta^2 x^2} \frac{x^n D(x)}{[1 + \frac{\pi^{1/2}}{\kappa^2} x D(x)]^2} \tag{7}$$

for $n = 0, 1$. The approximative analytical evaluation of the integral (8) makes use of the rational approximation [17] better than 2.5×10^{-5} given by $D(x) \simeq a_1 t - a_2 t^2 + a_3 t^3$ with $t = 1/(1 + px)$, $p = 0.47047$, $a_1 = 0.3480242$, $a_2 = 0.0958798$, and $a_3 = 0.7478556$. Given the smallness of a_2, we use as lower and upper limits $D_L(x) < D(x) < D_U(x)$ with

$$D_L(x) \simeq (a_0 + a_3 t^2)t, \qquad D_U(x) \simeq (a_1 + a_4 t^2)t, \tag{8}$$

where $a_0 = a_1 - a_2 = 0.2521444$ and $a_4 = a_3 - a_2 = 0.6519758$. The integral (8) then is well approximated by $J_n^L(\beta,\kappa) < J_n(\beta,\kappa) < J_n^U(\beta,\kappa)$, with

$$J_n^{U,L}(\beta,\kappa) = \int_0^\infty dx \frac{x^n D_{U,L}(x)}{[1+\beta^2x^2][1+\frac{\pi^{1/2}}{\kappa^2}xD_{L,U}(x)]^2}. \quad (9)$$

After straightforward but tedious algebra, we derive

$$J_0^{L,U} \simeq \frac{a_{0,1}}{p}\frac{Y_{L,U}+\ln(1+Y_{L,U})+\ln\frac{pe^{1/2}}{\beta}}{(1+Y_{L,U})^2} \simeq \frac{a_{0,1}}{p}\begin{cases}\ln(pe^{1/2}/\beta) & \text{for } Y_{L,U}\ll 1\\ \frac{Y_{L,U}+\ln(pY_{L,U}/\beta)}{Y_{L,U}^2} & \text{for } Y_{L,U}\gg 1\end{cases} \quad (10)$$

and

$$J_1^{L,U} \simeq \frac{a_{0,1}}{p^2(1+Y_{L,U})^2}\left[\frac{2p}{\beta}+\frac{Y_{L,U}-1}{Y_{L,U}+1}\ln\frac{p(1+Y_{L,U})}{\beta}\right] \simeq \frac{a_{0,1}}{p^2}\begin{cases}\frac{2p}{\beta} & \text{for } Y_{L,U}\ll 1\\ \frac{(2p/\beta)+\ln(pY_{L,U}/\beta)}{Y_{L,U}^2} & \text{for } Y_{L,U}\gg 1,\end{cases} \quad (11)$$

with $Y_{L,U}(\kappa) = \pi^{1/2}a_{1,0}/[p\kappa^2]$. The asymptotic expansions for small and large values of $Y_{L,U}$ correspond to large and small values of the normalized wave number, respectively.

According to Eq. (6), the wave number power spectra $k^2\langle\delta B^2\rangle_k^{L,U} \propto J_0^{U,L}(\beta,\kappa)$ to leading order increase $\propto \kappa^2$ at small normalized wavelength $\kappa \le (a_{1,0}\pi^{1/2}/p)^{1/2}$ and approach constants at large κ. The constants at large values of κ provide the dominating contribution to the remaining κ integral in Eq. (4). We find $\delta B_U^2 = 1.38\delta B_L^2$ with

$$\delta B_L^2 = \frac{4a_0}{\pi^{1/2}p}\ln\left(\frac{pe^{1/2}}{\beta_e}\right)m_ec^2\beta_e^2n_e^{1/3}\left(\frac{\omega_{p,e}}{c}\right)^2 = 4\pi\beta_e^2g^{2/3}W_e = 3.5\times10^{-32}T_4n_{-7}^{4/3}\ \mathrm{erg\,cm^{-3}}, \quad (12)$$

with the thermal energy density $W_e = 3n_ek_BT/2 = 2.1\times10^{-19}n_{-7}T_4\ \mathrm{erg\,cm^{-3}}$ and the plasma parameter $g = 2.3\times10^{-13}n_{-7}^{1/2}T_4^{-3/2}$. This magnetic field energy density corresponds to a minimum total fluctuating magnetic field strength of

$$|\delta B|_L = 3.5\beta_eg^{1/3}W_e^{1/2} = 1.8\times10^{-16}T_4^{1/2}n_{-7}^{2/3}\ \mathrm{G}, \quad (13)$$

providing $|\delta B| \simeq 2\times10^{-16}T_4^{1/2}$ G in cosmic voids and $|\delta B|\simeq 2\times10^{-10}T_4^{1/2}$ G in protogalaxies. These guaranteed magnetic fields in the form of randomly distributed fluctuations, produced by the spontaneous emission of the isotropic thermal IGM plasma, may serve as seed fields for possible amplification by later possible plasma instabilities from anisotropic plasma particle distribution functions, magnetohydrodynamic instabilities, and/or the magnetohydrodynamic dynamo process. Neither the dynamo process nor plasma instabilities generate magnetic fields without such seed fields. We also note that the strength of the guaranteed spontaneously emitted magnetic seed fields (15) is significantly larger than the seed fields from the Biermann battery process [18,19] (10^{-18} G) and cosmological phase transitions [20] (10^{-20} G). These spontaneously emitted fluctuations have typical plasma scale lengths $\le 10^{10}n_{-7}^{-1/2}$ cm, but, as argued below, the first hydrodynamical compression generates considerably longer correlation lengths of the compressed magnetic fields determined by the spatial scale of the compressor.

While generated continuously by spontaneous emission, the turbulent magnetic field also experiences strong dissipation at small scales by viscous damping from collisional processes with the damping rate [21] $\Gamma_v(k) = 0.06\beta_e^2k^2c^2\tau_C/\mu^2 = 0.18\beta_e^2k^2c^2/(\omega_{p,e}g\mu^2)$ Hz with the Coulomb collision time $\tau_C = 3(\omega_{p,e}g)^{-1} = 7.3\times10^{11}T_4^{3/2}n_{-7}^{-1}$ s. The equilibrium magnetic field fluctuation spectrum from continuous spontaneous emission and viscous damping is given by $\langle\delta B^2\rangle_{\mathrm{eq}}(k) = P(k)/\Gamma_v(k)$, where

$$P(k) = 2\int_0^\infty d\gamma\gamma U(k,\gamma) = \frac{\omega_{p,e}m_ec^2\beta_e^3}{2\pi^{5/2}\kappa}J_1(\beta_e,\kappa) \quad (14)$$

is the magnetic field power radiated per unit volume at k due to spontaneous emission and J_1 is the integral (7) for $n = 1$. Consequently, the equilibrium magnetic field fluctuation power spectrum is given by

$$\langle\delta B^2\rangle_{\mathrm{eq}}(k) = \frac{\mu^2m_ec^2g\beta_eJ_1(\beta_e,\kappa)}{0.36\pi^{5/2}\kappa^3}, \quad (15)$$

so that in this case

$$(\delta B)_{\mathrm{eq}}^2 = 2m_pc^2\beta_eg\left(\frac{\omega_{p,e}}{c}\right)^3\int_0^{2\pi cn_e^{1/3}/\omega_{p,e}} d\kappa\frac{J_1(\beta_e,\kappa)}{\kappa}. \quad (16)$$

With the asymptotics (11), we find for the maximum and minimum equilibrium magnetic field strength $|\delta B|_{\mathrm{eq},U} = 1.18|\delta B|_{\mathrm{eq},L}$ and

$$|\delta B|_{\mathrm{eq},L} = 1.46\times10^{-10}g^{1/2}n_e^{3/4}\left[15.1-\frac{\ln n_e}{6}\right]^{1/2} = 1.7\times10^{-21}n_{-7}T_4^{-3/4}\ \mathrm{G}, \quad (17)$$

providing $|\delta B|\simeq 2\times10^{-21}T_4^{-3/4}$ G in cosmic voids and $|\delta B|\simeq 2\times10^{-12}T_4^{-3/4}$ G in protogalaxies. Accounting for the additional viscous damping reduces the equilibrium magnetic field strength by about 5 and 2 orders of magnitude in cosmic voids and protogalaxies, respectively, as compared to the estimate (13).

In cosmic voids and protogalaxies, the spontaneously emitted seed magnetic fields are too weak to affect the dynamics of the IGM plasma, as the small values of the associated turbulent plasma beta $\beta_t \ge 10^{13}$ in cosmic voids and $\beta_t \ge 10^{10}$ in protogalaxies indicate. Because of these ultrahigh turbulent plasma beta values, the seed fields are tied passively to the highly conducting IGM plasma as frozen-in magnetic fluxes.

Now, we finally address how ordered magnetic field structures emerge from these randomly distributed magnetic fields. As we demonstrated, the unmagnetized, isotropic, thermal, and steady IGM plasma byaa spontaneous emission generates steady tangled fields, isotropically distributed in direction, on small spatial scales $\leq 10^{10} n_{-7}^{-1/2}$ cm (corresponding to $\kappa \geq 1$), which are passively tied to the highly conducting IGM plasma. Earlier analytical considerations and numerical simulations [22–26] showed that any shear and/or compression of the IGM medium not only amplifies these seed magnetic fields but also makes them anisotropic. Considering a cube containing an initially isotropic magnetic field, which is compressed to a factor $\eta \ll 1$ times its original length along one axis, these authors showed that the perpendicular magnetic field components are enhanced by the factor η^{-1}. Depending on the specific exerted compression and/or shear, even one-dimensional ordered magnetic field structures can be generated out of the original isotropically tangled field configuration [26]. Hydrodynamical compression or shearing of the IGM medium arises from the shock waves of the supernova explosions of the first stars at the ends of their lifetimes or from supersonic stellar and galactic winds. The IGM seed magnetic field upstream of these shocks is random in direction, and, by solving the hydrodynamical shock structure equations for oblique and conical shocks, it has been demonstrated [27,28] that the shock compression enhances the downstream magnetic field component parallel to the shock but leaves the magnetic field component normal to the shock unaltered. Consequently, a more ordered downstream magnetic field structure results from the randomly oriented upstream field. Such stretching and ordering of initially turbulent magnetic fields is also seen in the numerical hydrodynamical simulations of supersonic jets in radio galaxies and quasars [25]. Obviously, this magnetic field stretching and ordering occurs only in IGM regions overrun frequently by shock or wind. Each individual shock or wind (with speed V_s) compression orders the field on spatial scales R on time scales given by the short shock crossing time R/V_s, but significant amplification requires multiple compressions. The magnetic field filling factor is determined by the filling factors of the shock and wind, which are large (80 percent) in the coronal phase of interstellar media [29] and near shock waves in large-scale IGM structures [30]. In IGM regions with high shock or wind activity, this passive hydrodynamical amplification and stretching of magnetic fields continues until the magnetic restoring forces affect the gas dynamics, i.e., at ordered plasma betas near unity. As a consequence, magnetic fields with equipartition strength are not generated uniformly over the whole IGM by this process but only in localized IGM regions with high shock or wind activity. However, these compressions lead to ordered magnetic fields that will affect the motions of the IGM protons and electrons. Then, our original expressions for the spontaneously emitted fluctuation spectra (1) in unmagnetized plasmas no longer apply and have to be replaced by the fluctuation spectra in magnetized plasmas.

In protogalaxies, significant and rapid amplification of the spontaneously emitted aperiodic turbulent magnetic fields results from the small-scale kinetic dynamo process [31,32] generated by the gravitational infall motions during the formation of the first stars [19,33,34]. Additional gaseous spiral motion may stretch and order the magnetic field on large protogalactic spatial scales.

In principle, our suggested mechanism of spontaneously emitted aperiodic turbulent magnetic fields should also operate during earlier cosmological epochs before recombination. However, the nonrelativistic fluctuation spectra apply only for electron temperatures well below 10^9 K, corresponding to redshifts $z < 10^5$. It needs to be explored which traces on the cosmic microwave background that the magnetic field present at the moment of recombination ($z \simeq 10^3$) will leave.

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[1] L. Biermann and L. Davis, Z. Astrophys. **51**, 19 (1960).
[2] E. N. Parker, Astrophys. J. **145**, 811 (1966).
[3] E. N. Parker, in *Nebulae and Interstellar Matter*, edited by B. M. Middlehurst and L. H. Aller (University of Chicago Press, Chicago, 1968), p. 707.
[4] M. S. Longair, *Particles, Photons and their Detection*, High Energy Astrophysics Vol. 1 (Cambridge University Press, Cambridge, England, 1992).
[5] G. R. Burbidge, Astrophys. J. **124**, 416 (1956).
[6] N. Duric, *Galactic and Intergalactic Magnetic Fields: Proceedings of the 140th Symposium of the International Astronomical Union, Heidelberg, 1989*, edited by R. Beck *et al.* (Kluwer, Dordrecht, 1990), p. 235.
[7] R. Schlickeiser, A. Sievers, and H. Thiemann, Astron. Astrophys. **182**, 21 (1987); R. Schlickeiser, P. L. Biermann, and A. Crusius-Wätzel, *ibid.* **247**, 283 (1991).
[8] B. A. Maruca, J. C. Kasper, and S. D. Bale, Phys. Rev. Lett. **107**, 201101 (2011).
[9] R. Schlickeiser, M. J. Michno, D. Ibscher, M. Lazar, and T. Skoda, Phys. Rev. Lett. **107**, 201102 (2011).
[10] E. Weibel, Phys. Rev. Lett. **2**, 83 (1959).
[11] E. E. Salpeter, Phys. Rev. **120**, 1528 (1960); A. G. Sitenko, *Electromagnetic Fluctuations in Plasma* (Academic, New York, 1967); S. Ichimaru, *Basic Principles of Plasma Physics* (Benjamin, Reading, MA, 1973); W. Kegel, *Plasmaphysik* (Springer, Berlin, 1998).
[12] R. Schlickeiser and P. H. Yoon, Phys. Plasmas **19**, 022105 (2012).
[13] We corrected a typo in the expression for $\langle \delta E_{\parallel}^2 \rangle_{k,\omega}$.
[14] L. Hui and N. Y. Gnedin, Mon. Not. R. Astron. Soc. **292**, 27 (1997).

[15] L. Hui and Z. Haiman, Astrophys. J. **596**, 9 (2003).
[16] M. Stiavelli, *From First Light to Reionization: The End of the Dark Ages* (Wiley, New York, 2009).
[17] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (NBS, Washington, DC, 1972).
[18] L. Biermann, Z. Naturforsch. A **5**, 65 (1950).
[19] H. Xu, B. W. O'Shea, D. C. Collins, M. L. Norman, H. Li, and S. Li, Astrophys. J. **688**, L57 (2008).
[20] G. Sigl, A. V. Olinto, and K. Jedamzik, Phys. Rev. D **55**, 4582 (1997).
[21] S. I. Braginskii, in *Reviews of Plasma Physics*, edited by M. A. Leontovich (Consultants Bureau, New York, 1965), Vol. 1, p. 205.
[22] J. A. Högbom, Astron. Astrophys. Suppl. Ser. **36**, 173 (1979).
[23] R. A. Laing, Mon. Not. R. Astron. Soc. **193**, 439 (1980).
[24] P. A. Hughes, H. D. Aller, and M. F. Aller, Astrophys. J. **298**, 301 (1985).
[25] A. P. Matthews and P. A. G. Scheuer, Mon. Not. R. Astron. Soc. **242**, 616 (1990); **242**, 623 (1990).
[26] R. A. Laing, Mon. Not. R. Astron. Soc. **329**, 417 (2002).
[27] T. V. Cawthorne and W. K. Comb, Astrophys. J. **350**, 536 (1990).
[28] T. V. Cawthorne, Mon. Not. R. Astron. Soc. **367**, 851 (2006).
[29] C. McKee and J. P. Ostriker, Astrophys. J. **218**, 148 (1977).
[30] F. Miniati, D. Ryu, H. Kang, T. W. Jones, R. Cen, and J. P. Ostriker, Astrophys. J. **542**, 608 (2000).
[31] A. Brandenburg and K. Subramanian, Phys. Rep. **417**, 1 (2005).
[32] A. A. Shekochinin, S. A. Boldyrev, and R. M. Kulsrud, Astrophys. J. **567**, 828 (2002).
[33] D. R. G. Schleicher, R. Banerjee, S. Sur, T. G. Arshakian, R. S. Kleesen, R. Beck, and M. Spaans, Astron. Astrophys. **522**, A115 (2010).
[34] J. Schober, D. Schleicher, C. Federrath, S. Glover, R. S. Klessen, and R. Banerjee, Astrophys. J. **754**, 99 (2012).