Time Evolution and Dynamical Phase Transitions at a Critical Time in a System of One-Dimensional Bosons after a Quantum Quench

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A renormalization group approach is used to show that a one-dimensional system of bosons subject to a lattice quench exhibits a finite-time dynamical phase transition where an order parameter within a light cone increases as a nonanalytic function of time after a critical time. Such a transition is also found for a simultaneous lattice and interaction quench where the effective scaling dimension of the lattice becomes time dependent, crucially affecting the time evolution of the system. Explicit results are presented for the time evolution of the boson interaction parameter and the order parameter for the dynamical transition as well as for more general quenches.

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A fundamental and challenging topic of research is to understand nonequilibrium strongly correlated systems in general, and how phase transitions occur in such systems in particular. While the theory of equilibrium phase transitions is well developed and relies heavily on the renormalization group, the development of an equally powerful approach to studying nonequilibrium phase transitions is still in its infancy. Moreover, in any progress on this topic, it has always appeared that nonequilibrium phase transitions have one aspect in common with their equilibrium counterparts: Both occur by adiabatically tuning some parameter of the system, in the absence or presence of an external drive, and strictly speaking occur only in the limit of infinite time (steady state) [1–8].

In contrast, here we study a completely different kind of a nonequilibrium phase transition, one that occurs as a function of time. Employing a time-dependent renormalization group (RG) approach, we study the quench dynamics of interacting one-dimensional (1D) bosons in a commensurate lattice. This system in equilibrium shows the Berezinskii-Kosterlitz-Thouless (BKT) transition separating a Mott insulating phase from a superfluid phase (Fig. 1) [9]. For the nonequilibrium situation, we explicitly show the appearance of a dynamical phase transition where an order parameter grows as a nonanalytic function of time after a critical time. Such a behavior has no analog in equilibrium systems.

A dynamical transition in time was recently identified in the exactly solvable transverse-field Ising model where the Loschmidt echo was found to show nonanalytic behavior at a critical time, whereas the behavior of the order parameter was analytic [10]. In contrast, here we identify a situation where the order parameter itself can show nonanalyticities as a function of time. In addition, we generalize the study of dynamical transitions to models that are not exactly solvable and to low dimensions where strong fluctuations negate a mean-field analysis [11].

We identify the dynamical transition by studying an order parameter $\Delta(r, T_m)$ that due to the quench depends both on position r and a time T_m after the quench. The phase transition is associated with a nonanalytic behavior as a function of time T_m on the value of this order parameter spatially averaged within a light cone. Our results hold relevance not only to experiments in cold-atomic gases where system parameters can be tuned rapidly in time [12], but also to conventional solid-state materials where the time evolution of an order parameter may be probed with high precision using ultrafast pump-probe [13] and angle-resolved photoemission spectroscopy [14].

We model the 1D Bose gas as a Luttinger liquid, [9]

$$H_i = \frac{u_0}{2\pi}\int dx\left\{K_0[\pi\Pi(x)]^2 + \frac{1}{K_0}[\partial_x\phi(x)]^2\right\}, \quad (1)$$

where $-\partial_x\phi/\pi$ represents the density of the Bose gas, Π is the variable canonically conjugate to ϕ, K_0 is the dimensionless interaction parameter, and u_0 is the velocity of the sound modes. We assume that the bosons are initially in the ground state of H_i. The system is driven out of equilibrium via an interaction quench at $t = 0$ from $K_0 \to K$, with a commensurate lattice V_{sg} also switched on suddenly,

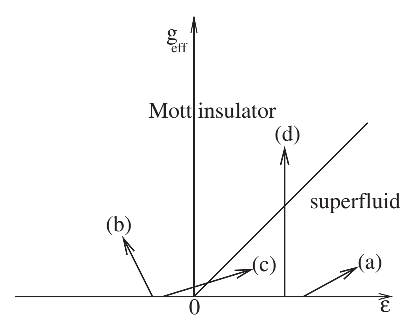

FIG. 1. The equilibrium BKT phase diagram. Arrows connect the Hamiltonians before (H_i) and after (H_f) the quench. A dynamical phase transition is found for case (d).

at the same time as the quench. This triggers a nontrivial time evolution from $t > 0$ due to a Hamiltonian $H_f = H_{f0} + V_{\rm sg}$, where $H_{f0} = H_i(K_0 \to K)$ and $V_{\rm sg} = -\frac{gu}{\alpha^2}\int dx \cos(\gamma\phi)$, with $g > 0$, and $\Lambda = \frac{u}{\alpha}$, a short-distance cutoff. We assume that the quench preserves Galilean invariance so that $uK = u_0K_0$; however, this is not critical for either the approach or the result. While $\gamma = 2$ for bosons, we keep it general so that the results may be generalized to other 1D systems.

In the absence of the lattice, the system is exactly diagonalizable in terms of the density modes $H_i = \sum_{p\neq 0} u_0|p|\eta_p^\dagger\eta_p$ and $H_{f0} = \sum_{p\neq 0} u|p|\beta_p^\dagger\beta_p$, where β and η are related by a canonical transformation. This fact has been used to study the dynamics of a Luttinger liquid exactly and has revealed interesting physics arising from a lack of thermalization in the system [15–18]. To study the system in the presence of the lattice employing a RG, we write the Keldysh action representing the time evolution from the initial pure state $|\phi_i\rangle$ (hence an initial density matrix $\rho = |\phi_i\rangle\langle\phi_i|$) corresponding to the ground state of H_i, $Z_K = {\rm Tr}[\rho(t)] = {\rm Tr}[e^{-iH_ft}|\phi_i\rangle\langle\phi_i|e^{iH_ft}] = \int \mathcal{D}[\phi_{cl},\phi_q]e^{i(S_0+S_{\rm sg})}$. S_0 is the quadratic part that describes the nonequilibrium Luttinger liquid, which at a time t after the quench is [19]

$$S_0 = \int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_2 \int_0^t dt_1 \int_0^t dt_2 (\,\phi_{cl}(1) \quad \phi_q(1)\,) \times \begin{pmatrix} 0 & G_A^{-1}(1,2) \\ G_R^{-1}(1,2) & -[G_R^{-1}G_KG_A^{-1}](1,2) \end{pmatrix} \begin{pmatrix} \phi_{cl}(2) \\ \phi_q(2) \end{pmatrix}, \tag{2}$$

where $1(2) = (x_{1(2)}, t_{1(2)})$ and $\phi_{cl,q} = \frac{\phi_- \pm \phi_+}{\sqrt{2}}$, with the minus (plus) sign representing a field that is time (antitime) ordered on the Keldysh contour [20]. The lattice potential is given by $S_{\rm sg} = \frac{gu}{\alpha^2}\int_{-\infty}^{\infty} dx_1 \int_0^t dt_1[\cos\{\gamma\phi_-(1)\} - \cos\{\gamma\phi_+(1)\}]$.

We define an order parameter $\Delta_m = \langle e^{im\gamma\phi_{cl}(x,t)}\rangle$ such that in equilibrium $\Delta_{m<1}$ is zero in the gapless phase and nonzero in the gapped phase and such that Δ_1, while always nonzero, is a nonanalytic function of g. We will show that after a quench Δ_1 can be a nonanalytic function of time. In order to understand the framework of the RG, let us study the two-point correlation function $C_{ab}(1,2) = \langle e^{i\gamma\phi_a(1)}e^{-i\gamma\phi_b(2)}\rangle$ (a, $b = \pm$) for $g = 0$ but for the nonequilibrium Luttinger liquid ($K_0 \neq K$). Denoting $1(2) = R + (-)\frac{r}{2}$, $T_m + (-)\frac{\tau}{2}$, C_{ab} depends both on the time difference τ and on the mean time T_m after the quench and is translationally invariant in space [19]. C_{ab} depends on three exponents: $K_{\rm eq} = \frac{\gamma^2 K}{4}$, $K_{\rm neq} = \frac{\gamma^2}{8}K_0(1 + \frac{K^2}{K_0^2})$, $K_{\rm tr} = \frac{\gamma^2}{8}K_0(1 - \frac{K^2}{K_0^2})$. Consider C_{ab} at equal time ($\tau = 0$) and at unequal positions. Then, for short times $T_m\Lambda \ll 1$ after the quench but long distances $r \gg u/\Lambda$, C_{ab} decays in position is a power law with the exponent $K_{\rm neq} + K_{\rm tr} = \gamma^2K_0/4$ (i.e., $C_{ab} \sim r^{-\gamma^2K_0/4}$). Hence the short time behavior is determined primarily by the initial wave function. In contrast, at long times, $T_m\Lambda \gg 1$, C_{ab} decays as a power law but with a new exponent $K_{\rm neq}$ (i.e., $C_{ab} \sim r^{-K_{\rm neq}}$). $K_{\rm tr}$ governs the transient behavior connecting these limits. Further, at long times after the quench $T_m\Lambda \gg 1$, C_{ab} also becomes translationally invariant in time (and hence independent of T_m).

The RG in equilibrium sums the leading logarithms. We use the same philosophy to employ a RG to study dynamics. In particular, at short times (but long distances), the RG will resum the logarithms $\frac{\gamma^2K_0}{4}\ln|r|$, whereas, at long times ($T_m\Lambda \gg 1$), it will resum the logarithms $K_{\rm neq}\ln\sqrt{(r\pm\tau)^2}$. Our approach generalizes the use of a RG to study quench dynamics near classical critical points [21,22] to quantum systems.

Derivation of RG equations.—We split the field $\phi_{0,\Lambda}$ into slow ($\phi^<_{0,\Lambda-d\Lambda}$) and fast ($\phi^>_{\Lambda-d\Lambda,\Lambda}$) components in momentum space $\phi_\pm = \phi^<_\pm + \phi^>_\pm$ and integrate out the fast fields perturbatively in g. Following this, we rescale the cutoff back to its original value and rescale position and time R, $T_m \to \frac{\Lambda}{\Lambda'}(R, T_m)$, where $\Lambda' = \Lambda - d\Lambda$. Following this, we write the action as $S = S_0^< + S_{\rm sg}^< + \delta S_0^< + \delta S_{T_{\rm eff}}^< + \delta S_\eta^<$, where $S_0^<$ is simply the quadratic action corresponding to H_{0f} with the rescaled variables, $S_{\rm sg}^<$ is the rescaled action due to the lattice, and $\delta S^<_{0,T_{\rm eff},\eta}$ are corrections to $\mathcal{O}(g^2)$:

$$S_{\rm sg}^< = g\left(\frac{\Lambda}{\Lambda'}\right)^2 \int_{-\infty}^{\infty} dR \int_0^{t\Lambda} dT_m[\cos\gamma\phi_-^<(R,T_m) - \cos\gamma\phi_+^<(R,T_m)]e^{-(\gamma^2/4)\langle[\phi_{cl}^>(T_m)]^2\rangle}, \tag{3}$$

$$\delta S_0^< = \frac{g^2\gamma^2}{2}\frac{d\Lambda}{\Lambda}\int_{-\infty}^{\infty} dR \int_0^{t\Lambda/\sqrt{2}} dT_m[-I_R(T_m)(\partial_R\phi_{cl}^<) \times (\partial_R\phi_q^<) - I_{ti}(T_m)(\partial_{T_m}\phi_{cl}^<)(\partial_{T_m}\phi_q^<)], \tag{4}$$

$$\delta S_{T_{\rm eff}}^< = \frac{ig^2\gamma^2}{2}\frac{d\Lambda}{\Lambda}\int_{-\infty}^{\infty} dR \int_0^{t\Lambda/\sqrt{2}} dT_m(\phi_q^<)^2 I_{T_{\rm eff}}(T_m), \tag{5}$$

$$\delta S_\eta^< = -\frac{g^2\gamma^2}{2}\frac{d\Lambda}{\Lambda}\int_{-\infty}^{\infty} dR \int_0^{t\Lambda/\sqrt{2}} dT_m\phi_q^<[\partial_{T_m}\phi_{cl}^<]I_\eta(T_m). \tag{6}$$

Equation (3) shows that the scaling dimension of the lattice depends on $\langle[\phi_{cl}^>(T_m)]^2\rangle$ and in particular is time dependent. To leading order ($g = 0$), $\frac{\gamma^2}{4}\langle[\phi_{cl}^>(T_m)]^2\rangle = \frac{d\Lambda}{\Lambda}[K_{\rm neq} + \frac{K_{\rm tr}}{1+(2T_m\Lambda)^2}]$. $\delta S_0^<$ shows that the quadratic part of the action acquires corrections that are also time dependent [19]. $\delta S^<_{\eta,T_{\rm eff}}$ indicates the generation of new terms, such as a time-dependent dissipation (η) and a noise ($\eta T_{\rm eff}$) whose physical meaning is the generation of inelastic scattering processes that will eventually relax the distribution function [5,6]. These time-dependent corrections lead to RG equations that depend on time T_m after the quench. Defining $\frac{d\Lambda}{\Lambda} = \frac{\Lambda - \Lambda'}{\Lambda} = d\ln(l)$, $I_{u,K} = I_R \pm I_{ti}$ [19] and dimensionless variables $T_m \to T_m\Lambda$, $\eta \to \eta/\Lambda$, $T_{\rm eff} \to T_{\rm eff}/\Lambda$, the RG equations are

$$\frac{dg}{d\ln l} = g\left[2 - \left(K_{\rm neq} + \frac{K_{\rm tr}}{1+4T_m^2}\right)\right], \tag{7}$$

$$\frac{dK^{-1}}{d\ln l} = \frac{\pi g^2\gamma^2}{8} I_K(T_m), \tag{8}$$

$$\frac{dT_m}{d\ln l} = -T_m, \tag{9}$$

$$\frac{1}{Ku}\frac{du}{d\ln l} = \frac{\pi g^2\gamma^2}{8} I_u(T_m), \tag{10}$$

$$\frac{d\eta}{d\ln l} = \eta + \frac{\pi g^2\gamma^2 K}{4} I_\eta(T_m), \tag{11}$$

$$\frac{d(\eta T_{\rm eff})}{d\ln l} = 2\eta T_{\rm eff} + \frac{\pi g^2\gamma^2 K}{8} I_{T_{\rm eff}}(T_m). \tag{12}$$

Note that T_m not only acts as an inverse cutoff in that modes of momenta $\Lambda < 1/T_m$ dominate the physics at a time T_m [23,24], but that it also governs the crossover from a short time behavior where the physics is determined primarily by the initial state and a long time behavior characterized by a new nonequilibrium fixed point. This crossover is most easily seen from Eq. (7), where the scaling dimension of the lattice $\epsilon(T_m) = [K_{\rm neq} + \frac{K_{\rm tr}}{1+4T_m^2} - 2]$ depends on time as follows: At short times ($T_m \ll 1$), it is $(-2 + \frac{\gamma^2 K_0}{4})$ and hence depends on the initial wave function, while, at long times ($T_m \gg 1$), a nonequilibrium scaling dimension $(-2 + K_{\rm neq})$ emerges.

Above, $I_{K,u,\eta,T_{\rm eff}}$ reach steady-state values at $T_m \gg 1$, whereas for short times, they vanish as $T_m \to 0$, as expected since the effect of the lattice potential vanishes at $T_m = 0$. For example, at short times, $I_K \sim \mathcal{O}(T_m^2)$ [19,25]. Equations (8) and (10) represent the renormalization of the interaction parameter and the velocity. The effects of the latter being small will be neglected, and in what follows we set $u = 1$. Equations (11) and (12) show the generation of dissipation and noise that represent inelastic scattering between bosonic modes [5,6].

In what follows, we do an analysis for a time $T_m < 1/\eta$, where $1/\eta$ is the time in which the distribution function first begins to change due to inelastic scattering. A perturbative calculation [5,6] shows that for small quenches ($|K_0 - K| \to 0$) and at steady state, $\eta \sim g^2(K_0 - K)^4$. Since $\eta \ll 1$, one may easily be in the regime of $T_m \gg 1$, but $T_m\eta \ll 1$, so that inelastic scattering may be neglected. At these times and in what follows, we will only use Eqs. (7)–(9).

The behavior of the system is very different depending upon K_0, K, and g. We discuss four cases (see Fig. 1). Case (a) is when the periodic potential is irrelevant at all times after the quench, case (b) is when the periodic potential is always relevant, case (c) is when the periodic potential is relevant at short times and irrelevant at long times, and case (d) is when the periodic potential is irrelevant at short times and relevant at long times. For case (d), we show that an order parameter behaves in a discontinuous way in time. There is a critical time T_m^* after which the order parameter begins to increase as a nonanalytic function of time, indicating a dynamical phase transition. In contrast, for case (c), the behavior of the order parameter is analytic in time.

We use ϵ_0, g_0, T_{m0}, and Λ_0 to denote bare physical values. From Eq. (8), we define an effective interaction $g_{\rm eff}(T_m) = g\sqrt{\frac{\pi\gamma^2}{8}}\sqrt{I_K(T_m)}\frac{\gamma K}{2}\sqrt{\frac{K}{K_0}}$, where $g_{\rm eff}$ goes to zero as $T_m \to 0$ and reaches a steady-state value for $T_m \gg 1$. Physically, this implies that at short times the particles have not had sufficient time to interact; therefore, however large g may be, any renormalization effects due to interactions are vanishingly small. The time dependence of $g_{\rm eff}(T_m)$ and $\epsilon(T_m)$ will be important for the results.

Case (a), in which the periodic potential is always irrelevant.—This case occurs for $\epsilon(T_m) > 0$ and a $g_{\rm eff}$ that is not too large [a condition to be made more precise when discussing case (d)]. Here, the periodic potential renormalizes to zero and one recovers a gapless theory that eventually looks thermal at $T_m \gg 1/\eta$ [5,6]. The RG predicts how quantities renormalize in time and in particular shows that at long times the steady-state state is approached as a power law with a nonuniversal exponent $\epsilon^* \xrightarrow{\Lambda_0 T_{m0} \gg 1} A + \mathcal{O}(\frac{1}{\Lambda_0 T_{m0}})^{2A}$, where $A = \sqrt{\epsilon_0^2(\infty) - g_{\rm eff,0}^2(\infty)}$.

Case (b), in which the periodic potential is always relevant.—This case occurs for $\epsilon(T_m) < 0$. Thus, we are always in the strong coupling regime. Here, we integrate the RG equations up to a scale $l^*(T_m)$, where the renormalized coupling is $O(1)$. Beyond this scale, our RG equations are not valid; however, the advantage of the bosonic theory is that at strong coupling $g\cos(\gamma\phi) \simeq g(1 - \gamma^2\phi^2/2 + \cdots)$ so that $\sqrt{g}$ may be identified with a gap. The physical gap or order parameter is then given by $\Delta = \sqrt{g}/l^* = 1/l^*$. Since $l^*(T_m)$ depends on time, it tells us how the order parameter evolves in time [26].

Let us first consider short times $T_{m0}\Lambda_0 \ll 1$. Here, perturbation theory is valid and gives $\Delta_1 \sim g_0 T_{m0}^2$, a result that is consistent with a lattice quench at the exactly solvable Luther-Emery point [27]. At long times after the quench, the scaling dimension is $2 - K_{\rm neq}$. Provided that $\Delta T_{m0} \gg 1$, we find the steady-state order parameter,

$\Delta_{ss} = (g_{\rm eff,0})^{1/(2-K_{\rm neq})}$. Compare this with the order parameter in the ground state of H_f [9] $\Delta_{\rm eq} = (g_{\rm eff,0})^{1/(2-K_{\rm eq})}$. Since $K_{\rm neq} > K_{\rm eq}$ and $g_{\rm eff,0} \ll 1$, the order parameter at long times after the quench is always smaller than the order parameter in equilibrium. The RG equations may also be solved at intermediate times [19] $\frac{1}{\Lambda_0} \ll T_{m0} \ll \frac{1}{\Delta}$. Here, we find that $\Delta = [(\Lambda_0 T_{m0})^{\gamma^2 K_0/4 - K_{\rm neq}} g_{\rm eff,0}]^{1/(2-\gamma^2 K_0/4)}$. Thus, at intermediate times, the gap decreases with time if $K_{\rm neq} > \frac{\gamma^2 K_0}{4}$ or increases with time for the reverse case. For $K_0 = K$, this intermediate time power-law dynamics is absent.

Case (c), in which the periodic potential is relevant at short times and irrelevant at long times.—This case occurs when $\epsilon(T_m)$ changes sign from negative to positive and $g_{\rm eff}$ is not too large. Here, the short time behavior is the same as case (b); however, at long times, the order parameter decreases with time as $\Delta \sim (\frac{1}{\Lambda_0 T_{m0}})^{1+A/(2-\gamma^2 K_0/4)}$. Figure 2 summarizes the behavior of the order parameter for cases (b), (c), and (d) [case (d) is discussed next]. The nonmonotonic dependence of the order parameter in time is due to the time dependence of the scaling dimension, which physically leads to a situation where quantum fluctuations are enhanced [suppressed] at a later time for $\epsilon(T_m = \infty) > \epsilon(T_m = 0)$ $[\epsilon(T_m = \infty) < \epsilon(T_m = 0)]$, causing the order parameter to decrease [increase].

Case (d), in which the periodic potential is irrelevant at short times and relevant at long times.—This case occurs under two conditions. Either $\epsilon(T_m)$ changes sign from positive to negative during the time evolution or $\epsilon(T_m)$ is always positive, but $g_{\rm eff}(T_m)$ becomes sufficiently large at some time T_m^*. The latter includes the case of a pure lattice quench ($K_0 = K$). For either condition, the RG treatment, which neglects the effect of irrelevant operators, shows that at long times, the order parameter reaches a steady-state value, while at short times it is zero. This indicates a nonanalytic behavior at a critical time T_m^*.

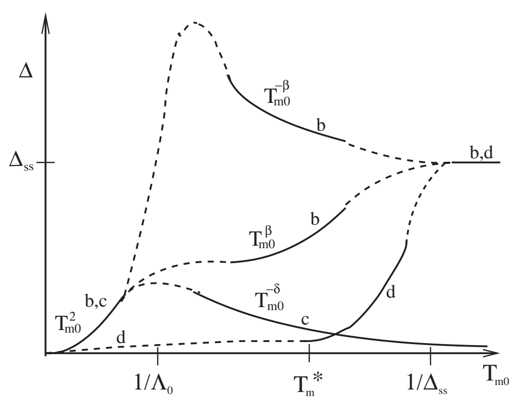

FIG. 2. Time evolution of the order parameter after the quench for cases (b)–(d). Solid lines show a short time behavior ($T_{m0} \ll \frac{1}{\Lambda_0}$), an intermediate time asymptotics ($\frac{1}{\Lambda_0} \ll T_{m0} \ll \frac{1}{\Delta_{ss}}$), and a long time behavior $T_{m0} \gg \frac{1}{\Delta_{ss}}$. At intermediate times, the order parameter increases as T_{m0}^{β} (decreases as $T_{m0}^{-\beta}$) when $K_{\rm neq} < \frac{\gamma^2 K_0}{4}$ ($K_{\rm neq} > \frac{\gamma^2 K_0}{4}$) for case (b) and eventually reaches a steady-state value Δ_{ss}. For case (c), the order parameter decreases for $T_{m0} \gg \frac{1}{\Lambda_0}$ as $\Delta \propto T_{m0}^{-\delta}$. For case (d), the order parameter increases after time T_m^* in a nonanalytic manner in time [Eq. (14)]. $\beta = \theta(|\frac{\gamma^2 K_0}{4} - K_{\rm neq}|)$, $\delta = 1 + A\theta$, $\theta = \frac{1}{2-\gamma^2 K_0/4}$, and $A = \sqrt{\epsilon_0^2 - g_{\rm eff,0}^2}$. Dashed lines are a guide to the eye for the crossover regimes.

Figure 1 contrasts case (d) with the previous cases considered where the order parameter behaved analytically. The renormalized interaction parameter $g_{\rm eff}(T_m)$ is vanishingly small right after the quench. For case (b), since infinitesimally small $g_{\rm eff}$ is a relevant perturbation, an order parameter starts growing immediately after the quench. On the other hand, for a quench corresponding to case (d), Fig. 1 shows that $g_{\rm eff}$ has to be larger than a critical value in order to be in the Mott phase. Thus, one has to wait some finite time before which renormalization effects become large enough for an order parameter to grow. We now discuss this physics in a more quantitative manner and for simplicity consider only the case of the pure lattice quench.

Let us suppose that $T_{m0}\Lambda_0 \gg 1$. Here, the RG equations are solved in two steps, one for $1 < l < T_{m0}\Lambda_0$ and the other for $T_{m0}\Lambda_0 < l$. For the first step, since I_K varies slowly at long times, eventually reaching a steady-state value, we may assume that $\frac{1}{2}|\frac{dI_K}{d\ln l}| \ll |\frac{d\ln g}{d\ln l}|$. Thus, the RG equations are the conventional ones of the equilibrium BKT transition $\frac{dg_{\rm eff}}{d\ln l} = -g_{\rm eff}\epsilon$, $\frac{d\epsilon}{d\ln l} = -g_{\rm eff}^2$. For the second step, ($\Lambda_0 T_{m0} < l$), since $I_K \sim T_m^2$, the RG equations become $\frac{dg_{\rm eff}}{d\ln \bar{l}} = -g_{\rm eff}\epsilon$, $\frac{d\epsilon}{d\ln \bar{l}} = -\frac{g_{\rm eff}^2}{\bar{l}^2}$, where $\bar{l} = \frac{l}{\Lambda_0 T_{m0}}$. The solution shows that there is a critical time T_m^* such that $g_{\rm eff}$ is irrelevant before this time and is a relevant perturbation after this time. We find that

$$\Lambda_0 T_m^* = e^{(1/D)\arctan(\epsilon_0/D) - (1/D)\arctan(D/2)}, \quad D^2 = g_{\rm eff,0}^2 - \epsilon_0^2. \tag{13}$$

The deeper one quenches into the Mott phase; the shorter is T_m^*. Moreover, T_m^* is longest along the critical line $g_{\rm eff,0} = \epsilon_0$. By identifying a length scale at which $g_{\rm eff}(\bar{l}^*) \sim 1$, we find that the order parameter grows as

$$\Delta \sim \Delta_{\rm smooth} + \theta(T_{m0} - T_m^*) \times \frac{1}{\Lambda_0 T_{m0}}[g_{\rm eff}(l = \Lambda_0 T_{m0})]^{f_2/(T_{m0} - T_m^*)}, \tag{14}$$

where $f_2 = \frac{1}{|d\epsilon(l = T_m)/dT_m|_{T_m = T_m^*}|}$ and $\Delta_{\rm smooth}$ is a background contribution arising from irrelevant operators whose effects may be treated perturbatively. Thus, while Δ is always nonzero after the quench due to the presence of irrelevant terms, due to the relevant terms it increases as a nonanalytic function of time after a critical time.

An important question concerns the spatial variation of the order parameter. Quenches in gapless systems are associated with light-cone dynamics where two points a position R apart get correlated after a time $T_m \sim R$ [28]. For our case, any two points separated by $R > T_m$ will behave primarily like the initial state with power-law correlations in position determined by K_0. The predictions for the order parameter made above are for a region within a light cone $R < T_m$. The dynamical transition at T_m^* is associated with the appearance of order in regions of size $R^* \sim T_m^*$, after which the ordered regions will begin to grow in size.

In summary, in employing a RG we have identified a novel dynamical phase transition in a strongly correlated system where an order parameter grows as a nonanalytic function of time after a critical time [Eq. (14)]. The order parameter shows rich dynamics both at the transition and for more general quenches (Fig. 2). Identifying similar dynamical transitions in higher dimensions where thermal fluctuations are less effective in destroying order is an important direction of research.

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