# Majorana-like Modes of Light in a One-Dimensional Array of Nonlinear Cavities

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The search for Majorana fermions in $p$-wave paired fermionic systems has recently moved to the forefront of condensed matter research. Here we propose an alternative route and show theoretically that Majorana-like modes can be realized and probed in a driven-dissipative system of strongly correlated photons consisting of a chain of tunnel-coupled cavities, where $p$-wave pairing effectively arises from the interplay between strong on-site interactions and two-photon parametric driving. The nonlocal nature of these exotic modes could be demonstrated through cross-correlation measurements carried out at the ends of the chain—revealing a strong photon-bunching signature—and their non-Abelian properties could be simulated through tunnel-braid operations.

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In recent years, strongly correlated photons have proved to be a remarkably rich platform for investigating phenomena traditionally regarded as pertaining to condensed matter physics. Tremendous theoretical and experimental efforts have made it possible to achieve strong optical nonlinearities at the single-photon level [1,2] and to demonstrate photon blockade effects [3–8]. Meanwhile, the pursuit of Majorana fermions has become a new focus of condensed matter research [9,10], and $p$-wave paired superfluids and superconductors have been promoted as paradigmatic systems for investigating the physics of Majorana modes [11]. Even though strongly interacting photons have been predicted to exhibit a typical fermionic behavior [12,13], optical systems have never been considered as candidates for realizing such exotic physics.

In this Letter, we show that Majorana-like modes (MLMs) can be obtained in a one-dimensional (1D) strongly correlated system of impenetrable (or "fermionized") photons. More specifically, we consider a chain of coupled cavities with strong on-site nonlinearities and introduce a drive mechanism based on parametric amplification which, in stark contrast to previous works, gives rise to an effective $p$-wave pairing between (fermionized) photons. We map our system to the 1D chain originally proposed by Kitaev as a toy model for Majorana fermions [14] and show the existence of zero-energy modes with properties similar to those of Majorana modes in solid-state systems. Owing to the intrinsic dissipative nature of the system, these "Majorana-like" modes do not benefit from parity (or "topological") protection against decoherence [15,16], and thus cannot serve as topological quantum memories [17]. However, they do behave as genuine Majorana modes on time scales shorter than the lifetime of a photon in the system, allowing Majorana physics to be probed.

To demonstrate the fact that MLMs can be detected via simple optical schemes, we propose a realistic experiment that takes full advantage of the optical nature of the system and allows for the direct observation of MLMs through second-order photon cross-correlation measurements. Although our proposal is strictly limited to 1D—since impenetrable photons do not behave as fermions in higher dimensions—we show that MLMs can effectively be exchanged using "tunnel-braid" operations [18], enabling us to simulate their non-Abelian properties.

*The model.*—The backbone of our system consists of a 1D chain of $N$ optical cavities coupled through nearest-neighbor photon tunneling (Fig. 1). Each cavity exhibits a large optical nonlinearity (i.e., is strongly coupled to an artificial atom) and supports a single mode that can be described as a Wannier function localized on site $i$ around the cavity center. Photon tunneling occurs as a result of the nonvanishing spatial overlap between nearest-neighboring Wannier modes [19], and the system Hamiltonian takes the generic form of a generalized Bose-Hubbard model:

$$H_0 = \omega_c \sum_{i=1}^{N} b_i^\dagger b_i + \frac{U}{2} \sum_{i=1}^{N} b_i^\dagger b_i^\dagger b_i b_i - J \sum_{i=1}^{N-1} (b_i^\dagger b_{i+1} + \text{H.c.}), \tag{1}$$

where $b_i$ ($b_i^\dagger$) are annihilation (creation) operators associated with the $i$th cavity of the chain with resonance

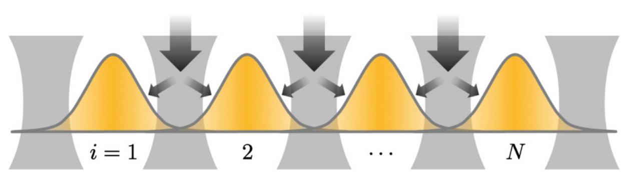


FIG. 1 (color online). Driven-dissipative chain of coupled cavities. Each cavity exhibits a large optical nonlinearity and sustains a single Wannier mode which weakly overlaps with its nearest neighbors, thus allowing for photon hopping between sites. Parametric pumps (depicted by arrows) couple to the weak intercavity field and inject photon pairs which, owing to strong photon-photon repulsion, split up into different cavities, effectively giving rise to $p$-wave pairing.

 

frequency $\omega_c$, $U$ is the strength of the on-site photon-photon repulsion (Kerr energy) due to the large optical nonlinearities, and $J$ denotes the tunneling amplitude between nearest-neighboring sites. In this work, we will focus exclusively on the strong-interaction regime, in which the energy cost $U$ of adding an extra photon to an occupied cavity is by far the largest of all relevant energy scales [20]. In this so-called "hard-core" limit, the occupation of each site is effectively restricted to 0 or 1, and the photons exhibit a characteristic fermionic behavior [12,13].

To achieve $p$-wave pairing, we introduce *parametric* pumps (or amplifiers) which, in stark contrast to usual coherent drives, are tailored to inject *pairs* of photons into the system through nonlinear optical processes (see Supplemental Material [21]). Assuming that these pumps drive the system locally through the intercavity field—which consists of a superposition of two neighboring Wannier modes—photons from a single pair can either be emitted into the same cavity (or Wannier mode) or settle into different, nearest-neighboring cavities. In the strong-interaction regime, the second process is strongly favored, and the drive Hamiltonian effectively reads

$$H_{\mathrm{drive}} = -|\Delta| \sum_{i=1}^{N-1} (e^{\mathrm{i}(2\omega_p t+\phi)} b_i b_{i+1} + \mathrm{H.c.}), \quad (2)$$

where $\Delta = |\Delta| e^{\mathrm{i}\phi}$ defines the amplitude and phase of the parametric pumps, and $\omega_p$ their frequency. We note that the amplitude $|\Delta|$ of the parametric drive is determined by the overlap of the Wannier modes in a way similar to the tunneling amplitude $J$ defined above. We thus expect to be able to reach a regime in which the two quantities are of the same order. Physically, the above Hamiltonian describes the coherent exchange of $p$-wave paired photons between the system and the classical pump field(s) [22]. It provides the optical counterpart of $p$-wave superconductivity that is crucial to access Majorana physics and compensates for losses by continuously replenishing the system with photons. The time evolution of the system including the drive and the photon losses is governed by the Lindblad master equation

$$\partial_t \rho = -\mathrm{i}[H_0 + H_{\mathrm{drive}}, \rho] + \Gamma \sum_{i=1}^{N} \left( b_i \rho b_i^\dagger - \frac{1}{2}\{b_i^\dagger b_i, \rho\} \right), \quad (3)$$

where $\rho$ is the density matrix of the system and $\Gamma$ the decay rate associated with the individual cavities.

*Mapping to a 1D Kitaev chain.*—In the strong-interaction regime ($U \gg J, |\Delta|$), the Hilbert space of the system effectively reduces to that of hard-core photons $\tilde{b}_i = \mathcal{P} b_i \mathcal{P}$, $\tilde{b}_i^\dagger = \mathcal{P} b_i^\dagger \mathcal{P}$, where $\mathcal{P}$ projects onto the subspace of single occupancy. Hard-core photons can be seen as spin-1/2 particles, with Pauli-type matrices $\sigma_i^- = 2\tilde{b}_i$, $\sigma_i^+ = 2\tilde{b}_i^\dagger$ ($\sigma_i^\pm = \sigma_i^x \pm \mathrm{i}\sigma_i^y$), and their fermionic nature can be unveiled by mapping the spin-1/2 particles to spinless fermions $a_i$, $a_i^\dagger$ using a Jordan-Wigner transformation [23] of the form $a_i = \frac{1}{2}\prod_{j=1}^{i-1}(-\sigma_j^z)\sigma_i^-$. Defining $\mu = \omega_p - \omega_c$ and moving to a rotating frame defined by $H_1 = \omega_p \sum_i \tilde{b}_i^\dagger \tilde{b}_i$, the Hamiltonian $H = H_0 + H_{\mathrm{drive}}$ of the full system becomes, in the fermionic picture,

$$H = -J \sum_{i=1}^{N-1} (a_i^\dagger a_{i+1} + \mathrm{H.c.}) + |\Delta| \sum_{i=1}^{N-1} (e^{\mathrm{i}\phi} a_i a_{i+1} + \mathrm{H.c.}) - \mu \sum_{i=1}^{N} a_i^\dagger a_i, \quad (4)$$

which corresponds to the 1D $p$-wave superconductor of spinless fermions originally introduced by Kitaev [14]. Here, the cavity frequency $\omega_c$ plays the role of a Fermi level, and the detuning $\mu = \omega_p - \omega_c$ between the pump and cavity frequencies that of a chemical potential. Assuming that $|\Delta| \neq 0$, two topologically distinct (gapped) phases can be identified [14]: a trivial phase corresponding to $|\mu| > |2J|$ and a nontrivial phase corresponding to $|\mu| < |2J|$, in which the system supports Majorana modes that are exponentially localized at both ends of the chain. The topological phenomena associated with the 1D Kitaev chain can be most easily understood for $J = \Delta > 0$ and $\mu = 0$ (i.e., $\omega_p = \omega_c$). In this illustrative case, the Hamiltonian reduces to

$$H = \mathrm{i}J \sum_{i=1}^{N-1} c_{2i} c_{2i+1} = -J \sum_{i=1}^{N-1} \sigma_i^x \sigma_{i+1}^x, \quad (5)$$

with Majorana operators defined as

$$\begin{aligned} c_{2i-1} &= a_i + a_i^\dagger = \prod_{j=1}^{i-1}(-\sigma_j^z)\sigma_i^x, \\ c_{2i} &= -\mathrm{i}(a_i - a_i^\dagger) = -\prod_{j=1}^{i-1}(-\sigma_j^z)\sigma_i^y, \end{aligned} \quad (6)$$

and can readily be diagonalized as $H = 2J\sum_{i=1}^{N-1}(\tilde{a}_i^\dagger \tilde{a}_i - 1/2)$ with Bogoliubov-Valatin quasiparticle operators $\tilde{a}_i = (c_{2i} + \mathrm{i}c_{2i+1})/2$. The associated spectrum is symmetric about the "Fermi level" $\omega_c$ and features a gap $2J$. Most importantly, it exhibits two Majorana zero-energy modes corresponding to the Majorana operators $c_1$ and $c_{2N}$ localized at the ends of the chain but absent from the Hamiltonian. These modes define a two-dimensional, *nonlocal* degenerate (zero-energy) subspace which we identify as a "Majorana qubit," with associated $\sigma^z$ operator

$$\sigma_M^z = \mathrm{i}c_1 c_{2N} = \prod_{j=1}^{N}(-\sigma_j^z)\sigma_1^x \sigma_N^x. \quad (7)$$

The stringlike operator $P = \prod_{j=1}^{N}(-\sigma_j^z)$ that appears in the above expression corresponds to the parity operator associated with the *total* number of (fermionized) photons. It commutes with the Hamiltonian of Eq. (4), but *anticommutes* with the collapse operators $\sigma_i^-$ entering the

Liouvillian in Eq. (3). Physically, this means that single-photon losses result in the breakdown of parity conservation, as expected, such that the Majorana qubit is not parity protected—or "topologically protected"—from decoherence [24]. Although this disqualifies Majorana modes of light for practical applications such as topological quantum memories, this crucially does *not* hinder the observation of their exotic physics, since the key features of Majorana physics—such as the existence of localized Majorana modes and the possibility of simulating non-Abelian braiding operations in the associated zero-energy subspace—do not require perfect parity protection to be realized. In fact, as we demonstrate below, Majorana physics does remain accessible within time scales much shorter than the lifetime $\sim 1/\Gamma$ of a photon in the system. However, the Majorana modes appearing in our optical framework only behave as genuine, *local* Majorana modes in the limit $\Gamma \to 0$. Only then does the stringlike operator $P$ that they carry—a remnant of the Jordan-Wigner mapping—reduce to a simple phase $P = \pm 1$. In this respect, we will refer to Majorana modes of light as "Majorana-like" modes.

Our proposal for MLMs, embodied in Eqs. (1)–(3), could be realized in any cavity quantum electrodynamics (cavity QED) system lying deep in the strong-coupling regime. In the Supplemental Material [21], we outline a potential implementation in circuit QED which we deem as closest to experimental realization and give ballpark figures for the relevant parameters and energy scales.

*Optical detection scheme.*—Multiple schemes have been proposed for detecting Majorana modes in solid-state systems (see, e.g., Ref. [10] for a review). Although we believe that most of them can be transposed to our optical setting, we will focus on the detection of Majorana-mediated (photonic) Cooper pair splitting, following the proposals of Refs. [25,26]. We start with a Kitaev chain in a topologically nontrivial regime defined by $J$, $\Delta > 0$, and $0 \leq \mu < 2J$, without loss of generality. In such a parameter range, exponentially localized MLMs are expected on both sides of the chain, with a length scale that increases with $\mu$ and diverges as $\mu \to 2J$ [14]. For finite $\mu < 2J$ and small enough system sizes, these modes weakly couple and the levels of the Majorana qubit that they form split in energy by an amount $\delta_M > 0$ [27], as captured by the Hamiltonian $\delta_M \sigma_M^z$ [the explicit form of $\sigma_M^z$ reduces to $\sigma_1^y \sigma_N^y$ when restricted to the end cavities and is given by Eq. (7) for $\mu = 0$]; as a key ingredient, we assume that $\delta_M \ll E_g$, where $E_g$ denotes the gap of the system [28]. Next we introduce two additional nonlinear cavities—one on each side of the Kitaev chain—which we refer to as the left ($L$) and right ($R$) probe cavities, respectively. We assume that the latter have resonance frequencies $\omega_{L,R} = \omega_p$ and that they couple to the end cavities of the Kitaev chain through weak tunneling only. In the rotating frame introduced above [in deriving Eq. (4)], the Hamiltonian describing the interaction with the probes then takes the form $H_{\text{probe}} = -J_L(\sigma_L^x \sigma_1^x + \sigma_L^y \sigma_1^y) - J_R(\sigma_N^x \sigma_R^x + \sigma_N^y \sigma_R^y)$, where $0 < J_{L,R} \ll \delta_M$ denotes the weak amplitude for tunneling into the left and right probe cavities, respectively. Since all energy scales associated with $H_{\text{probe}}$ are, by assumption, much smaller than $E_g$, the probe cavities only probe the low-energy physics associated with the MLMs of the chain. Owing to this energy selectivity, the terms of the form $\sigma_i^y \sigma_j^y$ appearing in $H_{\text{probe}}$—which mediate coupling to higher excited states—can safely be neglected, while the operators $\sigma_1^x$ and $\sigma_N^x$—which anticommute with $\sigma_M^z$ and thus effectively describe a spin flip of the Majorana qubit—can be replaced by $\sigma_M^x$. This results in the following low-energy effective Hamiltonian for the full system:

$$H_{\text{eff}} = \delta_M \sigma_M^z - J_L \sigma_L^x \sigma_M^x - J_R \sigma_M^x \sigma_R^x. \tag{8}$$

Physically, the above expression tells us that the nonlocal Majorana qubit formed by the "localized" (in the limit $\Gamma \to 0$) MLMs of the chain mediates a *nonlocal* coherent exchange of photons between the probe cavities. Clearly, the bottleneck of such an exchange is given by the time scale $t_M \sim 1/\delta_M$ over which the Majorana qubit evolves. We thus only expect to see nonlocal correlations between the probes if $t_M$ is the shortest time scale in Eq. (8), i.e., if $\delta_M \gg J_{L,R}$, as has been assumed. To detect these correlations, one can take advantage of the intrinsic dissipative nature of the system. Assuming that the decay rate of the probe cavities satisfies $\Gamma_{L,R} \sim J_{L,R} \ll \delta_M$, so that spontaneous emission occurs on a time scale much longer than the time scale $t_M$ over which correlations are generated, we expect a direct signature of MLMs to appear in the second-order photon cross correlations between the light emitted from the two probe cavities. In order to illustrate this, we consider the simple case $J_L = J_R \equiv \sqrt{2}\tilde{J}$, $\Gamma_L = \Gamma_R \equiv 8\tilde{\Gamma}$, in the limit where the decay rate $\Gamma$ associated with the cavities of the Kitaev chain vanishes. Following the method of Ref. [29] (see Supplemental Material [21]), we then obtain steady-state photon cross correlations between the probe cavities that read

$$\begin{aligned} g_{LR}^{(2)} &\equiv \frac{\langle \tilde{b}_L^\dagger \tilde{b}_R^\dagger \tilde{b}_R \tilde{b}_L \rangle}{\langle \tilde{b}_L^\dagger \tilde{b}_L \rangle \langle \tilde{b}_R^\dagger \tilde{b}_R \rangle} = 1 + \frac{\langle \sigma_L^z \sigma_R^z \rangle - \langle \sigma_L^z \rangle \langle \sigma_R^z \rangle}{(1 + \langle \sigma_L^z \rangle)(1 + \langle \sigma_R^z \rangle)} \\ &= 1 + \frac{\tilde{\Gamma}^2 \delta_M^2}{(\tilde{J}^2 + \tilde{\Gamma}^2)^2}. \end{aligned} \tag{9}$$

Remembering that $\tilde{\Gamma}^2 \sim \tilde{J}^2 \ll \delta_M^2$, we thus find $g_{LR}^{(2)} \gg 2$; in other words, the light emitted by the spatially separated probe cavities is strongly bunched [30]. To examine the effect of weak dissipation from the chain, we have carried out numerical simulations of the *full* Kitaev chain coupled to the probe cavities. Our studies confirm that a striking nonlocal photon-bunching signature of MLMs remains visible for small decay rates $\Gamma \sim \Gamma_{L,R} \ll \delta_M$ of the chain cavities and small enough system sizes, i.e., as long as the effective width of the Majorana levels is much smaller than their energy splitting (Fig. 2).

We remark that the above results closely parallel those obtained in Refs. [25,26] in the solid-state setting. Here, the probe cavities play a similar role to the metallic leads used in typical solid-state proposals: they provide a narrow band of states of effective width $\sim 1/\Gamma_{L,R}$ close to the Fermi level $\omega_c$ into which fermionized photons can be emitted from the system. Because of the large splitting $\delta_M$ between MLMs, nonlocal—or "crossed"—Andreev reflection is favored over its local analog and Cooper pairs of photons are split into separate leads (or probe cavities). We remark that one could in principle remove the probe cavities and observe directly the light emitted by the end cavities of the Kitaev chain. In that case, however, spectral filtering would be required in order to isolate the bunching signature associated with the low-energy MLMs from contributions of the bulk.

*Versatility of the optical proposal beyond detection.*—In contrast to previous proposals (such as Refs. [31–33]), our optical system provides a very versatile platform for simulating Majorana physics. In addition to being a conceptually simple realization of Kitaev's original lattice toy model—fermionized photons are intrinsically spinless, and pairing occurs between nearest-neighboring cavities only—it allows for single-site addressability and for local control of all parameters entering Kitaev's model: the "chemical potential" $\mu$ can be tuned easily and locally by modifying the frequency of the individual pumps (or, alternatively, the resonance frequency of the individual cavities), and the amplitude and phase of the "superconducting order parameter" $\Delta$ can similarly be adjusted by controlling the amplitude and phase of the parametric amplifiers. The tunneling amplitude $J$, on the other hand, can be tuned by introducing intermediate control devices between cavities (see, e.g., Ref. [34] and references therein). Such level of control is key to overcoming the crucial challenges currently facing most solid-state proposals, such as the tuning in and out of the topological phase and the suppression of disorder effects [33].

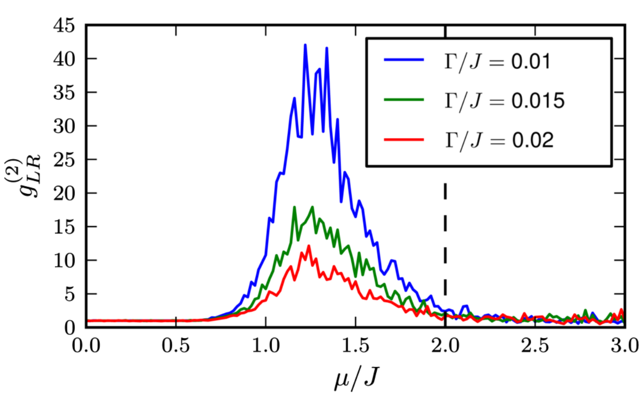


FIG. 2 (color online). Numerical results showing the second-order cross-correlation function $g^{(2)}_{LR}$ as a function of the detuning $\mu$ for $N = 12$ (including probe cavities), $\Delta/J = 1$, $J_{L,R}/J = 0.02$, and for different values of $\Gamma/J = \Gamma_{L,R}/J$. MLMs (with energy splitting $\delta_M$ determined by $\mu$) are expected in the region $0 \leq \mu/J < 2$ (delimited by a dashed vertical line), which corresponds to a topologically nontrivial phase (in the limit $N \rightarrow \infty$). As expected, a strong bunching signature is observed for large $\mu$ *inside* the topologically nontrivial region, while $g^{(2)}_{LR} \approx 1$ beyond the critical point $\mu/J = 2$, clearly signaling the absence of MLMs. All results were obtained using a Monte Carlo wave function approach (see, e.g., Ref. [37]) with 400 trajectories.

Despite its conceptual simplicity, versatility, and physical realizability, our optical proposal departs from being ideal in two respects: (i) it lacks topological protection (or parity conservation), and (ii) it cannot be scaled up to networks of 1D wires [35,36]. The first imperfection arises as a direct consequence of photon losses—unavoidable in photonic systems—and puts stringent constraints on the time scale over which Majorana physics can be observed; namely, MLMs must be manipulated and detected on a time scale much shorter than the lifetime of a photon in the system. We argue in the Supplemental Material [21], however, that state-of-the-art technologies in circuit QED could allow for the experimental realization of our proposal with a sufficient control over dissipation to meet these requirements. The second imperfection stems from the intrinsic nonlocal nature of the Jordan-Wigner mapping invoked in deriving Eq. (4), ruling out the possibility to observe non-Abelian exchange statistics in connected wire geometries. In 1D, the Jordan-Wigner string carried by the end MLMs of a chain essentially corresponds to the parity operator of the latter [see, e.g., Eq. (7)], such that end MLMs do behave as genuine, local Majorana modes on time scales over which parity is effectively constant. In higher dimensions, however, the situation changes drastically: when multiple 1D chains are contacted (not through their ends, so that the systems effectively is higher dimensional), the parity of the individual chains becomes a dynamical quantity and the nonlocal nature of the MLMs comes into play—with dramatic consequences such as the absence of non-Abelian exchange statistics. To avoid such complications, we simply strictly restrict ourselves to 1D systems. In that case, the exchange of Majorana modes is impossible in real-space, but can nevertheless be *simulated* using so-called "tunnel-braid" operations [18]. As shown in the Supplemental Material [21], these operations only preserve the degeneracy of the MLMs—as real-space braiding—provided that the relative phase between the latter is properly tuned, and therefore are not strictly speaking topologically protected. This, however, does not constitute an additional problem in our optical setting where parity is anyway not conserved. In the framework of our proposal, tunnel-braid operations crucially allow us to obviate the need for real-space braiding, hence providing us with a full-fledged 1D optical platform for Majorana physics.

*Conclusions.*—We anticipate that our proposal for realizing and detecting photonic $p$-wave pairing will allow for an exciting alternative avenue for the investigation of Majorana physics. An interesting possibility would be the investigation of Majorana modes in a continuum 1D model of strongly interacting optical photons [12].

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