

Analytic Structure of the Landau-Gauge Gluon Propagator

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The analytic structure of the nonperturbative gluon propagator contains information on the absence of gluons from the physical spectrum of the theory. We study this structure from numerical solutions in the complex momentum plane of the gluon and ghost Dyson-Schwinger equations in Landau gauge Yang-Mills theory. The resulting ghost and gluon propagators are analytic apart from a distinct cut structure on the real, timelike momentum axis. The propagator violates the Osterwalder-Schrader positivity condition, confirming the absence of gluons from the asymptotic spectrum of the theory.

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Introduction.—One of the fundamental properties of QCD is the absence of its elementary degrees of freedom, the quarks and gluons, from the physical spectrum of the theory. The associated problem of quark confinement is a much debated issue [1]. In this discussion it is useful to distinguish between two notions of confinement. One is in terms of color confinement, i.e., the absence of colored states from the asymptotic, physical state space of the theory. The other is strictly related to the center symmetry of Yang-Mills theory. Both notions are not equivalent. If center symmetry is unbroken, there exists a linear rising potential between static color charges in the fundamental representation of the gauge group, stretching out to arbitrary distances r of the charges. In QCD, the mere presence of fundamental dynamical charges breaks this symmetry. Consequently, string breaking sets in at a sufficiently large separation R of the fundamental test charges and the potential becomes flat for $r > R$. Thus, if confinement is defined in terms of unbroken center symmetry, QCD is not a confining theory [1]. Gluons are also not confined in this sense, since they live in the adjoint representation of the gauge group. When two gluons are separated far enough from each other, they pull two additional gluons out of the vacuum and dress up to form colorless bound states. This brings us back to the other notion of confinement: the absence of colored asymptotic states. A possible explanation of this absence are positivity violations. By definition, the asymptotic Hilbert space $\mathcal{H}_{\text{phys}}$ of colorless physical particles has to be positive (semi-) definite, otherwise a probabilistic interpretation of its S -matrix elements would not be possible. Thus, positivity violations in the gluon propagator constitute a sufficient signal for the absence of gluons from the asymptotic spectrum of the theory.

Another source of interest in the analytic structure of the gluon propagator comes from heavy-ion collisions. Currently, there is great activity both from theory and experiment at RHIC and ALICE/LHC to shed light on the properties of the quark-gluon plasma, i.e., strongly interacting matter at large temperatures and/or density.

Transport models like the parton-hadron-string dynamics approach [2] analyze the dynamics of quarks and gluons in the quark-gluon plasma. An important input into these calculations is the temperature-dependent spectral function of the gluon, a quantity directly related to its analytical structure. Whether and how this structure changes below and above the deconfinement transition is currently an open question [3].

What can the zero temperature analytic structure of the gluon propagator look like? Based on studies of the gauge fixing problem, Gribov [4] and later on Zwanziger [5] suggested a form for the gluon propagator with complex conjugate poles at purely imaginary squared Euclidean momenta. A generalization with complex conjugate poles in the negative half-plane of squared complex momenta has been suggested by Stingl in Ref. [6] and has been recently explored in detail in the refined Gribov-Zwanziger framework Ref. [7]. These forms have in common that they may pose a problem for the analytic continuation of the theory from Minkowski space to the Euclidean formulation used by lattice gauge theory and functional methods. An alternative form with a branch cut structure for real and timelike squared momenta has been proposed in Ref. [8] and found to compare well with numerical results for the propagator and its Schwinger functions in the Dyson-Schwinger approach.

All explicit calculations of the gluon propagator so far have been restricted to the real and spacelike Euclidean momentum domain. Clearly, in order to pin down the analytic structure of the gluon propagator, an extension of these calculations into the complex momentum domain is highly desirable. In this Letter we report the first results of such a calculation. Within the continuum formulation of Landau-gauge Yang-Mills theory, we solve the coupled system of Dyson-Schwinger equations for the nonperturbative gluon and ghost propagators in the complex momentum plane and extract the analytic structure at timelike momenta. As a main result we find analytic propagators everywhere, apart from cuts on the real, timelike momentum axis.

The framework.—The Dyson-Schwinger equations (DSEs) for the ghost and gluon propagators are shown in Fig. 1. They form a coupled set of integral equations with renormalized, bare, and dressed propagators and vertices. In Landau gauge, the explicit form of the propagators is given by

$$D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{Z(p^2)}{p^2}, \quad D_G(p) = -\frac{G(p^2)}{p^2}, \quad (1)$$

with the gluon dressing function $Z(p^2)$ and the ghost dressing function $G(p^2)$. These functions can be numerically determined from their DSEs provided explicit expressions for the dressed ghost-gluon, three-gluon, and four-gluon vertices are known. Since these satisfy their own DSEs containing unknown higher n -point functions, in practice one needs to truncate this tower to generate a closed and solvable system of equations. Certainly, meaningful results can only be achieved by careful control of the quality of such a truncation.

A scheme which maintains multiplicative renormalizability and transversality has been devised in Ref. [9]; for transverse projection of the gluon DSE, this scheme is equivalent to the one used previously in Ref. [10]. It uses a bare ghost-gluon vertex, a choice that is close to the results of corresponding lattice calculations [11], an *Ansatz* for the dressed three-gluon vertex in terms of the propagator functions, see Refs. [9,10] for details, and a vanishing four-gluon interaction. Given this choice, the coupled system of DSEs can be solved numerically. The resulting solution for the gluon dressing function $Z(p^2)$ has been discussed in Refs. [9,10] and, for the convenience of the readers, is shown again in Fig. 2 together with corresponding lattice results [12]. In the large momentum region, where the perturbative behavior sets in, both approaches agree very well. This is also true in the low-momentum region. In the deep infrared, the gluon dressing function displays the

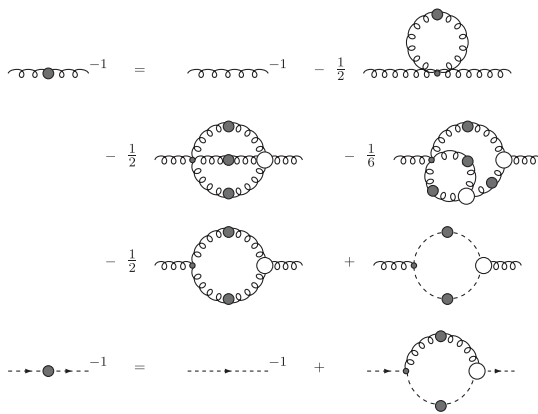


FIG. 1. Dyson-Schwinger equations for the gluon and ghost propagator. Filled circles denote dressed propagators and empty circles denote dressed vertex functions.

“massive” behavior $Z(p^2) \sim p^2$. Such “decoupling” types of solutions, as opposed to “scaling,” [13] have been suggested long ago [14] and have been revived in Refs. [15–17]. Large-volume lattice results agree with this type of solution [18], although it remains a matter of current debate whether problems with gauge fixing in the context of Gribov copies are already well under control [19–22].

In the midmomentum region around one GeV, there are differences between the DSE and the lattice result on the 20% level, which have to be attributed to the above-discussed vertex truncations for the ghost-gluon, three-gluon, and four-gluon vertex. Improvements for the dressed ghost-gluon vertex have been discussed in Refs. [23,24]. Furthermore, first studies of other types of *Ansätze* for the dressed three-gluon vertex are available [24], and studies in the background gauge Pinch-technique scheme emphasize the importance of poles in the longitudinal parts of the three-gluon vertex, see Ref. [25] for a review. While all these studies are interesting on systematic grounds, the resulting solutions for the gluon dressing function are not closer to the lattice result than the one shown in Fig. 2. The remaining difference may therefore very well be attributed to the missing two-loop diagrams involving the four-gluon interaction. Indeed, pointwise agreement with the lattice data has been achieved within the framework of functional renormalization group equations [9], where such contributions can be taken into account due to the exact one-loop structure of the equations. Within the DSE framework, the technical complications arising from the two-loop diagrams only allowed for phenomenological treatments of these contributions so far [26]. We therefore prefer to defer a study of the influence of these terms to future work and use the well-established truncation scheme of Refs. [9,10] in the following.

The numerical techniques necessary to solve a coupled set of DSEs in the complex plane have been explored up to now only in the context of the fermion propagator, see Ref. [27] and the Appendix of Ref. [8]. The basic idea is to shift the contour of radial integration in the loop integral into the complex plane such that singularities in the angular

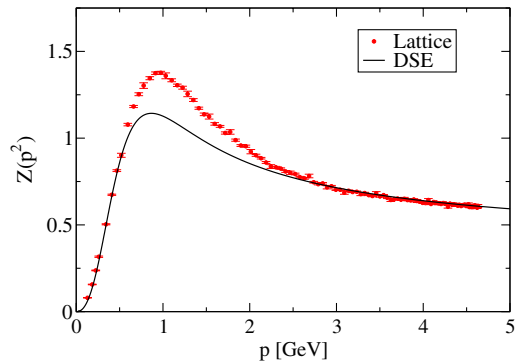


FIG. 2 (color online). Results for the gluon dressing function $Z(p^2)$ from lattice calculations [12] compared to the result from DSEs [9].

integral are meliorated. For this work we adapted this method for the gluon and gluon system. Details will be given elsewhere. We have cross-checked our numerics also by employing a different method which solves the DSEs directly on a grid of complex momenta without any shifts in the integrals [28]. The results of both methods agree well for a large range of complex momenta. However, close to the timelike momentum axis, our first method clearly delivered much more stable results and is therefore preferred.

Results and discussion.—Our results for the analytic structure of the gluon and ghost propagator in $SU(N)$ Yang-Mills theory are shown in Figs. 3 and 4. Let us first discuss the real parts of the propagators. In the lower diagram of Fig. 3, we see a spike structure of the ghost dressing function, which is located at the origin of the complex momentum plane. Since we have chosen a decoupling type of solution for the ghost-gluon system, the ghost dressing function is finite at $p^2 = 0$. At our numerical infrared cutoff $|\epsilon^2| = 10^{-5}$, the value $G(\epsilon^2)$ depends slightly on the direction from which zero is approached. In our calculation, $G(\epsilon^2) = 5$ when $p^2 \rightarrow 0^+$, but $G(-\epsilon^2) = 5.005 \mp i0.004$ when $p^2 \rightarrow 0^-$ on the real axis. Thus, the real part of the ghost dressing function is almost symmetric around the origin of the complex momentum plane. For the real part of the gluon propagator in the upper diagram of Fig. 3, the situation is entirely different. Again, the propagator is finite at $p^2 = 0$ but shows large positive values for complex momenta and negative structures close to the negative real momentum axis. The positive spikes extend up to $\Re D(p^2) = 40 \text{ GeV}^2$ staying

definitely finite. Closer to the negative real momentum axis, the propagator becomes negative for $|p^2|$ larger than some finite value on the negative real momentum axis. The corresponding narrow dip is finite in depth and sizably extends 0.3 GeV^2 out into the complex momentum plane. The minimum value of the dip is approximately $\Re D(p^2) = -105 \text{ GeV}^2$.

Now let us discuss the imaginary part in Fig. 4. Here we clearly see a cut structure emerging for both the gluon propagator and the ghost dressing function along the negative real momentum axis. No further structure is seen in the complex momentum plane. We thus arrive at an important result of our study: the ghost and gluon propagators have nontrivial analytic structure only on the timelike real momentum axis. This is in contrast to the expectations from the studies of Gribov, Zwanziger, and Stingl [4–7], which all assumed singularities away from the real momentum axis. We find no evidence for these. Within the present numerical accuracy, the cuts are sharply peaked but finite. Whether an even more precise treatment leads a singularity in $\Im D(p^2)$, as assumed in the fits to the DSE results in Ref. [8], remains an open question.

The cuts in the imaginary part of the ghost and gluon propagators are directly related with the corresponding spectral functions,

$$\rho_{G,g}(p^2) = -\Im\{D_{G,g}(p^2)\}/\pi, \quad (2)$$

with $D_G(p^2) = -G(p^2)/p^2$, $D_g(p^2) = Z(p^2)/p^2$, and the momentum p^2 on the negative real momentum axis of the upper complex half plane. We therefore show them more closely in Fig. 5. Our numerical results are obtained on a

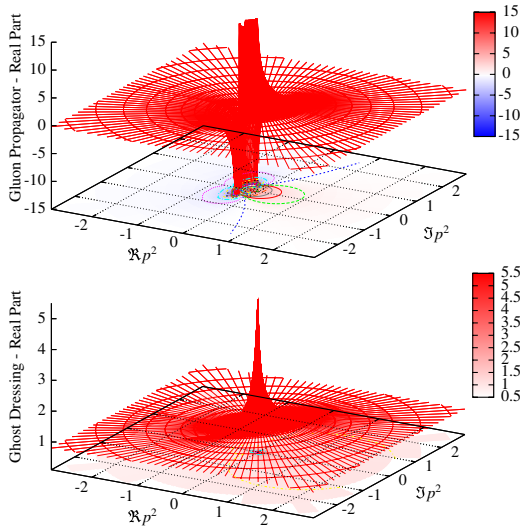


FIG. 3 (color online). Results for the real part of the gluon propagator function $D(p^2)$ and the ghost dressing function $G(p^2)$ in the complex momentum plane, including colored contour maps and lines. The displayed range of the gluon propagator is restricted in order to resolve smaller structures. See text for the extrema of $\Re D(p^2)$.

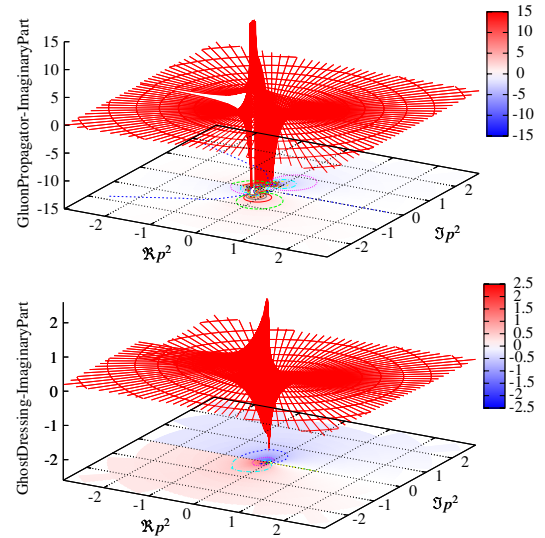


FIG. 4 (color online). Results for the imaginary part of the gluon propagator function $D(p^2)$ and the imaginary part of the ghost dressing function $G(p^2)$ in the complex momentum plane including colored contour maps and lines. The displayed range of the gluon propagator is restricted in order to resolve smaller structures. See also Fig. 5 for the full scale.

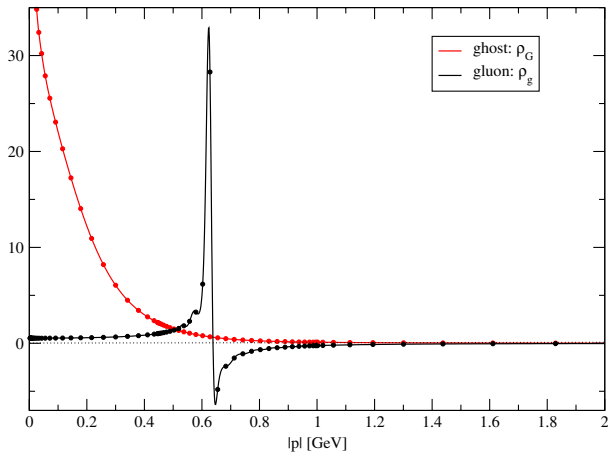


FIG. 5 (color online). Results for the gluon spectral function and the ghost spectral function as a function of momentum.

grid of momentum points which are displayed explicitly, whereas the interpolation is done via Chebychev polynomials and serves to guide the eye. For the ghost spectral function (and most of the gluon), this interpolation clearly works also on a quantitative level; however, the interpolation for the gluon in the region $0.5 < |p| < 0.7$ GeV can only be regarded as a qualitative one. Whereas the ghost spectral function is dominated by the massless $1/p^2$ pole in the propagator, the gluon is clearly different. Its imaginary part rises first, turns over and crosses through zero, turns again, and then approaches zero from below at large time-like momenta. The exact locations and heights of the maxima in the positive and negative region cannot be determined precisely within the present numerical accuracy. Nevertheless, the qualitative behavior is fixed from the explicitly calculated points shown in the plot. The gluon spectral function obeys the Oehme-Zimmermann normalization condition [29]

$$Z_3^{-1} = \int \rho_g(s) ds, \quad (3)$$

where Z_3 denotes the gluon wave function renormalization constant, with a deviation of 10%. On top of this we expect a systematic error of 20% based on the quality of our truncation in the spacelike momentum region. This provides a measure on the accuracy achieved in the present computation. The negative contributions to the gluon spectral function indicate its absence from the asymptotic spectrum of the theory. In general, the cuts in the ghost and gluon propagators signal the radiation of unphysical particles (ghosts and gluons) from unphysical particles. We expect such a structure to be gauge-parameter independent. Moreover, the gluon is certainly not a massive particle in the usual sense. Nevertheless, one may be tempted to define something like an “effective mass” m_g for the gluon from the location of the positive peak in the spectral function. Within the present accuracy, we find

$$600 \text{ MeV} < m_g < 700 \text{ MeV}. \quad (4)$$

We stress again, however, that m_g is not a measurable quantity; strictly speaking, it is just the scale where positivity violations in the gluon set in.

In this Letter we presented the first nonperturbative solution of the gluon and ghost propagators in the complex momentum plane together with an extraction of their respective spectral functions. Our results agree with expectations based on the corresponding Schwinger functions discussed in Refs. [8,9]. We presented solutions for the decoupling case; a comprehensive comparison with scaling will be given elsewhere. Besides the considerable theoretical interest in these functions, they are also a necessary input into the calculations of glueball masses within the framework of Bethe-Salpeter equations. Corresponding results will be detailed in a subsequent work.

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