Interaction-Mediated Surface-State Instability in Disordered Three-Dimensional Topological Superconductors with Spin SU(2) Symmetry

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We show that arbitrarily weak interparticle interactions destabilize the surface states of 3D topological superconductors with spin SU(2) invariance (symmetry class CI) in the presence of nonmagnetic disorder. The conduit for the instability is disorder-induced wave function multifractality. We argue that time-reversal symmetry breaks spontaneously at the surface, so that topologically protected states do not exist for this class. The interaction-stabilized surface phase is expected to exhibit ferromagnetic order, or to reside in an insulating plateau of the spin quantum Hall effect.

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The existence of novel delocalized surface states is a key signature of 3D topological phases of matter [1–3]. These states envelop a fully gapped, yet topologically "twisted" bulk and can display exceptional properties such as the quantized magnetoelectric effect and Majorana fermions [1]. A complete classification [2,3] for (effectively) noninteracting particles has demonstrated that only five classes of topological phases and associated surface states arise in 3D.

An important development [2] has been the incorporation of disorder effects on 2D surface states. This is crucial because the terminating facets of a bulk 3D crystal inevitably host structural defects and impurities. The topologically nontrivial bulk is linked [2] to an effective low-energy surface theory of 2D Dirac fermions, perturbed by random impurity potentials [4,5]. Each of the five classes of 3D topological phases is "protected" from the effects of time-reversal invariant (i.e., nonmagnetic) disorder, in the sense that at least one surface Dirac wave function escapes Anderson localization [2,5].

Unlike uniform plane waves, the extended 2D energy eigenstates enveloping a surface-disordered topological phase exhibit wild spatial amplitude fluctuations. These arise from quantum interference due to multiple impurity scattering, and manifest in the local density of states (LDOS). The pattern of LDOS fluctuations presents an intricate structure, characterized by an infinite set of scaling dimensions associated to interwoven fractal measures of the surface, a feature known as *multifractality* [5]. The evasion of localization in favor of multifractal scaling is rare in 2D, and is a signature of topological protection in the presence of disorder [6].

In this Letter, we show that topological protection can be undermined by interparticle interactions. In particular, we study the *combined* effects of multifractal LDOS fluctuations *and* interactions upon the surface Andreev bound states of 3D topological superconductors. Because the bulk condensate screens the long-ranged Coulomb force, surface quasiparticles interact only via short-ranged potentials. In the clean limit, the vanishing density of states for the 2D Dirac surface band implies that weak short-ranged interactions are irrelevant, i.e., the surface states remain "protected." However, it is known that disorder-induced LDOS multifractality can amplify interaction effects, such as pairing correlations near the superconductor-insulator transition [7]. With physics dominated by its surface, the complete picture of a 3D topological phase must incorporate *both* disorder-induced quantum interference and interactions [8].

Specifically, we demonstrate that arbitrarily weak interactions (consistent with bulk symmetries) destabilize the noninteracting surface states of 3D topological superconductors with spin SU(2) symmetry (class CI [2,3]), in the presence of nonmagnetic disorder. Multifractal LDOS fluctuations enhance the interactions, facilitating the instability. We argue that time-reversal symmetry breaks spontaneously at the surface, so that "protected" surface states *do not exist* in this class. Depending upon the sign of the relevant interaction coupling U [see Fig. 1, Eqs. (10) and (11)], the surface should develop ferromagnetic order, or enter an insulating plateau state of the spin quantum Hall effect [9]. Our result provides the impetus to identify a suitable material for the class CI bulk as an avenue to realize the spin quantum Hall phase. A similar analysis for the 3D topological superconductor class AIII will be published elsewhere [10].

Three of the five 3D topological symmetry classes can be realized as time-reversal invariant superconductors, distinguished by the degree of electronic spin conservation. In a 3D topological superconductor, Cooper pairing leads to a fully gapped quasiparticle band in the bulk, associated to an integer-valued winding number ν [2,3]. The modulus

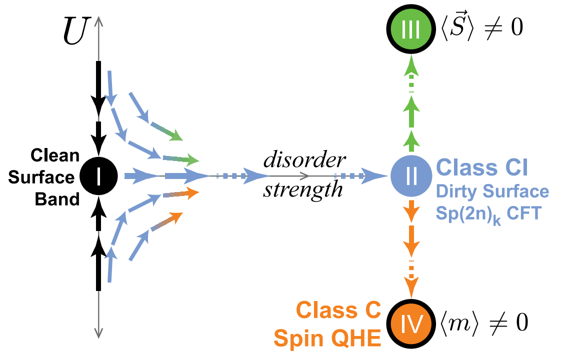

FIG. 1 (color online). Phase portrait sketch for the surface physics of a 3D time-reversal invariant, spin SU(2) symmetric topological superconductor. The vertical axis is the interaction strength U [Eq. (10)], while the horizontal axis measures nonmagnetic disorder. Although the noninteracting system has a disorder-stabilized phase with delocalized ("protected") surface states (II), it is destroyed by arbitrarily weak interactions [Eq. (11)]. Instead, at zero temperature, we expect that the surface exhibits broken spin symmetry [$U > 0 \Rightarrow$ (III)], or the spin quantum Hall effect [$U < 0 \Rightarrow$ (IV)]. In either scenario, interactions break time-reversal symmetry spontaneously.

$|\nu|$ equals the number of flavors (or "valleys") of 2D quasiparticle bands that appear at the sample surface, with energies that infiltrate the bulk gap. In the clean limit, the surface states exhibit a massless Dirac character at low energies; the Dirac point appears precisely at the bulk chemical potential (inside the gap) due to particle-hole symmetry.

In this paper, we study a universal low-energy model for the 2D surface states of a 3D class CI topological superconductor. In contrast to the spin-orbit-coupled $\mathbb{Z}_2$ topological insulators, a CI superconductor has full spin SU(2) symmetry. The nontrivial topology arises through the entwining of orbital degrees of freedom, including non-(simple) s-wave pairing [11,12]. For class CI, ν is even because Dirac surface bands appear in time-reversal conjugate pairs [2,12,13]. We consider the generic case with $|\nu| = 2k$, $k \in \{1, 2, 3, \ldots\}$. Neglecting interactions, the Hamiltonian for the surface theory is

$$H_D = \int d^2\mathbf{r}\, \psi^\dagger \{-\hat{\boldsymbol{\sigma}} \cdot [i\boldsymbol{\nabla} - \mathbf{A}_i(\mathbf{r})\hat{\mathrm{t}}^i_\kappa]\}\psi. \tag{1}$$

The fermion field ψ is a complex Dirac spinor with pseudospin $\sigma \in \{1, 2\}$ and valley $v \in \{1, \ldots, 2k\}$ indices, i.e., $\psi \to \psi_{\sigma,v}$ when all indices are displayed. The pseudospin components $\psi_{1,v}$ and $\psi_{2,v}$ are linear combinations of the Nambu elements $c_{\uparrow,v,\lambda}$ and $c^\dagger_{\downarrow,\bar{v},\lambda'}$. These annihilate (create) spin up (down) electrons in valley v ($\bar{v}$). (Under time reversal, v and $\bar{v}$ interchange.) The indices $\{\lambda, \lambda'\}$ label additional orbital (e.g. sublattice) degrees of freedom, whose precise interpretation depends upon bulk microscopics. A 3D class CI lattice model with $\nu = \pm 2$ appeared in Ref. [12].

For a 3D topological superconductor, a key consequence of the nontrivial bulk is the special form that time-reversal symmetry adopts on the surface. If we write $H_D \equiv \psi^\dagger \hat{h} \psi$, with $\hat{h}$ the single-particle Hamiltonian operator, then the usual time-reversal symmetry for spin-1/2 electrons in the bulk translates into the following *chiral* condition on the surface [2,10,12,13]:

$$-\hat{\sigma}^3 \hat{h} \sigma^3 = \hat{h}. \tag{2}$$

This implies that any surface disorder that does not break time reversal (including nonmagnetic impurities) can manifest only as a random vector potential in the low-energy Dirac description. [Recall that ψ in Eq. (1) carries U(1) spin, rather than electric charge. In this language, vector potentials couple to time-reversal invariant spin and valley currents.] A homogeneous perturbation such as a chemical potential shift, or a time-reversal invariant pairing of the *surface* quasiparticles can be eliminated by a gauge transformation. Moreover, an energy gap (Dirac mass term) cannot appear at the surface of a topological superconductor unless time reversal is broken. For class CI, nonmagnetic disorder induces scattering between the $2k$ valleys in the form of the non-Abelian valley vector potential $\mathbf{A}_i(\mathbf{r})\hat{\mathrm{t}}^i_\kappa$ in Eq. (1). Here $\hat{\mathrm{t}}^i_\kappa$ denotes a $2k \times 2k$ generator of the group Sp($2k$). (The group is symplectic due to the spin symmetry [13].) In the absence of interactions, elastic scattering due to vector potential disorder begets delocalized, multifractal surface states, many properties of which can be computed exactly via conformal field theory (CFT) [4,14,15].

We first consider the effects of disorder upon the noninteracting surface states. Below we describe the physics and main idea of the CFT method. A technical summary can be found in Ref. [13], while a more comprehensive discussion will appear elsewhere [10]. The spatial character of the surface-state wave functions (localized versus extended) can be ascertained via disorder-averaged moments of physical observables, such as the conductance or the LDOS. To facilitate this, we replicate $\psi_{\sigma,v} \to \psi_{\sigma,v,a}$, where the replica index $a \in \{1, \ldots, n\}$, and we are to take $n \to 0$ at the end of the calculation [5]. Symmetry is the primary tool employed in the following, so we will rewrite Eq. (1) in a manifestly symmetric form. We decompose ψ and $\psi^\dagger$ into "left" $\mathcal{L}$ and "right" $\mathcal{R}$ fields,

$$\begin{aligned}\{\mathcal{L}_{\uparrow,v,a}, \mathcal{L}_{\downarrow,v,a}\} &\equiv \{\psi_{1,v,a}, \psi^\dagger_{2,v',a}(\hat{\kappa}^2)_{v',v}\},\\ \{\mathcal{R}_{\uparrow,v,a}, \mathcal{R}_{\downarrow,v,a}\} &\equiv \{\psi_{2,v,a}, \psi^\dagger_{1,v',a}(\hat{\kappa}^2)_{v',v}\}.\end{aligned} \tag{3}$$

Here and below, repeated indices are summed. $\mathcal{L}_{s,v,a}$ denotes a $4nk$-component spinor; the index s (v) transforms in the fundamental representation of the spin SU(2) [valley Sp($2k$)] symmetry. We also define

$$\mathsf{L} \equiv \mathcal{L}^{\mathsf{T}} i\hat{s}^2 \hat{\kappa}^2 \to \mathsf{L}^{s,v}_a, \qquad \mathsf{R} \equiv \mathcal{R}^{\mathsf{T}} i\hat{s}^2 \hat{\kappa}^2 \to \mathsf{R}^{s,v}_a. \tag{4}$$

$\mathsf{L}^{s,v}_a$ and $\mathsf{R}^{s,v}_a$ transform in the conjugate representations of the spin and valley symmetry groups; $\hat{s}^2$ and $\hat{\kappa}^2$ are spin and valley antisymmetric Pauli matrices [16]. Equation (1) can be rewritten as $H_D = H_0 + \delta H_D$, where

$$H_0 = i\int d^2\mathbf{r}[\mathsf{L}\bar{\partial}\mathcal{L} + \mathsf{R}\partial\mathcal{R}], \tag{5}$$

$$\delta H_D = \int d^2\mathbf{r}[J^i_\kappa \bar{A}_i + \bar{J}^i_\kappa A_i]. \tag{6}$$

In Eq. (5), we have switched to complex spatial coordinates $\{z, \bar{z}\} = x \pm iy$, $\{\partial, \bar{\partial}\} \equiv \frac{1}{2}(\partial_x \mp i\partial_y)$. The valley disorder appears in Eq. (6), where $\{A_i, \bar{A}_i\} \equiv -i(A^x_i \mp iA^y_i)$. This couples to the valley Sp$(2k)$ current, which has the holomorphic component $J^i_\kappa \equiv -(i/2)\mathsf{L}\hat{\mathrm{t}}^i_\kappa\mathcal{L}$.

Eq. (5) is manifestly invariant under chiral (independent left and right) spin SU(2), valley Sp$(2k)$, and replica SO(n) transformations. The symmetry group enlarges to SO$(4nk)$ if we include operations that mix all three index types. This free fermion theory is equivalent to the SO$(4nk)_1$ Kac-Moody CFT [17]. The latter has a special property known as a conformal embedding rule [17–19], which gives a decomposition into a "product" of two other CFTs: Sp$(2n)_k$, associated to the (spin) $\times$ (replica) invariance of Eq. (5), and Sp$(2k)_n$, associated to the valley symmetry.

The delocalization physics of the noninteracting surface with Hamiltonian $H_0 + \delta H_D$ is the same as in Refs. [14,15], which dealt with 2D Dirac fermions coupled to a random SU(N) vector potential. Disorder is a relevant perturbation to the clean fermion theory [14]. Crucially, the impurity potential couples only to the valley current J^i_κ in Eq. (6). This leads to a "fractionalization" of the original SO$(4nk)_1$ CFT: the valley Sp$(2k)_n$ sector localizes, leaving behind the "critical" (delocalized) spin-replica Sp$(2n)_k$ sector [14]. The latter is used to compute the scaling behavior disorder-averaged observables. Even in the absence of interactions, disorder localizes all surface states away from zero energy [20]; this is different from the case of a single Dirac fermion on the surface of a 3D topological insulator [21]. However, the localization length diverges upon approaching the chemical potential, so that the zero energy state at the Dirac point remains completely delocalized ("topologically protected").

The disorder-induced spatial fluctuations of the LDOS $\nu(\varepsilon, \mathbf{r})$ are encoded in the *multifractal spectrum* $\tau(q)$ [5,6]. The $\tau(q)$ spectrum measures the sensitivity of extended wave functions to the sample boundary. A large $L \times L$ area of the surface is finely partitioned into a grid of boxes of size $a \ll L$. One then defines the box probability μ_n and inverse participation ratio $\mathcal{P}_q$,

$$\mu_n(\varepsilon) \equiv \int_{\mathcal{A}_n} d^2\mathbf{r}\,\nu(\varepsilon, \mathbf{r}), \qquad \mathcal{P}_q(\varepsilon) \equiv \sum_n \left[\frac{\mu_n(\varepsilon)}{\bar{\nu}}\right]^q, \tag{7}$$

where $\mathcal{A}_n$ denotes the nth box and $\bar{\nu} = \sum_i \mu_i$ is the global DOS. When ε is tuned to a critical delocalization energy (such as a mobility edge), $\mathcal{P}_q \sim (a/L)^{\tau(q)}$, where the exponent $\tau(q)$ is both self-averaging and universal [22]. The multifractal spectrum thus provides a unique fingerprint for spatial fluctuations in a particular symmetry class. In the field-theoretic description, the qth moment of the disorder-averaged LDOS ($q \in 1, 2, 3, \ldots,$) is associated to a particular composite operator $\mathcal{O}_q$, with scaling dimension Δ_q. The set of such dimensions determines the multifractal spectrum via $\tau(q) = 2(q-1) + \Delta_q - q\Delta_1$ [5,6]. By contrast, localized states are insensitive to the sample boundary for sufficiently large L and have $\tau(q) = 0$.

For the class CI surface, we have identified the operators that represent disorder-averaged LDOS moments; these are a subset of the primary fields in the Sp$(2n)_k$ CFT. As a result, we obtain the exact disorder-averaged multifractal spectrum at zero energy [10,13],

$$\tau(q) = (q-1)\left[2 - \frac{q}{2(k+1)}\right]. \tag{8}$$

For $k = 1$, Eq. (8) agrees with previous calculations [15]; the form for general k is new. One of the main results of this paper, Eq. (8), proves that the noninteracting surface states at the bulk chemical potential remain delocalized, a consequence of the bulk topological order.

Now we turn to interparticle interactions. Robust surface states must be protected from the *combined* effects of both disorder and interactions. In a weakly interacting fermion gas, the low-energy behavior of the density of states completely determines the importance of short-ranged interactions. The lowest-order (tree level) renormalization group (RG) equation for a generic four-fermion coupling U is [10]

$$d\ln U/dl = \Delta_1 - \Delta_2^{(U)} + O(U), \tag{9}$$

where l denotes the log of the RG length scale such as the system size. In a *clean* 2D system, $\Delta_2^{(U)} = 2\Delta_1$, with Δ_1 the scaling dimension of the LDOS. For the clean Dirac surface band, $\Delta_1 = 1$, so that weak short-ranged interactions are strongly irrelevant. By contrast, a negative Δ_1 (due, e.g., to a van Hove singularity) would imply that U is relevant, signaling a potential instability. With impurities present, the exponents Δ_1 and $\Delta_2^{(U)}$ denote scaling dimensions of the disorder-averaged LDOS and four-fermion interaction, respectively. The latter satisfies the lower bound $\Delta_2^{(U)} \geq \Delta_2$ [10], where Δ_2 is the dimension of the second LDOS moment that determines $\tau(2)$. The crucial point is that Δ_2 is independent of, and *strictly less than* $2\Delta_1$ for a multifractal delocalized state in a disordered system [23]. Impurity-mediated LDOS fluctuations can therefore amplify short-ranged interaction effects by increasing the overlap of single-particle wave functions in local regions. This is particularly relevant for an interaction U that saturates the bound $\Delta_2^{(U)} = \Delta_2 < 2\Delta_1$.

Physically, we expect that the important interactions include a spin exchange channel (because spin is a conserved hydrodynamic mode) and a Cooper pairing interaction (because disorder respects time reversal). The former is written $-\vec{S}\cdot\vec{S}$, where $\vec{S}$ denotes the spin density. As discussed below Eq. (2), the pairing of the surface quasiparticles does not open a gap unless time reversal is simultaneously broken. The latter occurs when the Dirac mass operator $m \equiv \psi^\dagger \hat{\sigma}^3 \psi$ develops an expectation value. This can be understood explicitly in the 3D CI topological superconductor lattice model of Ref. [12], which features real d-wave pairing in the bulk. In that model, m is interpreted as a sum of pairing operators: $m \sim -ic_\uparrow^\dagger c_\downarrow^\dagger + ic_\downarrow c_\uparrow$, where c_s annihilates a lattice electron. Crucially, m is odd under time reversal, due to the factors of i. Thus, a nonzero expectation $\langle m\rangle \neq 0$ would imply ("$d + is$") pairing of the surface quasiparticles, opening an energy gap and breaking time-reversal symmetry. The resulting state is an insulating plateau of the spin quantum Hall effect (see below). An attractive Cooper pairing interaction can be written as $-m^2$.

To keep the analysis general, we enumerate all four-fermion interactions that preserve the bulk symmetries [time-reversal invariance, spin SU(2), and valley Sp($2k$) symmetry]. This necessitates the incorporation of a third interaction channel $J_S^\gamma \bar{J}_S^\gamma$, where J_S^γ is the holomorphic spin current. The replicated interaction Hamiltonian for the CI surface is [13]

$$H_I = \sum_{a=1}^{n} \int d^2\mathbf{r}\Big[U(m_a m_a - 4\vec{S}_a\cdot\vec{S}_a) + V J_{Sa}^\gamma \bar{J}_{Sa}^\gamma + W\Big(3m_a m_a + 4\vec{S}_a\cdot\vec{S}_a - \frac{1}{k}J_{Sa}^\gamma \bar{J}_{Sa}^\gamma\Big)\Big]. \quad (10)$$

The interaction strengths $\{U, V, W\}$ are defined so as to couple to RG eigenoperators in the presence of disorder. In the minimal two valley realization ($k = 1$), the W-channel interaction does not exist. For that case only, $J_{Sa}^\gamma \bar{J}_{Sa}^\gamma = 3m_a m_a + 4\vec{S}_a\cdot\vec{S}_a$.

Our task is to evaluate Eq. (9) in the disordered, noninteracting CI surface theory for the three interaction operators in Eq. (10) [24]. Using the Sp$(2n)_k$ CFT, we have found that one particular operator controls the scaling of *both* the second LDOS moment and the interaction U, leading to $\Delta_2^{(U)} = \Delta_2 = 0$, while $\Delta_1 = 1/2(k+1)$ [10,13]. The main result of this paper follows:

$$\frac{dU}{dl} = \frac{U}{2(k+1)} + O(U^2), \quad (11)$$

which implies that the interaction U in Eq. (10) grows at longer wavelengths, destabilizing the noninteracting, dirty surface for any number of $2k$ valleys. By contrast, the other interactions V and W remain irrelevant for any k, satisfying $\frac{d\ln V}{dl} = -\frac{4k+3}{2(k+1)}$, $\frac{d\ln W}{dl} = -\frac{3}{2(k+1)}$ [10,13]. We conclude that while weak interactions are suppressed in the clean limit by the vanishing density of states at the Dirac point, surface disorder strongly renormalizes the interaction channel U, making it relevant.

Equation (11) can be understood as an enhancement of interaction matrix elements in the eigenbasis of the disordered theory: local accumulations of the DOS due to wave function multifractality induce stronger interactions between the surface quasiparticles. The amplification of the particular interaction channel U over the others signals the instability of the noninteracting surface to spontaneous time-reversal symmetry breaking. From Eq. (10), we anticipate (at least local) ferromagnetic order $\langle\vec{S}\rangle \neq 0$ when $U \to +\infty$. Without time-reversal symmetry, the surface is not "topologically protected" [1–3], and we expect Anderson localization of all surface states [2,5]. However, we cannot rule out an exotic metallic phase when spin symmetry is also broken [25]. By contrast, $U \to -\infty$ should cause Cooper pairing of the surface quasiparticles. Treating the relevant interaction in mean field theory, one replaces $m^2 \to 2\langle m\rangle \psi^\dagger \hat{\sigma}^3 \psi$ in Eq. (10). A nonzero Dirac mass opens an energy gap, producing an insulating surface. Time-reversal symmetry is broken because $\langle m\rangle \neq 0$ implies surface pairing at a nonzero superfluid phase angle with respect to the bulk.

To lowest order in $(1/k)$, Eq. (11) agrees with a perturbative result [26] obtained using the nonlinear sigma model [5,13]. The calculations in Ref. [26] were performed in the context of a nontopological 2D system of gapless superconductor quasiparticles, subject to disorder and interactions with spin SU(2) symmetry and time-reversal invariance. The Sp$(2n)_k$ CFT employed here has a sigma model representation with the same structure, but augmented with a Wess-Zumino-Witten (WZW) term [17]. In the $k \gg 1$ limit, this model becomes weakly coupled, and the WZW term can be ignored. The results of Ref. [26] therefore provide a nontrivial check of our analysis in the many-valley limit. In addition, at one loop in the sigma model calculation, RG flow equations beyond linear order in the interaction strengths can be obtained because the sigma model treats interactions nonperturbatively via RPA and BCS-type summations. For the 2D class CI quasiparticle system, the sigma model generically predicts an instability of the "metallic" phase signaled by the divergence of the spin exchange or BCS pairing interaction strengths [13,26]. This provides evidence for the absence of an interacting, time-reversal invariant fixed point.

The insulating state that occurs for $\langle m\rangle \neq 0$ preserves spin SU(2) symmetry. This state resides in a plateau of the so-called spin quantum Hall effect [9], analogous to the "half-integer" quantum Hall phase at the surface of a 3D $\mathbb{Z}_2$ topological insulator with broken time-reversal symmetry [1,12]. The quantized spin Hall conductance [9] is $\sigma_{xy}^s = \frac{1}{h}(\frac{\hbar}{2})^2 p$, with $p = k\,\mathrm{sgn}\langle m\rangle$ if valley symmetry is unbroken *on average* (i.e., after disorder averaging). If valley symmetry remains broken even after disorder averaging, then $p \in \{-k, -k+2, \ldots, k-2, k\}$; see also Ref. [9]. Our results are summarized in Fig. 1.

In conclusion, we have demonstrated that interactions destabilize class CI disordered surface states in 3D.

We have argued that time-reversal symmetry breaks spontaneously, and that the CI topological superconductor surface enters into either a ferromagnetic or a spin quantum Hall phase. These are expected to be interaction stabilized Anderson insulators. The other 3D topological superconductor classes AIII and DIII also admit WZW CFT descriptions [2]. The minimal surface state (single Dirac valley) realization for each of these is stable against disorder and short-ranged interactions [2,10]. Results for class AIII with multiple valleys will appear elsewhere [10], while class DIII is an important topic for future work.

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