Tachyon Condensation Due to Domain-Wall Annihilation in Bose-Einstein Condensates

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We show theoretically that a domain-wall annihilation in two-component Bose-Einstein condensates causes tachyon condensation accompanied by spontaneous symmetry breaking in a two-dimensional subspace. Three-dimensional vortex formation from domain-wall annihilations is considered a kink formation in subspace. Numerical experiments reveal that the subspatial dynamics obey the dynamic scaling law of phase-ordering kinetics. This model is experimentally feasible and provides insights into how the extra dimensions influence subspatial phase transition in higher-dimensional space.

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A tachyon is a hypothetical superluminal particle that violates causality in special relativity. Most physicists deny its existence because it is inconsistent with the known laws of physics. However, in quantum field theories, a tachyon *field* can exist due to the instability of quantum vacuum. Here, the "instability" means that the state is at a maximum of an effective potential $V(T)$ for the tachyon field T [Fig. 1(a)]. The tachyon field grows exponentially with time and rolls down toward a minimum of the potential as the true vacuum. This process is called tachyon condensation [1].

Tachyon condensation is a key concept used for describing the dynamics of string theory, a promising candidate for a "theory of everything" that describes all fundamental forces and forms of matter in nature [2]. In this theory, a tachyon exists in a system containing a Dirichlet(D)-brane and an anti-D-brane, where the former is an extended solitonic object [2] and the latter its antiobject. The two annihilate in a collision similarly to particles-antiparticles annihilation in a collision. The annihilation process is described by tachyon condensation, and the system falls into the true vacuum after complete annihilation [1].

A remarkable application of tachyon condensation is in brane cosmology [3–6], in which the big bang is hypothesized to occur as a result of a collision of a brane and an antibrane. After this collision, lower-dimensional branes remain as relics of tachyon condensation [1], which correspond to cosmic strings in brane cosmology [7–9] [see Fig. 1(b)]. This situation resembles conventional phase transitions accompanied by spontaneous symmetry breaking (SSB), resulting in the formation of topological defects via the Kibble-Zurek mechanism [10,11]. This mechanism produces topological defects in the early Universe due to phase transitions [12], which has been tested in several condensed matter systems [13–21]. In contrast, tachyon condensation as a SSB phenomenon has not yet been well understood. Because it may lead to defect nucleation in a restricted lower-dimensional subspace, the dynamics should be affected by the degree of freedom associated with the extra dimension. However, the influence of the extra dimension has never been discussed, partly because such phenomena are absent in actual systems.

Here, we provide a ground-breaking system to tackle this problem, using atomic Bose-Einstein condensates (BECs). Tachyon condensation is simulated by considering vortex formations from a pair annihilation of domain walls, i.e., branes, in binary BECs. This system is advantageous in that we can theoretically and experimentally address the nonlinear dynamics of branes, such as defect nucleation and subsequent dynamics, which is difficult in string theory. Anderson *et al.* [22] observed the creation of vortex rings via the dynamic (snake) instability of a dark soliton in two-component BECs, where the nodal plane of a dark soliton in one component was filled with the other component and then, the filling component was selectively removed using a resonant laser beam. Recently, we proposed that domain walls in phase-separated, two-component BECs correspond to D-branes in the sense that vortex lines (strings) can terminate on them [23]. Therefore, the experiment in Ref. [22] may be interpreted as the demonstration of defect formations via brane annihilation [24], although those phenomena have never been understood as SSB phenomena in a restricted lower-dimensional subspace. In this Letter, we theoretically show that a domain-wall annihilation causes spontaneous Z_2 symmetry breaking and phase-ordering dynamics in the two-dimensional subspace, where a tachyon field is introduced by projecting the original order parameters onto the branes in three-dimensional space. Our theory is justified by demonstrating the scaling law of phase-ordering kinetics [25] in numerical experiments [see Figs. 1(d) and 2(b)]. Although the analogue of the brane annihilation was simulated

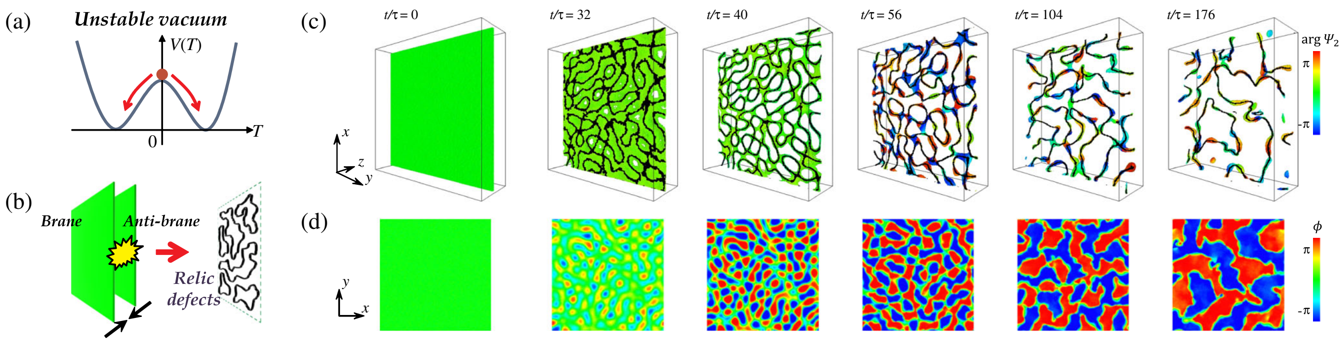

FIG. 1 (color online). (a) Schematic diagrams of tachyon condensation. (b) Schematic diagrams of an annihilation of a brane pair and the relic lower-dimensional objects. (c) Dynamics of a domain wall pair annihilation in a numerical experiment for $\nu = 0.84$ and $\Delta\Theta = \pi$. Panels show the time development of the isosurface $|\Psi_1| = |\Psi_2|$. Black curves represent the cores of vortices in the Ψ_1 component (not shown for $t/\tau = 0$). The phase $\arg\Psi_1$ strongly fluctuates in the region $|\Psi_1| \sim 0$ between the branes, where there are many vortices with little vorticity of mass current ($t/\tau = 32$). The initial fluctuation grows into a meshed structure ($t/\tau = 40$). The structure yields serpentine vortices, which trap the Ψ_2 component along the vortex cores. The Ψ_2 component propagates along the cores and causes varicose oscillations of the isosurface. Reconnections between vortices accidentally create vortex rings or disk-shaped density pulses, which escape to the bulk. (d) Panels show the time development of the projected field $\phi(x, y, t)$ of Eq. (5), calculated from the results of (c). The box sizes are $80\xi \times 80\xi \times 25.6\xi$ for (c) and $102.4\xi \times 102.4\xi$ for (d).

experimentally on the AB phase boundary of superfluid ^{3}He [26], its theoretical explanation remains lacking. This work provides the first theory of brane-annihilation phenomena in condensed matter systems.

We consider two-component BECs, which consist of condensations of two distinguishable Bose particles. Two-component BECs are well described by two complex order parameters, Ψ_j ($j = 1, 2$), in the Gross-Pitaevskii model at zero temperature [27]. The order parameters in a uniform system obey the action $\mathcal{S} = \int dt \int d^3x (i\hbar \sum_j \Psi_j^* \partial_t \Psi_j - \mathcal{K} - \mathcal{V})$ with kinetic energy density $\mathcal{K} = \sum_j \frac{\hbar^2}{2m_j} |\nabla \Psi_j|^2$ and potential energy density $\mathcal{V} = \sum_j (\sum_k \frac{g_{jk}}{2} |\Psi_k|^2 - \mu_j) |\Psi_j|^2$. Here, we have expressed the coupling constant $g_{jk} = 2\pi\hbar^2 a_{jk}/m_{jk}$ ($j, k = 1, 2$) with the reduced mass $m_{jk}^{-1} = m_j^{-1} + m_k^{-1}$ and the s-wave scattering length a_{jk} between atoms in the Ψ_j and Ψ_k components. The chemical potential μ_j is introduced as the Lagrange multiplier for the conservation of the norm $N_j = \int d^3x |\Psi_j|^2 =$ const, which gives the particle number of the Ψ_j component. We consider strongly segregated BECs and set the parameters as $m_1 = m_2 \equiv m$, $g_{11} = g_{22} \equiv g$, $\mu_2/\mu_1 \equiv \nu$, and $g_{12}/g \equiv \gamma = 2$; this parameter setting is experimentally feasible, e.g., Ref. [28]. The time and length scales of our system are characterized by $\tau \equiv \hbar/\mu_1$ and $\xi \equiv \hbar/\sqrt{m\mu_1}$, respectively.

Let us consider the annihilation of a domain wall (brane) at $z = -R/2$ and an antidomain wall (antibrane) at $z = R/2$ perpendicular to the z axis, between which the Ψ_2 component is sandwiched by the two domains occupied with the Ψ_1 component. We define the interbrane distance R as the distance between the two planes defined by $|\Psi_1| = |\Psi_2|$. The distance R increases with N_2, which is controllable experimentally [22]. Because the "penetration" of the amplitude $|\Psi_1|$ ($|\Psi_2|$) decays exponentially with distance into the Ψ_2 (Ψ_1) domain, the short-range interaction between the branes works effectively only when R is comparable to the penetration depth, or the brane thickness, and then the annihilation process can start substantially. The trivial process of pair annihilation is that the branes collide to leave the trivial state $\Psi_1 =$ const. However, the annihilation processes become nontrivial depending on the phase difference $\Delta\Theta \equiv \arg\Psi_1(z \gg R/2) - \arg\Psi_1(z \ll -R/2)$.

Since the essential mechanism of the nontrivial annihilation has been discussed partly in Ref. [24], we explain it briefly here [29]. To capture the essence of the annihilation process, we assume that two branes with large $R (\gg \xi)$ are brought rapidly to a small distance $R \lesssim \xi$, but the two Ψ_1 domains are disconnected at $t = 0$ [30]. In the annihilation process, junctions connecting the two Ψ_1 domains emerge in various places on the x-y plane around $z = 0$. For $\Delta\Theta \neq 0$, a junction causes a superfluid current along the z axis with a current velocity $v_\perp \sim \frac{\hbar}{m}\Delta\Theta/R > 0$ or $v_\perp \sim \frac{\hbar}{m}(\Delta\Theta - 2\pi)/R < 0$. If the current velocities through the two neighboring junctions are parallel, the annihilation is completed between the junctions. On the other hand, the two junctions with opposite velocities leave a single-quantum vortex in the Ψ_1 component, where the Ψ_2 component is trapped in the vortex cores so that N_1 and N_2 are conserved.

Although the growth rates of junctions with $v_\perp > 0$ and $v_\perp < 0$ are generally different, they are statistically equivalent for $\Delta\Theta = \pi$. Then the junctions grow in a random fashion from initial random fluctuations, and vortices emerge as serpentine curves along the boundary between the two opposite junctions. These scenarios are demonstrated numerically as shown in Fig. 1(c) [29]. The snake instability observed by Anderson *et al.* corresponds to the coincident limit of the two branes ($R \to 0$) with $\Delta\Theta = \pi$, where the vortex ring nucleation results from the spherical geometry of the external potential [22]. More generally, we can regard the interbrane distance R and the phase difference $\Delta\Theta$ as two parameters to characterize the dynamics of the brane annihilation.

To describe the annihilation process systematically, we construct an effective field theory parametrized with R and $\Delta\Theta$. Tachyon condensation in string theory is explained by introducing a growing field, that is, the tachyon field, in the lower-dimensional space spanned by the coordinates along the branes [1]. In a similar manner, we consider an effective tachyon field $T(x, y, t)$ as a real scalar field in a two-dimensional space parametrized by x and y. On the basis of the vortex formation mechanism explained above, we introduce the variational ansatz,

$$\Psi_1 = \sqrt{\mu_1/g}\left[\sqrt{\tanh^2(z/\xi_\perp)e^{\pm i\Delta\Theta}} + T\mathrm{sech}^2(z/\xi_\perp)\right], \quad (1)$$

$$\Psi_2 = \sqrt{n_2(T)}\mathrm{sech}(z/\xi_\perp), \quad (2)$$

where $T(t=0) = 0$ and the sign $\pm$ in Eq. (1) takes $+$ ($-$) sign for $z < 0$ ($z > 0$) [31]. This ansatz describes the annihilation starting from a small interbrane distance, where the population of the Ψ_2 component is small between the branes at $t = 0$. The parameters $n_2^0 \equiv n_2(t=0)(\geq 0)$ and $\xi_\perp$ are determined to minimize the energy $\int d^3x(\mathcal{K} + \mathcal{V})$ at $t = 0$. The ansatz at $t = 0$ reasonably reduces to the solution of a dark soliton with $\xi_\perp \to \xi$ and $R \to 0$ for $n_2^0 \to 0$ and $\Delta\Theta = \pi$. The growth of the tachyon field from $T = 0$ into $T > 0$ ($T < 0$) induces a superflow with $v_\perp > 0$ ($v_\perp < 0$). Although the dynamics may be described more precisely with additional variational parameters, our simplest ansatz is enough to capture the essence of the vortex formation dynamics.

The effective potential for the tachyon field T, namely the tachyon potential V, is defined as a function of T, $V(T) = \int_{-\infty}^{+\infty} dz(\mathcal{K} + \mathcal{V})$ with $\partial_x\Psi_j = \partial_y\Psi_j = 0$. We assume that the density parameter $n_2(T) \geq 0$ is determined so as to minimize V. It is straightforward to obtain the form

$$V = \frac{\mu_1^2\xi_\perp}{g}\sum_{n=0}^{4} F_n T^n, \quad (3)$$

where $F_n = A_n(\Delta\Theta, \nu, \gamma) + B_n(\Delta\Theta, \nu, \gamma)\theta(n_2)$ with a step function, $\theta(n_2 > 0) = 1$ and $\theta(n_2 \leq 0) = 0$. The contribution from the gradient $\nabla_\parallel \equiv (\partial_x, \partial_y)$ along the coordinates parallel to the brane is calculated similarly. The original energy is then reduced to the form

$$E_{\mathrm{2D}} = \int dxdy[G(T)(\xi\nabla_\parallel T)^2 + V(T)]. \quad (4)$$

Here, the coefficient $G(T) > 0$ in the gradient term depends on T (see Ref. [29] for details).

Equation (4) represents the effective energy for the field T in the projected two-dimensional system. The potential V is symmetric $V(-T) = V(T)$ for $\Delta\Theta = \pi$, because the coefficients F_1 and F_3 are proportional to $\cos(\Delta\Theta/2)$. The Z_2 symmetry comes from the degeneracy of the kinetic energy $\propto v_\perp^2 \sim (\hbar\pi/mR)^2$ for $\pm T$ owing to a symmetry in the initial configuration; $\mathrm{Re}\Psi_1(t=0) = -\mathrm{Re}\Psi_1(t=0)$ with $\Delta\Theta = \pi$. In terms of relativistic quantum field theory, we find a particlelike state with a mass $m_T \equiv \sqrt{V''(T=0)/2} = \mu_1\sqrt{\xi_\perp F_2/g}$ by expanding the potential V around $T = 0$. For $F_2 > 0$, this describes a conventional particle: however, for $F_2 < 0$, this describes a particle with purely imaginary mass $m_T^2 < 0$, i.e., a tachyon. The existence of a tachyon implies instability of the system, because a tachyon rolls down from the potential maximum at $T = 0$ toward a potential minimum with $T > 0$ or $T < 0$ [32]. Figure 2(a) shows the tachyon potential $V(T)$ for $\Delta\Theta = \pi$ and $\gamma = 2$. The coefficient F_2 increases with $\nu = \mu_2/\mu_1$, and the tachyon potential is convex upward around $T = 0$ for $\nu \leq 1$. Because R is an increasing function of ν, the instability becomes stronger as the interbrane distance R decreases.

The tachyon field T is an analogue of the order-parameter field, e.g., the magnetization density, of a ferromagnetic system [33] in a continuum description. The rolling tachyon corresponds to the spontaneous Z_2 symmetry breaking from the zero magnetization $T = 0$ toward a macroscopic magnetization $T > 0$ or $T < 0$ in the ordered phase for $\Delta\Theta = \pi$ [34]. In this sense, the interbrane distance R and the phase difference $\Delta\Theta$ ($\neq \pi$) play the roles of the temperature and external magnetic field, respectively. The coefficient F_2 increases with the temperature R, implying that the instability becomes weak for a small interbrane interaction for large R. Because the interbrane interaction decays exponentially with R for large distance and the instability vanishes precisely for $R \to \infty$, the infinity distance may correspond to the transition temperature. Although F_2 reached zero for finite R in our effective model, the tachyon potential $V(T)$ well reflects the nature of the instability for small R. On the other hand, the field T feels the magnetic field for $0 \leq \Delta\Theta < \pi$ ($\pi < \Delta\Theta \leq 2\pi$), and then, magnetization $T > 0$ ($T < 0$)

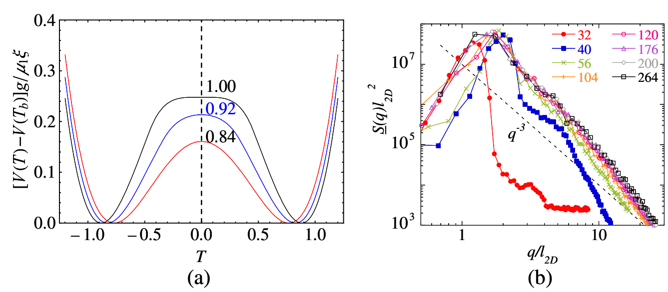

FIG. 2 (color online). (a) Plots of the tachyon potential with $\Delta\Theta = \pi$ for $\nu = 0.84$, 0.92, and 1.00. The potential $V(T)$ takes its minimum value $V(T_b)$ at $T = \pm T_b$. (b) Structure factor of ϕ in the numerical experiment [Fig. 1(d)]. The factor $\underline{S}(q, t)$ is the average of $S(\boldsymbol{q}, t)$ over directions of $\boldsymbol{q}$. The dashed line is the expected law, $S(q) \propto q^{-3}$ for $q/l_{\mathrm{2D}} \gg 1$, which is not observed in the small system.

is energetically favorable. In the original three-dimensional system, this is energetically explained from the difference in the kinetic energy induced by the rolling tachyon, $\propto v_\perp^2 \sim (\frac{\hbar\Delta\Theta}{mR})^2$ for $T > 0$ and $\propto v_\perp^2 \sim [\frac{\hbar(\Delta\Theta - 2\pi)}{mR}]^2$ for $T < 0$.

A vortex between junctions with $v_\perp > 0$ and $v_\perp < 0$ is considered a kink between the regions with $T > 0$ and $T < 0$ in the projected two-dimensional space. The validity of the effective theory is confirmed by evaluating the defect nucleation rate. The effective theory neglects the transfer of particles in the x and y directions in the defect formation process, violating the law of particle number conservation. However, the violation influences little on the rate evaluation for small R. According to the evaluations with several analytical and numerical methods in Ref. [29], the rate consistently decreases with R.

Finally, we provide the most important evidence showing that the field T behaves actually as a two-dimensional order parameter. According to the scaling law in the phase-ordering kinetics [25], the spatial structure of the order parameter is characterized by a single length scale, i.e., the mean interdefect distance, after a rapid quench from the disordered phase into the ordered phase. In the simulations of the three-dimensional system, the density of kinks in the field T is calculated from the line density l_{2D} of the projection of vortices onto the x-y plane by assuming that there is no overlap between the projection lines of different vortices. To visualize the dynamics of T, we introduce the projected field

$$\phi(x, y, t) \equiv \frac{m}{\hbar} \int_{-\infty}^{+\infty} dz v_\perp(\mathbf{r}, t), \tag{5}$$

with the superfluid current velocity $v_\perp = \frac{\hbar \sum_j |\Psi_j|^2 \partial_z \arg \Psi_j}{m \sum_k |\Psi_k|^2}$. Far from kinks in the projected two-dimensional space, we have $\Psi_2 \sim 0$ and then both fields ϕ and T are constant, $\phi(x, y) \sim \pm\pi$ and $T(x, y) = \pm T_b$, where $V(T)$ takes its minimum value at $T = \pm T_b$. The nonzero Ψ_2 component around the kink cores could yield different spatial dependences of the two fields. Thus, a spatial structure of the projected field $\phi(x, y, t)$ represents that of $T(x, y, t)$ on a length scale larger than the kink width ζ, which is sufficient for the following analysis.

If the field ϕ obey the scaling law, its structure factor $S(\boldsymbol{q}, t)$ for the wave number $|\boldsymbol{q}| \ll 1/\zeta$ is written with a time-independent function $\mathcal{F}$ as $\mathcal{F}(q/l_{2D}) = S(\boldsymbol{q}, t) l_{2D}^2$. Here, the scaling form is expected to follow the universal law $S(\boldsymbol{q}, t) \sim (l_{2D}/q)^{d+1}/l_{2D}^d$, known as the Porod law, with the spatial dimension $d = 2$ for $q/l_{2D} \gg 1$ [25].

Figure 2(b) shows scaling plots of $S(\boldsymbol{q}, t)$ for the numerical experiment of Fig. 1(c). The dynamics of ϕ in Fig. 1(d) resembles ferromagnetic relaxations after rapid quenching. In fact, the scaling plots almost coincide with each other after the domain structures of ϕ become clear. The scaling behavior is seen from the similarity between the patterns of $t/\tau = 104$ and $t/\tau = 176$ in Fig. 1(d). These facts show the domain-wall annihilation is regarded as phase ordering in the projected two-dimensional space.

The time dependence of l_{2D} contains a statistical information. The density l_{2D} follows a power law, $1/l_{2D} \propto t^{1/2}$ [29], which indicates that the projected two-dimensional system is dissipative. The dissipation may come from some degrees of freedom, neglected in the effective theory, such as the motions of the Ψ_2 component along the vortex core, emissions of vortex rings, and density pulses to the extra dimension induced by vortex reconnection [see Fig. 1(c) and the caption]. It is interesting that strings stretched between the brane and the antibrane cause a complex-scalar tachyon field in string theory [1], while the real-scalar field comes from fluctuating fields in the wall-antiwall background in our system. If vortices are stretched between the domain walls, a "vorton," a pointlike defect in three dimensions, can be nucleated due to superflow of the Ψ_2 component along the vortex core [35]. These issues will be discussed elsewhere.

Our proposal gives the first realistic example of nonrelativistic tachyon condensation due to the brane annihilation phenomena. Because the instability is essentially caused by the spatially inhomogeneous junctions between two domains described with the same order parameter, the phenomenon may be generalized to a problem of a sudden connection between two media in the same ordered phase. Therefore, it is expected that our theory can be extended to various condensed matter systems. The ^{3}He-brane experiment [26] should be retested from the viewpoint proposed in this work. Because techniques for detecting the vortex line density in the ^{3}He system are well developed, some statistical information about the brane annihilation could be observed from the power-law decay of the defect density.

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[29] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.109.245301 for supplemental data and discussions.
[30] This assumption is intended to simulate the phase-ordering dynamics after a rapid quench in the two-dimensional subspace described later.
[31] To focus our discussion mainly on similar situations in the experiment [22], we used this ansatz with a discontinuous derivative $\partial_z \Psi_1(t=0)$ at $z=0$, which is realized as a solution under the boundary condition $\Psi_1(z=0)=0$.
[32] The squared mass becomes positive in the stable vacuum obtained after the instability. In the effective theory for $\Delta\Theta=\pi$, the stable vacuum corresponds to a minimum, $T=T_b$ or $T=-T_b$, as seen in Fig. 2(a).
[33] See, for example, L. D. Landau and E. M. Lifshitz, *Statistical Physics: Course of Theoretical Physics* (Pergamon, New York, 1980), 3rd ed., Vol. 5, Part 1, Chap. 14.
[34] The effective energy E_{2D} in the projected two-dimensional system is invariance under inversion $T=-T$ for $\Delta\Theta=\pi$. The unstable vacuum (the initial state at $t=0$) with zero magnetization ($T=0$) is invariant under the transformation $T=-T$. This Z_2 symmetry is spontaneously broken in the stable vacuum ($T=T_b$ or $T=-T_b$) of the effective theory.
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