Can a Metal Surface Repel Electric Charges?

Primož Rebernik Ribič
Faculté des Sciences de Base, Ecole Polytechnique Fédérale de Lausanne (EPFL), 1015 Lausanne, Switzerland
(Received 7 August 2012; published 10 December 2012)

We show that the interaction between a surface and a charge packet moving parallel to it can become repulsive above a critical relativistic energy. We find that this is true for a lossless dielectric surface and also for a Drude metallic surface—in apparent contrast with such common notions as image charge. This counterintuitive phenomenon occurs for packets larger in the transverse than in the longitudinal (parallel to the motion) direction. The repulsion does not occur for a point charge that is instead attracted at all energies. In addition to the above attractive or repulsive transverse force, there is a longitudinal decelerating force, which for a dielectric corresponds to the Čerenkov effect. Once again, the behavior of a line packet differs from that of a point charge: for a packet with infinite transverse size, the decelerating field decreases to zero as the relativistic factor $\gamma \to \infty$, whereas, for a point charge, the asymptotic value is finite. These findings have a potential impact not only on fundamental electrodynamics but also on accelerator physics and electron spectroscopy.

DOI: 10.1103/PhysRevLett.109.244801 PACS numbers: 41.75.−i, 41.20.−q, 41.60.−m

The interaction between still or moving charges and solid surfaces was extensively studied for point charges and for relativistic bunches elongated in the longitudinal direction, parallel to the motion. Not much is known about transverse bunches: we discovered that their properties are rather counterintuitive and depart from basic notions of elementary electromagnetism.

For point charges, one can refer to the classic article of Bolotovskii [1] on the rigorous theory of Čerenkov radiation in finite-size media. A particle moving in vacuum at speed v parallel to the surface of a lossless dielectric experiences a decelerating longitudinal force, which for $v = c$ has magnitude equal to twice the static (transverse) image charge force produced by a metal surface [2]. The moving particle also experiences a transverse attractive force for all speeds of magnitude similar to the decelerating force. Specific nonrelativistic cases were later studied because of their importance for electron spectroscopy [3–8].

De Zutter and De Vleeschauwer [9] and Schieber and Schächter [10–12] expanded the analysis of relativistic behavior for point charges near dielectric and metal surfaces. It was specifically shown [10] that the transverse force decreases with the relativistic gamma factor as $1/\gamma$. Accelerator physics stimulated the analysis of longitudinal relativistic packets, e.g., the issue of wakefield acceleration [13–16]. So far, however, the analysis of transverse packets was rather limited [11].

We found that transverse packets are very interesting and lead to the unexpected and counterintuitive result of repulsive transverse forces at high energies—both for dielectric and metal surfaces. This discovery is illustrated in Fig. 1. The center of a charge bunch, with dimensions L_b, W_b, and H_b in its own reference frame and a total charge q, moves in vacuum ($z > 0$) at a distance z_0 from the flat surface of a solid with a dielectric function $\epsilon(\omega)$. The bunch velocity, of magnitude v, corresponding to $\gamma = (1 - v^2/c^2)^{-1/2}$, is parallel to the surface (x axis).

The evaluation of the electromagnetic field created by the packet is based on Maxwell's equations: the treatment is rather tedious and can be found in more detail in the Supplemental Material [17]. The fields are written as Fourier series, i.e., as sums of electromagnetic waves with amplitudes and phases depending on the $\mathbf{k}$-vector components.

We start with the simple case of a lossless dielectric with $\epsilon(\omega) = \epsilon = \text{const}$. The transverse $\mathbf{k}$-vector components in vacuum (1) and in the dielectric (2) are

$$\begin{aligned} k_{z1}(k, \psi) &= ik\sqrt{\frac{\cos^2\psi}{\gamma^2} + \sin^2\psi}, \\ k_{z2}(k, \psi) &= ik\sqrt{(1 - \beta^2\epsilon)\cos^2\psi + \sin^2\psi}, \end{aligned} \tag{1}$$

where $\beta = v/c$, $k = \sqrt{k_x^2 + k_y^2}$, and $\psi = \arctan(k_y/k_x)$; here, k_x and k_y are the $\mathbf{k}$-vector components parallel to the surface. From Eq. (1), the waves in vacuum are always evanescent; i.e., k_{z1} is imaginary: a charge bunch moving with a constant velocity parallel to a flat vacuum-dielectric interface cannot emit radiation in vacuum.

However, it can emit propagating Čerenkov waves in the dielectric. When the Čerenkov condition $\beta > 1/\sqrt{\epsilon}$ [or $\gamma > \sqrt{\epsilon/(\epsilon - 1)}$] is satisfied, k_{z2} can be either real or imaginary. It is real if the quantity under the square root in Eq. (1) is negative, leading to the well-known Čerenkov cone:

$$\cos\psi_c = \frac{1}{\beta\sqrt{\epsilon}}. \tag{2}$$

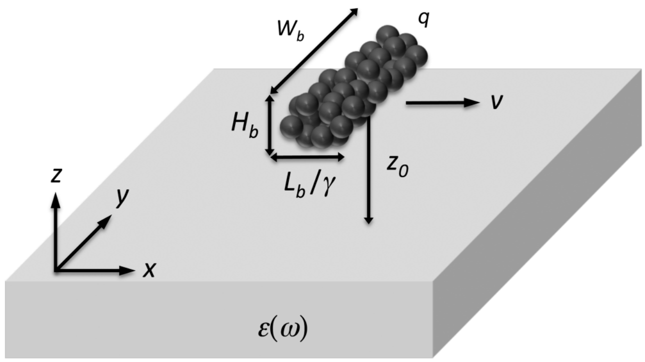

FIG. 1. The geometry of our analysis for a charge packet moving parallel to the surface of a solid.

Below the critical angle, ψ_c, the Čerenkov radiation propagates in the dielectric away from the interface. Above the critical angle, k_{z2} is imaginary and the waves in the dielectric decay with the distance from the interface. If $\beta < 1/\sqrt{\epsilon}$, k_{z2} is imaginary for all angles and there are only evanescent waves.

Based on the above discussion, we can consider two contributions to the total interaction between the bunch and the dielectric: one from evanescent waves and the other from propagating (Čerenkov) waves. We calculated the electromagnetic field in the center of the packet using the charge-normalized Lorentz force

$$\mathbf{F} = (\mathbf{E} + \mathbf{v} \times \mathbf{B})_{(x=vt, y=0, z=z_0)}, \quad (3)$$

where z_0 is the distance from the interface to the bunch center.

Within the packet, we assume a homogeneous charge distribution. For a line charge with infinite transverse size, the field defined above is proportional to the force per unit length. For a finite-size line charge, the calculated field reflects the force applied to the center; treating the detailed force for each part of the line is beyond the scope of this work, although the corresponding complications do not affect our qualitative conclusions.

The calculations concern the x (longitudinal) and transverse-z $\mathbf{F}$ field components. The transverse-y component is zero due to symmetry.

First, we show the results for a point particle (i.e., $z_0 \gg L_b, W_b, H_b$); we selected $\epsilon = 3$. Fig. 2 (top) presents the transverse field as a function of energy. The result is normalized with respect to the absolute value of the static field ($\gamma = 1$), which equals $[(\epsilon - 1)/(\epsilon + 1)] \times [q/(16\pi\epsilon_0 z_0^2)]$, where q is the particle charge. This case was already studied in Ref. [10]; however, since the results provide the background for the rest of our analysis, we summarize here the main findings.

The force contribution from evanescent waves in the dielectric is attractive (negative) for all energies. The contribution from Čerenkov waves is of course zero for $\gamma < \sqrt{\epsilon/(\epsilon - 1)}$. For $\gamma > \sqrt{\epsilon/(\epsilon - 1)}$, the figure shows that the Čerenkov-related force contribution is not always attractive but becomes repulsive above $\gamma \sim 4.5$.

Schieber and Schächter [10] used the language of quantum mechanics to explain this point. Čerenkov photons moving away from the interface carry a certain transverse momentum. For low energies, the dielectric can accommodate this momentum, whereas, for high energies, the

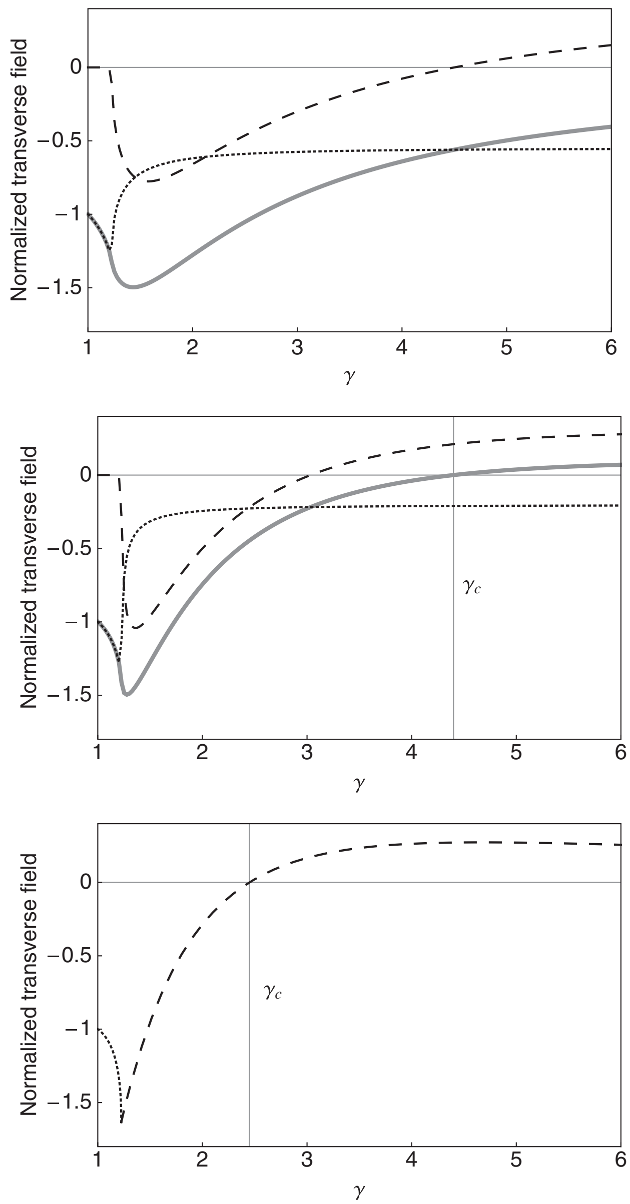

FIG. 2. Transverse field for a point charge (top), for a line charge with $W_b = 10z_0$ (middle), and for a line charge with infinite transverse size (bottom) moving parallel to the surface of a dielectric with $\epsilon = 3$. The interaction is divided into the Čerenkov (dashed line) and evanescent (dotted line) contributions. The thick solid gray line in the top and middle panels is the total interaction. For the line charge with infinite transverse size, the two contributions occur in different energy ranges. γ_c marks the transition from a total attractive to a total repulsive interaction.

particle has to balance it, which gives a repulsive interaction.

At low energies, the total transverse force is dominated by the attractive component and reaches a maximum above the Čerenkov condition. For high energies, the force decreases and approaches zero as $q/(16\pi\epsilon_0 z_0^2\gamma)$, independent of ϵ. Although the Čerenkov contribution becomes repulsive, the total force remains attractive at all energies.

Are these results for point charges also valid for charge distributions? This is our discovery: there are qualitative differences, and the total force can be made repulsive by manipulating the geometry of the charge packet. Equation (1) hints how: repulsive Čerenkov interaction comes from propagating waves with $\psi < \psi_c$. Above ψ_c, the waves are evanescent and contribute to the attractive interaction. If one could suppress the evanescent waves, this would decrease the attractive interaction, even to the point of producing a repulsive total interaction.

How can this be accomplished? The Fourier transform of the charge density determines the wave-vector range (see the Supplemental Material [17] for details). From the Fourier transform properties, we can estimate this range simply as $\Delta k_x \sim \gamma/L_b$ and $\Delta k_y \sim 1/W_b$ (the γ factor is present due to length contraction of the bunch along the x axis in the laboratory frame). The maximum wave-vector angle is therefore $\arctan[L_b/(\gamma W_b)]$. By increasing the ratio of the transverse to the longitudinal size of the bunch, we can suppress large-angle contributions for all γ values. The range of angles is reduced to zero when the point charge is replaced with a line charge of infinite transverse size ($W_b \to \infty$).

This formal explanation has a simple physical ground: the waves at oblique-incidence angles destructively interfere due to the symmetry of the problem. The interference is constructive only in the forward and backward directions, which, above the Čerenkov condition, give rise only to the Čerenkov part of the interaction.

This point is shown in Fig. 2 (bottom), where we plot the transverse field for a line charge with infinite transverse size moving parallel to a dielectric with $\epsilon = 3$. The analytical solution for this case is

$$F_z = \begin{cases} \frac{\rho_l}{4\pi\epsilon_0 z_0}\frac{\gamma^2(\epsilon-1)-\epsilon(\epsilon+1)+2\epsilon\sqrt{\epsilon-\gamma^2(\epsilon-1)}}{\gamma(\gamma^2+\epsilon)(\epsilon-1)} & \text{for } \gamma < \sqrt{\frac{\epsilon}{\epsilon-1}} \\ \frac{\rho_l}{4\pi\epsilon_0 z_0}\frac{\gamma^2(\epsilon-1)-\epsilon(\epsilon+1)}{\gamma(\gamma^2+\epsilon)(\epsilon-1)} & \text{for } \gamma > \sqrt{\frac{\epsilon}{\epsilon-1}} \end{cases} \tag{4}$$

$$F_x = \begin{cases} 0 & \text{for } \gamma < \sqrt{\frac{\epsilon}{\epsilon-1}} \\ -\frac{\rho_l}{2\pi\epsilon_0 z_0}\frac{\epsilon\sqrt{\gamma^2(\epsilon-1)-\epsilon}}{(\gamma^2+\epsilon)(\epsilon-1)} & \text{for } \gamma > \sqrt{\frac{\epsilon}{\epsilon-1}}, \end{cases} \tag{5}$$

where ρ_l is the linear charge density. By reducing the availability of large-angle wave vectors, one can completely eliminate the evanescent attractive interaction at energies above the Čerenkov condition. The interaction becomes repulsive for energies above a critical value given by $\gamma_c = \sqrt{\epsilon(\epsilon+1)/(\epsilon-1)}$. The repulsive force increases with energy, goes through a maximum, and then decreases as $1/\gamma$ for high energies, independent of ϵ.

The above picture is strictly valid if $W_b \gg z_0$. Finite-size effects are shown in Fig. 2 (middle), reporting the field for $W_b = 10z_0$. The evanescent attractive interaction is not zero above the Čerenkov condition as for an infinite line; however, it is weaker than for a point charge, and the repulsive Čerenkov interaction eventually prevails. The critical energy increases with respect to the infinite line charge. We also performed calculations for additional cases of packets with nonzero length and height. The fields show rather similar trends, provided that H_b, $L_b < W_b$; even if L_b is comparable to W_b, the evanescent waves are suppressed due to length contraction and the total interaction can still become repulsive.

The effect of a finite bunch size is further illustrated in Fig. 3, where we plot the critical energy as a function of the z_0/W_b ratio. For low ratios, the critical energy does not change significantly. When W_b becomes comparable to z_0, the critical energy strongly increases. This is reasonable since the dielectric "sees" the bunch as a point particle for high z_0/W_b.

At this point, we must raise a fundamental question. Are the above effects caused only by the packet geometry, or are they also influenced by the nature of the solid? Specifically, can they be extended from a lossless dielectric to a metal? To address this issue, we calculated the field for a point charge and a line charge with infinite transverse size moving parallel to the surface of a Drude metal, Fig. 4, with values of the plasma frequency ω_p and the damping coefficient γ_0 corresponding to copper.

For the point charge, the field magnitude decreases as the energy increases; it specifically goes as $1/\gamma$ for low and high energies and deviates slightly for intermediate energies. Numerical analysis shows that (in the range of interest $z_0 \sim 1$ μm–1 mm and in the vicinity of ω_p) the deviations

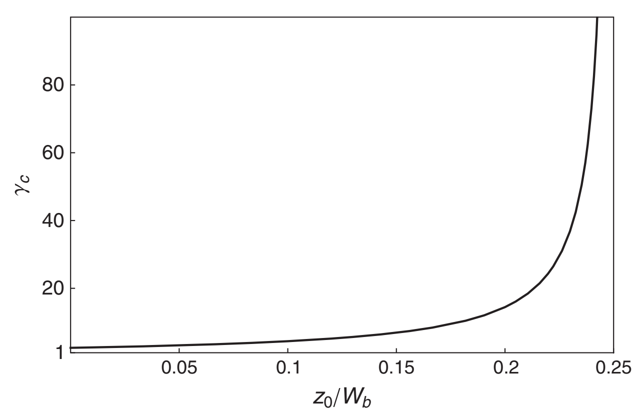

FIG. 3. Critical energy, above which the interaction of a transverse line charge with the surface of a dielectric ($\epsilon = 3$) becomes repulsive, plotted as a function of the z_0/W_b ratio.

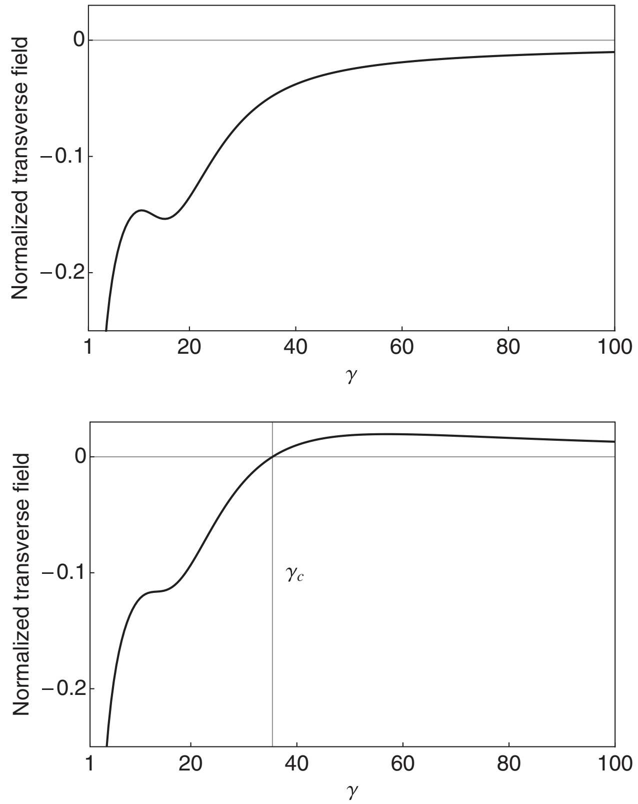

FIG. 4. Transverse field for a point charge (top) and for a line charge with infinite transverse size (bottom) moving parallel to the surface of a metal (with copper parameters) at a distance $z_0 = 10\ \mu\text{m}$. The field magnitude deviates from the $1/\gamma$ behavior at $\sim\sqrt{\omega_p z_0/c}$. The critical energy for the line charge is proportional to $\sqrt{\omega_p z_0/c}$.

occur close to $\sqrt{\omega_p z_0/c}$. Such deviations can be understood in the following way. The maximum frequency excited in the metal is approximately $\omega_{\max} = \gamma v/z_0$ (since for a relativistic point charge the electric and magnetic fields are non-negligible only in a cone defined by the angle $1/\gamma$). For small γ values, only low frequencies are excited and the material behaves similarly to a dielectric with $\epsilon \to \infty$. The field, therefore, is given by the (relativistic) image charge model, as $q/(16\pi\epsilon_0 z_0^2 \gamma)$. When γ increases, higher frequencies are excited in the metal and the real part (absolute value) of its dielectric function decreases. The metal starts behaving like a dielectric with finite ϵ, and the field deviates from the $1/\gamma$ behavior. As γ increases further, the image charge model clearly does not apply anymore; however, the field again exhibits the $1/\gamma$ trend. This is not surprising since this behavior is also observed for a dielectric as $\gamma \to \infty$, independent of ϵ.

The field magnitude of Fig. 4 for a line charge with infinite transverse size exhibits the above qualitative behavior only for low energies. At high energies, the interaction becomes repulsive, as for a dielectric. The critical transition energy is proportional to $\sqrt{\omega_p z_0/c}$. This demonstrates that the repulsive interaction is indeed caused by geometry and that it also occurs for metals.

Note that the transition from a total attractive to a total repulsive interaction would not occur for a metal if the real part (absolute value) of its dielectric function would not decrease with frequency. The transverse field for this particular case can be calculated from Eq. (4) by letting $\epsilon \to \infty$. This reproduces the interaction obtained with the (relativistic) image charge method, which is always attractive. In essence, using the image charge method is wrong since it unrealistically neglects the frequency dependence of the dielectric function.

What about the decelerating longitudinal field? We have seen [2] that for a point charge it asymptotically approaches a finite value whether the material is a dielectric or a metal, and that for a dielectric this value is twice the static image charge force, $-2q/(16\pi\epsilon_0 z_0^2)$. Equation (5) shows that, for a line charge, as $\gamma \to \infty$, the longitudinal field actually decreases as $1/\gamma$. For a metal surface, we found that the longitudinal field also asymptotically approaches zero.

These results could be interesting, e.g., for particle accelerators. When the charge bunch gets close to a surface, it produces longitudinal and transverse wakefields that can severely affect the beam properties [18]. This phenomenon persists at high energies. Our findings suggest that a manipulation of the packet geometry could alleviate this undesired effect.

In general terms, our results indicate that the largely neglected effects of the charge distribution geometry merit a more extensive and detailed analysis. This is true not only for fundamental reasons but also for the possible practical consequences.

I would like to thank G. Margaritondo for a critical reading of the manuscript. I appreciate advice from D. Djokić on how to avoid some problems in the mathematical derivation. I would also like to thank D. Veberič for pointing me in the right direction on how to optimize the numerical integration. Interesting discussions with L. Schächter are gratefully acknowledged. The research is supported by the Fonds National Suisse (FNS) de la Recherche Scientifique and by the CIBM.

[1] B. M. Bolotovskii, Sov. Phys. Usp. 4, 781 (1962). This paper is a review of the theory of Čerenkov radiation in bounded media; the reader should consult the exhaustive list of references herein.
[2] A. I. Morozov, Sov. Phys. JETP 5, 1028 (1957).
[3] N. Takimoto, Phys. Rev. 146, 366 (1966).
[4] D. L. Mills, Phys. Rev. B 15, 763 (1977).
[5] J. P. Muscat and D. M. Newns, Surf. Sci. 64, 641 (1977).
[6] N. Barberán, P. M. Echenique, and J. Viñas, J. Phys. C 12, L111 (1979).

[7] J. Mahanty and P. Summerside, J. Phys. F **10**, 1013 (1980).
[8] D. L. Mills, Solid State Commun. **84**, 151 (1992).
[9] D. De Zutter and D. De Vleeschauwer, J. Appl. Phys. **59**, 4146 (1986).
[10] D. Schieber and L. Schächter, Phys. Rev. E **57**, 6008 (1998).
[11] L. Schächter and D. Schieber, Nucl. Instrum. Methods Phys. Res., Sect. A **440**, 1 (2000).
[12] L. Schächter and D. Schieber, Phys. Lett. A **293**, 17 (2002).
[13] K. L. F. Bane, P. Chen, and P. B. Wilson, IEEE Trans. Nucl. Sci. **32**, 3524 (1985).
[14] M. Rosing and W. Gai, Phys. Rev. D **42**, 1829 (1990).
[15] S. Y. Park and J. L. Hirshfield, Phys. Rev. E **62**, 1266 (2000).
[16] I. N. Onishchenko, D. Y. Sidorenko, and G. V. Sotnikov, Phys. Rev. E **65**, 066501 (2002).
[17] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.109.244801 for a derivation of the electromagnetic fields from Maxwell's equations.
[18] A. W. Chao, *Physics of Collective Beam Instabilities in High Energy Accelerators* (Wiley, New York, 1993).