No-Go Theorems for Generalized Chameleon Field Theories

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The chameleon, or generalizations thereof, is a light scalar that couples to matter with gravitational strength, but whose manifestation depends on the ambient matter density. A key feature is that the screening mechanism suppressing its effects in high-density environments is determined by the local scalar field value. Under very general conditions, we prove two theorems limiting its cosmological impact: (i) the Compton wavelength of such a scalar can be at most $\simeq 1$ MPc at the present cosmic density, which restricts its impact to nonlinear scales; and (ii) the conformal factor relating Einstein- and Jordan-frame scale factors is essentially constant over the last Hubble time, which precludes the possibility of self-acceleration. These results imply that chameleonlike scalar fields have a negligible effect on the linear-scale growth history; theories that invoke a chameleonlike scalar to explain cosmic acceleration rely on a form of dark energy rather than a genuine modified gravity effect. Our analysis applies to a broad class of chameleon, symmetron, and dilaton theories.

The Λ cold dark matter (ΛCDM) standard model, featuring a cosmological constant as the dark energy component driving cosmic acceleration, will come under increased scrutiny in this decade. Upcoming large-scale surveys, such as the Dark Energy Survey, the Euclid mission, and the Large Synoptic Survey Telescope, will measure the expansion and growth histories of our universe with unprecedented accuracy. These observations may well reveal new physics beyond ΛCDM, in the form of new dynamical degrees of freedom in the dark sector.

It is natural to expect that such degrees of freedom (generally scalar fields) couple to both dark matter [1,2] and baryonic matter. Gravitationally (and universally) coupled scalars are ubiquitous in string theory. To ensure consistency with local tests of gravity, however, the effects of the scalars must be suppressed locally. This is achieved through screening mechanisms [3]: in high-density regions, such as in the solar system, the scalars develop nonlinear interactions, which in turn decouple them from matter.

Following are the two broad classes of screening mechanisms:

(i) The chameleonlike mechanism, in which the scalar interactions are governed by a potential $V(\phi)$. Whether an object is screened or not is determined by the local value of the scalar field. This mechanism is at play in chameleon [4–7], $f(R)$ [8–10], symmetron [11–13], and dilaton theories [14]. These theories generally enjoy no particular symmetries; hence, radiative corrections can be important [15]. On the other hand, since chameleon screening does not rely on derivative interactions, unlike the Vainshtein mechanism described below, there is in principle no obstruction to a UV completion in string theory [16].

(ii) The Vainshtein mechanism, in which the scalar nonlinearities result from derivative interactions. Whether an object is screened or not in this case is determined by derivatives of the scalar field. This mechanism is central to the Dvali-Gabadadze-Porrati model [17] and massive gravity theories [18], which involve a scalar with the Galileon symmetry [19]. Galileon interactions are not renormalized at any order in perturbation theory [20–22]. On the other hand, since Galileons generally propagate superluminally on certain backgrounds, their UV completion is not a local Lorentz-invariant quantum field theory or perturbative string theory [23]. Another possibility is a shift, but not Galileon, symmetric scalar [24].

In this paper we focus solely on chameleonlike theories. We will prove, under very general conditions, two theorems limiting the extent to which these theories can impact cosmological observations. The theorems apply to a broad class of these chameleon, symmetron, and dilaton theories. The key input is demanding that the Milky Way galaxy, or the Sun, be screened, which is a necessary condition to satisfy local tests of gravity.

The first theorem is an upper bound on the chameleon Compton wavelength at present cosmological density [25],

$$m_0^{-1} \lesssim \text{Mpc}. \tag{1}$$

Since the chameleon force is Yukawa suppressed on scales larger than m_0^{-1}, this implies that its effects on the large-scale structure are restricted to nonlinear scales. Any cosmological observable probing linear scales, such as redshift-space distortions, should therefore see no deviation from general relativity in these theories. While the

bound (1) also appeared independently in Ref. [26], the proof presented here follows a different approach.

The second theorem pertains to the possibility of self-acceleration. Assuming that the scalar field couples universally to matter, the theories of interest therefore involve two metrics, related by a conformal rescaling,

$$g^{\mathrm{J}}_{\mu\nu} = A^2(\phi)g^{\mathrm{E}}_{\mu\nu}. \qquad (2)$$

The Jordan-frame metric $g^{\mathrm{J}}_{\mu\nu}$ is the metric to which matter fields couple minimally. The Einstein-frame metric $g^{\mathrm{E}}_{\mu\nu}$ is by definition governed by the Einstein-Hilbert action, with constant Planck mass. Although the two frames are physically equivalent, cosmological observations are implicitly performed in the Jordan frame, where the masses of particles are constant. Meanwhile, the statement that we need some form of dark energy (a component with an equation of state $P/\rho < -1/3$) to drive cosmic acceleration is an Einstein frame statement, where the Friedmann equation takes its standard form.

By self-acceleration, we mean accelerated expansion in the Jordan frame, while the Einstein-frame expansion rate is not accelerating. This is a sensible definition, for the lack of acceleration in the Einstein frame—where the Einstein, and therefore the standard Friedmann, equations hold—is equivalent to the lack of dark energy. In self-accelerating theories, the observed (Jordan-frame) cosmic acceleration stems entirely from the conformal transformation (2), i.e., a genuine modified gravity effect. The Galileon provides such an example among its solutions, where the Einstein frame metric is Minkowski and the Jordan frame metric is de Sitter [19]. The well-known self-accelerating solution in the Dvali-Gabadadze-Porrati model [17] can be interpreted in such a manner.

Clearly a necessary condition for self-acceleration is that the conformal factor $A(\phi)$ varies by at least $\mathcal{O}(1)$ over the last Hubble time [27]. We will instead find for chameleon-like theories

$$\frac{\Delta A}{A} \ll 1, \qquad (3)$$

ruling out the possibility of self-acceleration. Jordan- and Einstein-frame metrics are indistinguishable, and cosmic acceleration requires a negative-pressure component.

Taken together, (1) and (3) imply that chameleonlike scalar fields have a negligible effect on density perturbations on linear scales and cannot account for the observed cosmic acceleration except as some form of dark energy. This applies to a broad class of chameleon, symmetron, and dilaton theories, including the popular example of $f(R)$. In other words, any such model that purports to explain the observed cosmic acceleration, and passes solar system tests, must be doing so using some form of quintessence or vacuum energy; the modification of gravity has nothing to do with the acceleration phenomenon. Nonetheless, the generalized chameleon mechanism remains interesting as a way to hide light scalars suggested by fundamental

theories. The way to test these theories is to study small scale phenomena. Astrophysically, chameleon scalars affect the internal dynamics [28,29] and stellar evolution [30–32] in dwarf galaxies in void or mildly overdense regions.

Setup.—Consider a general scalar-tensor theory in the Einstein frame,

$$S = \int d^4x \sqrt{-g^{\mathrm{E}}}\left(\frac{R^{\mathrm{E}}}{16\pi G_{\mathrm{N}}} - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right) + S_{\mathrm{m}}[g^{\mathrm{J}}]. \quad (4)$$

Matter fields described by S_{m} couple to ϕ through the conformal factor (positive) $A(\phi)$ implicit in $g^{\mathrm{J}}_{\mu\nu}$. The acceleration of a test particle is influenced by the scalar

$$\vec{a} = -\vec{\nabla}\Phi_{\mathrm{N}} - \frac{d\ln A(\phi)}{d\phi}\vec{\nabla}\phi = -\vec{\nabla}(\Phi_{\mathrm{N}} + \ln A(\phi)), \quad (5)$$

where Φ_{N} is the (Einstein frame) Newtonian potential. The fields Φ_{N} and ϕ obey

$$\vec{\nabla}^2\Phi_{\mathrm{N}} = 4\pi G_N A\rho; \qquad \Box\phi = V_{,\phi} + A_{,\phi}\rho, \quad (6)$$

where the matter is assumed to be nonrelativistic, and ρ is related to the Einstein- and Jordan-frame matter densities by $\rho = \rho_{\mathrm{E}}/A = A^3\rho_{\mathrm{J}}$, defined such that ρ is conserved in the usual sense in the Einstein frame [33]. An alternative form of the ϕ equation of motion is useful for comparing against the Poisson equation for Φ_{N},

$$\Box\varphi = 8\pi G_N(V_{,\varphi} + \alpha A\rho); \qquad \alpha \equiv \frac{d\ln A}{d\varphi} = M_{\mathrm{Pl}}\frac{d\ln A}{d\phi}, \quad (7)$$

where $\varphi \equiv \phi/M_{\mathrm{Pl}}$, $M_{\mathrm{Pl}} \equiv (8\pi G_{\mathrm{N}})^{-1/2}$, and α quantifies the dimensionless scalar-matter coupling, with $\alpha \sim \mathcal{O}(1)$ meaning gravitational strength.

A scalar solution of interest is one where ϕ takes the equilibrium value, $V_{,\phi} + A_{,\phi}\rho = 0$; i.e., ρ varies sufficiently slowly with space and time such that gradients of ϕ can be neglected. An example is cosmology, with the cosmic mean ϕ adiabatically tracking the minimum ϕ_{min} of the effective potential $V_{\mathrm{eff}}(\phi) \equiv V(\phi) + A(\phi)\rho$ as the universe evolves. For simplicity, we assume this minimum is unique, within the field range of interest [34]. Further, it is assumed ϕ_{min} varies monotonically with ρ, say, $d\phi_{\mathrm{min}}/d\rho \leq 0$; this is useful for implementing the idea that properties of the scalar field vary systematically with the ambient density [35]. Differentiating $V_{,\phi} + A_{,\phi}\rho = 0$ with respect to ϕ_{min}, it is straightforward to show that $d\phi_{\mathrm{min}}/d\rho = -A_{,\phi}(\phi_{\mathrm{min}})/m^2$, where

$$m^2 \equiv V_{\mathrm{eff},\phi\phi}(\phi_{\mathrm{min}}) = V_{,\phi\phi}(\phi_{\mathrm{min}}) + A_{,\phi\phi}(\phi_{\mathrm{min}})\rho \quad (8)$$

is assumed non-negative for stability. This means A must be monotonically increasing—hence V must be monotonically decreasing—with ϕ, at least over the field range of interest. A corollary is that $V(\phi_{\mathrm{min}}(\rho))$ and $A(\phi_{\mathrm{min}}(\rho))$ are,

respectively, monotonically increasing and decreasing functions of ρ.

We are particularly interested in the equilibrium $\phi_{\min}$ at cosmic mean density between redshifts $z = 0$ and $z \simeq 1$, the period during which the observed cosmic acceleration commences. Let us refer to the respective equilibrium values $\phi_{z=0}$ and $\phi_{z\simeq1}$. We are interested in theories with interesting levels of modified gravity effects during this period; we therefore assume

$$\alpha(\phi) \gtrsim \mathcal{O}(1) \quad \text{for } \phi_{z\simeq1} \le \phi \le \phi_{z=0}. \qquad (9)$$

Note that our setup automatically guarantees $\phi_{z\simeq1} \le \phi_{z=0}$. Hence, $A(\phi)$ grows with time, which is a necessary condition for self-acceleration.

Generalized screening condition.—Consider a spherically symmetric overdense object that is screened, meaning it sources a scalar force that is everywhere suppressed relative to the gravitational force. According to (5),

$$\frac{d \ln A(\phi)}{dr} \lesssim \frac{d\Phi_N}{dr}. \qquad (10)$$

Both sides of the inequality are positive. The positivity of the right-hand side is guaranteed by the positivity of $A\rho$; positivity of the left will be established below. Integrating from inside to outside the object, we have

$$\ln\left[\frac{A(\phi_{\text{out}})}{A(\phi_{\text{in}})}\right] \lesssim \Delta\Phi_N. \qquad (11)$$

Here, "inside" means the origin $r = 0$; "outside" means sufficiently far out such that ϕ_{out} is the equilibrium value at today's cosmic mean density: $\phi_{\text{out}} = \phi_{z=0}$. To satisfy solar system tests, we typically demand that the sun (and also the Milky Way [36]) is screened. Both have a gravitational potential $\Phi_N \sim -10^{-6}$; thus, the screening condition is

$$\ln\left[\frac{A(\phi_{z=0})}{A(\phi_{\text{in-MW}})}\right] \lesssim 10^{-6}. \qquad (12)$$

This inequality will be key in proving (1) and (3). It makes clear that it is the gravitational potential of the object in question, as opposed to its density alone, that ultimately determines whether it is screened or not.

Proof of theorems.—We first rule out self-acceleration by proving (3). To do so requires a closer examination of the static and spherically symmetric equation of motion,

$$\phi'' + \frac{2}{r}\phi' = V_{,\phi} + A_{,\phi}\rho, \qquad (13)$$

where $' \equiv d/dr$. This is subject to the boundary conditions $\phi'|_{r=0} = 0$ and $\phi \to_{r\to\infty} \phi_{z=0}$. Although ϕ tends to its equilibrium value asymptotically, we make no such assumption at the origin; i.e., $\phi|_{r=0} \equiv \phi_{\text{in}}$ need *not* coincide with $\phi_{\min}(\rho_{\text{in}})$. We distinguish the following three cases:

(i) *Case 1*: Suppose $V_{,\phi} + A_{,\phi}\rho \simeq 0$ at $r = 0$, that is, $\phi_{\text{in}} \simeq \phi_{\min}(\rho_{\text{in}})$. This is the thin-shell case of

standard chameleons [4]. Since $\rho_{\text{MW}} \gg \rho_{z\simeq1}$, our monotonicity assumptions imply $A(\phi_{z\simeq1}) \ge A(\phi_{\text{in-MW}})$; thus,

$$\ln\left[\frac{A(\phi_{z=0})}{A(\phi_{z\simeq1})}\right] \le \ln\left[\frac{A(\phi_{z=0})}{A(\phi_{\text{in-MW}})}\right] \lesssim 10^{-6}. \qquad (14)$$

This proves (3) in this case.

(ii) *Case 2*: Suppose $A_{,\phi}\rho \gg -V_{,\phi}$ at $r = 0$, which is the case relevant to symmetrons [11]. Given our assumption that $V_{\text{eff}} = V(\phi) + A(\phi)\rho$ has a unique minimum, this implies $\phi_{\text{in}} \ge \phi_{\min}(\rho_{\text{in}})$. Because $\phi'|_{r=0} = 0$, it follows from (13) that $\phi''|_{r=0} > 0$, and thus $\phi'|_{r>0} > 0$. And since ϕ' is continuous at the surface of the object, to satisfy $\phi \to_{r\to\infty} \phi_{z=0}$ we must therefore have $\phi_{\text{in}} < \phi_{z=0}$. In other words, case 2 corresponds to

$$\phi_{\min}(\rho_{\text{in}}) \le \phi_{\text{in}} < \phi_{z=0}. \qquad (15)$$

Unlike case 1, $\phi_{\text{in-MW}}$ is not *a priori* constrained to be smaller (or greater) than $\phi_{z\simeq1}$. If $\phi_{z\simeq1} \ge \phi_{\text{in-MW}}$, then as in case 1 we are led to (14), and self-acceleration is ruled out. The other possibility, $\phi_{z\simeq1} < \phi_{\text{in-MW}}$, is inconsistent with screening the Milky Way. Indeed, in this case ϕ falls within the range (9), where $\alpha(\phi) \gtrsim \mathcal{O}(1)$, and (7) can be approximated by $\nabla^2\varphi \sim 8\pi G_N \alpha A\rho$. Comparing with the Poisson equation $\nabla^2\Phi_N = 4\pi G_N A\rho$, it is clear the resulting scalar force is not small compared to the gravitational force, thus invalidating the screening of the Milky Way.

(iii) *Case 3*: Suppose $A_{,\phi}\rho \ll -V_{,\phi}$ at $r = 0$, that is, $\phi_{\text{in}} \le \phi_{\min}(\rho_{\text{in}})$. In this case, all inequalities are reversed relative to case 2, and instead of (15) we conclude $\phi_{\min}(\rho_{\text{in}}) \ge \phi_{\text{in}} > \phi_{z=0}$. But this is inconsistent with our assumption that $\phi_{\min}(\rho)$ is monotonically decreasing; hence, we can ignore this case.

To summarize, the only phenomenologically viable possibilities are case 1 and case 2 with $\phi_{z\simeq1} \ge \phi_{\text{in-MW}}$. In both cases we are led to (14). The very small $\Delta A/A$ over cosmological time scales precludes self-acceleration.

To establish the bound (1) on m_0^{-1}, consider the (Einstein-frame) cosmological evolution equation

$$\ddot{\phi} + 3H\dot{\phi} = -V_{,\phi} - A_{,\phi}\rho, \qquad (16)$$

where ρ is the total (dark matter plus baryonic) nonrelativistic matter component, and $H \equiv \dot{a}_E/a_E$ is the Einstein-frame Hubble parameter. Since $A(\phi)\rho \sim H^2 M_{\text{Pl}}^2$ from the Friedmann equation, the density term in (16) exerts a significant pull on $\phi(t)$. The potential prevents a rapid roll-off of ϕ by canceling the density term to good accuracy, $V_{,\phi} \simeq -A_{,\phi}\rho$. This cancellation must be effective over at least the Hubble time; i.e., ϕ must track adiabatically the minimum of the effective potential. Differentiating this relation with respect to time, and using (8) together with

the conservation law $\dot{\rho} = -3H\rho$ and the Friedmann relation, we find

$$m^2 \simeq \frac{3HA_{,\phi}\rho}{\dot{\phi}} \sim H\frac{dt}{d\ln A}\alpha^2(\phi)H^2. \qquad (17)$$

The factor of $H^{-1}d\ln A/dt$ is the change of $\ln A$ over the last Hubble time, which from (14) is less than 10^{-6}. Thus,

$$m^2 \gtrsim 10^6\alpha^2(\phi)H^2. \qquad (18)$$

Using (9), it follows that $m_0^{-1} \lesssim 10^{-3}H_0^{-1} \sim$ Mpc, as we wanted to show.

Discussion.—As with any no-go theorem, the key question is which of its assumptions can be circumvented? One option is to relax the assumption of adiabatic tracking for the cosmological scalar field. While this opens up a wider range of possibilities, a generic outcome is that ϕ undergoes large field excursions on cosmological time scales. Indeed, if the two terms on the right-hand side of (16) do not (approximately) cancel each other, the resulting acceleration $|\ddot{\phi}| \sim M_{\rm Pl}H^2$—either positive, if V dominates, or negative, if $A\rho$ dominates—will drive the field by an amount $|\Delta\phi| \sim M_{\rm Pl}(\Delta z)^2$ over a redshift difference Δz. In particular, $|\Delta\phi| \sim M_{\rm Pl}$ for $\Delta z \simeq 1$. It is unclear whether such large field excursions are consistent with the Milky Way being screened.

Another possibility is to relax the screening condition, which assumes the existence of test particles moving on Jordan-frame geodesics. If all astronomical objects used as dynamical tracers are screened [7], then there is no need to enforce (10). But this drastic measure to hide nearly all modified gravity effects leads to a strong backreaction: with even the smallest bodies being screened, the effective density sourcing ϕ cosmologically must be a tiny fraction of the total matter density, and thus the cosmological evolution needs to be reconsidered.

A further possibility is to relax the assumption of a single scalar field. It is likely possible to extend our no—self-acceleration theorem to a multifield version if V and A continue to be monotonic functions of ρ at equilibrium. It is unclear, however, how the mass bound would be modified in a multifield context. Fluctuations around the effective minimum would be described by a mass matrix, whose eigenvalues can span a wide range of scales. We leave a detailed investigation to future work.

Let us close with an observation on how theories that screen by the Vainshtein mechanism circumvent our no-go theorems: They replace the potential $V(\phi)$ by derivative interactions. A key effect is that the screening condition (10) needs only hold up to some radius, the so-called Vainshtein radius, of the object, thus decoupling $\phi_{\rm out}$ from $\phi_{z=0}$. It would be interesting to investigate whether chameleonlike theories can also achieve such decoupling.

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[25] Throughout this paper, we will use natural units: $\hbar = c = 1$.

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[27] Relating the Jordan- and Einstein-frame scale factors by $a_{\rm J} = Aa_{\rm E}$, it is straightforward to show $[a\ddot{a}]_{\rm J} - [a\ddot{a}]_{\rm E} = (A'/A)'$, where $\dot{}$ denotes derivative with respect to (Jordan- or Einstein-frame) proper time, and $0000'$

denotes derivative with respect to conformal time. Thus, if $[a\ddot{a}]_E \lesssim 0$, we must have $[a\ddot{a}]_J \lesssim (A'/A)'$, implying $1 \lesssim \Delta A/A$ over a (Jordan frame) Hubble time.

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[33] More precisely: $U \cdot \nabla \rho = -\rho \nabla \cdot U$ for a pressureless fluid. Here, $\rho = \rho_E/A$, U^μ is the fluid 4-velocity in Einstein frame ($= A \times$ Jordan frame velocity), and indices are contracted with the Einstein frame metric.

[34] Symmetrons seem to violate this assumption, by having more than one minimum: different parts of the universe might inhabit different domains. This case is nonetheless covered by our arguments, as long as one interprets the "field range of interest" as that within one domain.

[35] For instance, one would like to demand the scalar mass to be large, or the coupling α to be small, at high ρ. But our theorems do not rely on these additional assumptions.

[36] The screening of the Sun and the Milky Way Galaxy go hand in hand for no other reason than that their Newtonian potential happens to be comparable.