New Type of Crossover Physics in Three-Component Fermi Gases

Yusuke Nishida

Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA

(Received 30 July 2012; published 10 December 2012)

A three-component Fermi gas near a broad Feshbach resonance does not have a universal ground state due to the Thomas collapse, while it does near a narrow Feshbach resonance. We explore its universal phase diagram in the plane of the inverse scattering length $1/ak_F$ and the resonance range R_*k_F. For a large R_*k_F, there exists a Lifshitz transition between superfluids with and without an unpaired Fermi surface as a function of $1/ak_F$. With decreasing R_*k_F, the Fermi surface coexisting with the superfluid can change smoothly from that of atoms to trimers ("atom-trimer continuity"), corresponding to the quark-hadron continuity in a dense nuclear matter. Eventually, there appears a finite window in $1/ak_F$ where the superfluid is completely depleted by the trimer Fermi gas, which gives rise to a pair of quantum critical points. The boundaries of these three quantum phases are determined in regions where controlled analyses are possible and are also evaluated based on a mean-field plus trimer model.

DOI: 10.1103/PhysRevLett.109.240401 PACS numbers: 67.85.Lm, 03.75.Ss, 05.30.Rt

Introduction.—The physics of a dilute two-component Fermi gas with a short-range interaction becomes universal in the vicinity of a broad Feshbach resonance. In the limit of the vanishing potential range r_0 compared to the s-wave scattering length a and the Fermi wavelength k_F^{-1}, the physics is characterized solely by their dimensionless ratio $1/ak_F$. With increasing $1/ak_F$, a two-component Fermi gas exhibits a crossover from a Bardeen-Cooper-Schrieffer- (BCS-)type superfluid to a Bose-Einstein condensate (BEC) of diatomic molecules [1–3]. The BCS-BEC crossover and its variations with mass and density imbalances have been subject to extensive research [4–7].

This research field can be further extended to multicomponent Fermi gases. In particular, a three-component Fermi gas has attracted considerable interest partially because of its intriguing analogy to a matter of quarks with three colors [8–10], and, more recently, because of its experimental realization with ^{6}Li atoms [11–17]. Superfluid pairing and the BCS-BEC "crossover" of a three-component Fermi gas have been studied theoretically [18–31], typically in mean-field approximations.

However, a serious problem arises when a many-body ground state of a three-component Fermi gas is considered beyond the mean-field approximation: Three distinguishable fermions can form an infinitely deep bound state in the zero-range limit $r_0 \to 0$, which is known as the Thomas collapse [32]. Therefore, a three-component Fermi gas does not have a many-body ground state, or even if it exists, it is set by a nonzero r_0 and hence not universal. This is in sharp contrast to a two-component Fermi gas where the Thomas collapse does not take place due to the Pauli exclusion principle.

The lack of a universal ground state of a three-component Fermi gas can be cured in the vicinity of a narrow Feshbach resonance where both a and the resonance range R_* are much larger than r_0 [33]. Because R_* sets a ground state of three fermions even in the zero-range limit $r_0 \to 0$, a dilute three-component Fermi gas now becomes universal in the sense that its physics is completely characterized by the two dimensionless ratios, $1/ak_F$ and R_*k_F. Recently, a two-component Fermi gas near a narrow Feshbach resonance was studied experimentally using ^{6}Li atoms [34] or ^{6}Li-^{40}K mixture [35] as well as theoretically [36–39]. Its extension to three components of fermions is quite feasible. Multicomponent Fermi gases can also be realized with alkaline-earth-like atoms such as ^{173}Yb [40].

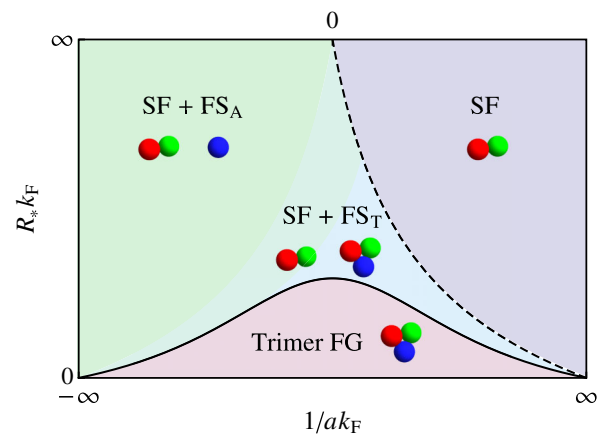

FIG. 1 (color online). Minimal phase structure of a three-component Fermi gas with equal masses, densities, and interactions at zero temperature. There are three quantum phases consisting of superfluids (SF) with and without an unpaired Fermi surface and a Fermi gas of trimers (Trimer FG). The Fermi surface coexisting with the superfluid can change smoothly from that of atoms (FS$_A$) to trimers (FS$_T$). The phase boundaries are asymptotically given by Eqs. (7) and (9) for the superfluid transition (solid curve) and by Eqs. (4) and (10) for the Lifshitz transition (dashed curve). See Fig. 2 for an evaluation based on a mean-field plus trimer model.

 © 2012 American Physical Society

The purpose of this work is to explore the universal phase diagram of a three-component Fermi gas in the vicinity of a narrow Feshbach resonance. For simplicity, we shall consider the case of equal masses, densities, and interactions [10,41], and our findings are summarized in Fig. 1. There are three regions in the plane of $1/ak_F$ and R_*k_F where quantum phases are easily located: a BCS superfluid with an unpaired Fermi surface in the weak coupling limit $1/ak_F \ll -1$, a BEC superfluid with no unpaired Fermi surface in the strong coupling limit $1/ak_F \gg 1$, and a Fermi gas of deeply bound three fermions (trimers) in the broad resonance limit $R_*k_F \ll 1$. In what follows, their phase boundaries will be determined in two regions where controlled analyses are possible: One is the narrow resonance limit $R_*k_F \to \infty$ with fixed $1/ak_F$ (upper side of Fig. 1) and the other is the dilute limit $k_F \to 0$ with fixed R_*/a (lower left and right corners). A similar approach was used in Ref. [42] to investigate the phase diagram of a bilayer Fermi gas. We also evaluate the phase boundaries based on a mean-field plus trimer model (see Fig. 2) and conclude that the minimal phase structure is already rich, as shown in Fig. 1.

Narrow resonance limit.—A three-component Fermi gas near a narrow Feshbach resonance with $\mathrm{SU}(3) \times \mathrm{U}(1)$ invariance is described by a Lagrangian density (hereafter, $\hbar = 1$):

$$\mathcal{L} = \psi_i^\dagger\left(i\partial_t + \frac{\nabla^2}{2m} + \mu\right)\psi_i + \phi_i^\dagger\left(i\partial_t + \frac{\nabla^2}{4m} - \nu + 2\mu\right)\phi_i + g\frac{\epsilon_{ijk}}{2}(\phi_i^\dagger\psi_j\psi_k + \psi_k^\dagger\psi_j^\dagger\phi_i). \tag{1}$$

Here ψ_i and ϕ_i represent fermionic atoms and bosonic molecules, respectively, ϵ_{ijk} is the antisymmetric tensor, and sums over repeated indices i, j, $k = 1, 2, 3$ are assumed. The bare couplings ν and g are related to the s-wave scattering length and the resonance range by

$$\frac{\nu}{g^2} = -\frac{m}{4\pi a} + \frac{m\Lambda}{2\pi^2} \quad \text{and} \quad g^2 = \frac{4\pi}{m^2 R_*}, \tag{2}$$

where the momentum cutoff Λ should be sent to infinity. A controlled analysis is possible in the narrow resonance limit $R_* \to \infty$ where the Feshbach coupling g is vanishingly small and thus the zero-temperature mean-field theory becomes exact [43].

In order for all three components of fermions to have the same number density $n_i \equiv k_F^3/(6\pi^2)$, three order parameters $\langle\phi_i\rangle$ have to be equal up to arbitrary phases. By introducing a gap parameter $\Delta \equiv \sqrt{3}g|\langle\phi_i\rangle|$, the grand potential density is given by

$$\Omega_{\mathrm{MF}} = -\left(\frac{m}{4\pi a} + \frac{m^2\mu R_*}{2\pi}\right)\Delta^2 - \frac{(2m\mu)^{5/2}}{30\pi^2 m}\theta(\mu) - \int\frac{d\boldsymbol{p}}{(2\pi)^3}\left[E_{\boldsymbol{p}} - (\epsilon_{\boldsymbol{p}} - \mu) - \frac{\Delta^2}{2\epsilon_{\boldsymbol{p}}}\right], \tag{3}$$

where $\epsilon_{\boldsymbol{p}} \equiv \boldsymbol{p}^2/(2m)$, $E_{\boldsymbol{p}} \equiv \sqrt{(\epsilon_{\boldsymbol{p}} - \mu)^2 + \Delta^2}$, and $\theta(\cdot)$ is the step function. The second term is a contribution of unpaired fermions and the last one is that of paired fermions, both of which are superpositions of the original fermion components $|i\rangle$. For example, the unpaired fermion represented by a blue particle in Fig. 1 is an equal superposition, $|\text{unpaired}\rangle = |1\rangle + |2\rangle + |3\rangle$. This uniform state is energetically favored over the phase separation discussed in Refs. [28,44]. While the superfluid always exists $\Delta \neq 0$, the coexisting unpaired Fermi surface of atoms appears only for $\mu > 0$ and disappears for $\mu < 0$. Therefore, a Lifshitz transition takes place at $\mu = 0$ [18].

By simultaneously solving the gap equation $\partial\Omega_{\mathrm{MF}}/\partial\Delta = 0$ and the number density equation $3n_i = -\partial\Omega_{\mathrm{MF}}/\partial\mu$, a location of the Lifshitz transition is found to be

$$R_*k_F = \frac{2^{10}\pi}{\Gamma(1/4)^8}(ak_F)^4 - \frac{4}{3\pi}ak_F\bigg|_{ak_F>0}. \tag{4}$$

This equation determines the asymptotic behavior of the dashed curve in Fig. 1 toward the narrow resonance limit $R_*k_F \to \infty$. We remind the reader that the zero-temperature mean-field theory (3) is correct only up to the leading order of the systematic large R_*k_F expansion [45]. Accordingly, the coefficient of the last term in Eq. (4) should be modified by beyond-mean-field corrections. The same Lifshitz transition has been observed at $1/ak_F = 0.633195$ in the broad resonance limit $R_*k_F \to 0$ [20,28,30], but the mean-field analysis breaks down here.

Dilute limit.—A different controlled analysis is possible in the dilute limit $k_F \to 0$ where the problem reduces to a few-body problem. In vacuum, two distinguishable fermions form a bound state (dimer) with binding energy $E_2 < 0$:

$$\sqrt{m|E_2|} = \frac{-1 + \sqrt{1 + 4R_*/a}}{2R_*}\bigg|_{a>0}. \tag{5}$$

On the other hand, a binding energy of three distinguishable fermions is determined by three coupled integral equations [46]:

$$\left[\sqrt{\frac{3}{4}\boldsymbol{p}^2 + m|E_3|} - \frac{1}{a} + R_*\left(\frac{3}{4}\boldsymbol{p}^2 + m|E_3|\right)\right]\chi_i(\boldsymbol{p}) = \sum_{j\neq i}\int\frac{d\boldsymbol{q}}{(2\pi)^3}\frac{4\pi\chi_j(\boldsymbol{q})}{\boldsymbol{p}^2 + \boldsymbol{q}^2 + \boldsymbol{p}\cdot\boldsymbol{q} + m|E_3|}. \tag{6}$$

Two types of solutions are potentially allowed: One is $\chi_1(\boldsymbol{p}) = -\chi_2(\boldsymbol{p})$ with $\chi_3(\boldsymbol{p}) = 0$ where Eq. (6) becomes equivalent to a problem of two-component fermions and hence no bound state solution; the other is $\chi(\boldsymbol{p}) \equiv \chi_i(\boldsymbol{p})$ for all $i = 1, 2, 3$ where Eq. (6) becomes equivalent to a problem of three identical bosons [33,47]. A ground state trimer with binding energy $E_3 < 0$ is found in the s-wave channel, $\chi(\boldsymbol{p}) = \chi(|\boldsymbol{p}|)$ [48]. While there is also an infinite tower of excited states due to the Efimov effect [49], they are irrelevant to the present purpose to determine the many-body ground state. We also note that bound states formed with more than three fermions are unlikely due to the Pauli exclusion principle.

With increasing R_*/a on the side of $a < 0$, the trimer appears from the three-atom threshold at

$$E_3 = 0 \quad \Leftrightarrow \quad \frac{R_*}{a} = -0.0917249. \tag{7}$$

On the left (right) side of this point, the dilute limit of a three-component Fermi gas reduces to a system of atoms (trimers). Because different components of fermions with equal densities can form Cooper pairs by an infinitesimal attraction, this is actually a quantum critical point to separate the atomic superfluid with an unpaired Fermi surface from a Fermi gas of trimers. Therefore, Eq. (7) determines the asymptotic behavior of the solid curve in Fig. 1 toward the dilute limit $|ak_F|$, $R_*k_F \to 0$.

We now turn to the side of $a > 0$ but away from the point for $E_3 = E_2 \Leftrightarrow R_*/a = 2.18151$ where the trimer disappears into the atom-dimer threshold. In this region, the dilute limit of a three-component Fermi gas reduces to noninteracting dimers and trimers because their sizes become negligible compared to a mean interparticle distance. Since all dimers, if they exist, condense at zero temperature, the chemical potential is fixed by their binding energy, $2\mu = E_2$. Then the total number density is a sum of contributions of condensed dimers and trimers:

$$\frac{k_F^3}{2\pi^2} = n_{\text{dimer}} + \frac{[6m(3\mu - E_3)]^{3/2}}{2\pi^2}\theta(3\mu - E_3). \tag{8}$$

With increasing R_*/a, dimers appear at $n_{\text{dimer}} = 0$, which gives

$$E_3 + \frac{k_F^2}{6m} = \frac{3}{2}E_2. \tag{9}$$

This is a quantum critical point to separate a Fermi gas of trimers on its left side from the trimer Fermi gas coexisting with the dimer superfluid on its right side. With increasing R_*/a further, the Fermi surface of trimers disappears at $n_{\text{dimer}} = k_F^3/(2\pi^2)$, which gives

$$E_3 = \frac{3}{2}E_2 \quad \Leftrightarrow \quad \frac{R_*}{a} = 0.359011. \tag{10}$$

This is a Lifshitz transition, and the system beyond this point is the dimer superfluid with no unpaired Fermi surface. Therefore, Eqs. (9) and (10) determine the asymptotic behaviors of the solid and dashed curves in Fig. 1, respectively, toward the dilute limit ak_F, $R_*k_F \to 0$.

Phase diagram.—Finally, we combine the above results to establish the universal phase diagram of a three-component Fermi gas. Because we developed controlled understanding on quantum phases in all available limits of $1/ak_F$ and R_*k_F, one emergent phase boundary has to end up as another phase boundary of the same type. Accordingly, the quantum critical lines for the superfluid transition found in Eqs. (7) and (9) are connected, and those for the Lifshitz transition found in Eqs. (4) and (10) are also connected. This leads to the minimal but rich phase structure shown in Fig. 1.

An interesting observation is that the unpaired Fermi surface coexisting with the superfluid is that of atoms in the weak coupling $1/ak_F \ll -1$ or dense region $R_*k_F \gg 1$, while it is that of trimers in the strong coupling and dilute region, $1/ak_F \gg 1$ and $R_*k_F \ll 1$. Because there is no sharp distinction between them in terms of symmetries or topologies, the Fermi surfaces of atoms and trimers in the superfluid phase can be smoothly connected. If we associate three components of fermions with three colors of quarks [8–10], this observation corresponds to a smooth crossover from deconfined quarks to confined baryons with decreasing density, which is known as the quark-hadron continuity [50,51]. Accordingly, the analogous new feature in a three-component Fermi gas shall be termed an "atom-trimer continuity."

Regarding low-lying excitations in the superfluid phase, the breaking of the $\mathrm{SU}(3) \times \mathrm{U}(1)$ symmetry down to $\mathrm{SU}(2) \times \mathrm{U}(1)$ generates three Nambu-Goldstone bosons [20]. Furthermore, in the presence of an unpaired Fermi surface, there exists a gapless fermionic excitation which has the character of an atom on one side and that of a trimer on the other side. Because only the parity of a particle number is conserved in the superfluid phase, an unpaired atom and a trimer cannot be distinguished by their particle numbers. Accordingly, the gapless fermionic excitation can also exhibit a crossover with its effective mass changing smoothly from m to $3m$, which signals the atom-trimer continuity [52].

In addition to the controlled analyses, we also develop a model analysis to quantify the phase diagram. A guiding principle to construct the model is that it must incorporate the correct asymptotic behaviors discussed above. The simplest possibility is just to add a contribution of noninteracting trimers to the mean-field grand potential density (3):

$$\Omega_{\text{MF+T}} = \Omega_{\text{MF}} - \frac{[6m(3\mu - E_3)]^{5/2}}{90\pi^2 m}\theta(3\mu - E_3). \tag{11}$$

By solving the gap equation $\partial\Omega_{\text{MF+T}}/\partial\Delta = 0$ and the number density equation $3n_i = -\partial\Omega_{\text{MF+T}}/\partial\mu$ at $\Delta \to 0$, we find that the superfluid transition takes place at

$$E_3 + \frac{k_F^2}{6m} = \frac{3}{2}E_2\theta(a). \tag{12}$$

The obtained quantum critical line is plotted in Fig. 2 by the solid curve, which continuously interpolates the correct asymptotic behaviors [Eqs. (7) and (9)] in the weak and strong coupling limits $ak_F \to \pm 0$. In particular, we find $R_*k_F = 0.288325$ at $1/ak_F = 0$ and its maximum $R_*k_F = 0.381739$ reached at $1/ak_F = 0.314545$.

On the other hand, in the range of Fig. 2, the Lifshitz transition takes place at $E_3 = 3\mu$ where the Fermi surface of trimers disappears with increasing $1/ak_F$. The obtained quantum critical line is plotted by the dashed curve, which again yields the correct asymptotic behavior [Eq. (10)] in the strong coupling limit $ak_F \to +0$. Similarly, the Fermi surface of trimers appears at $E_3 = 0$ and that of atoms

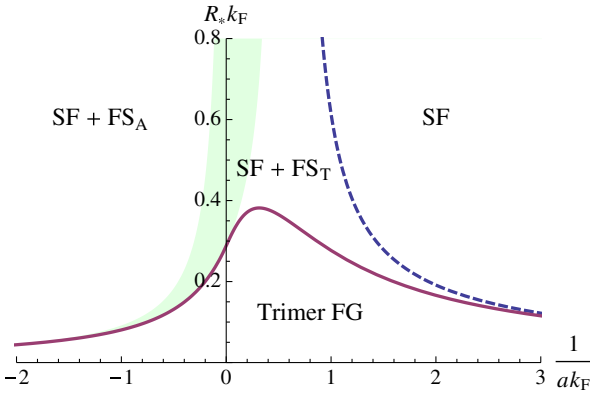

FIG. 2 (color online). Quantum critical lines for the superfluid transition (solid curve) and the Lifshitz transition (dashed curve) based on a mean-field plus trimer model (11). The shaded region indicates where the smooth crossover from the Fermi surface of atoms (left side) to trimers (right side) takes place.

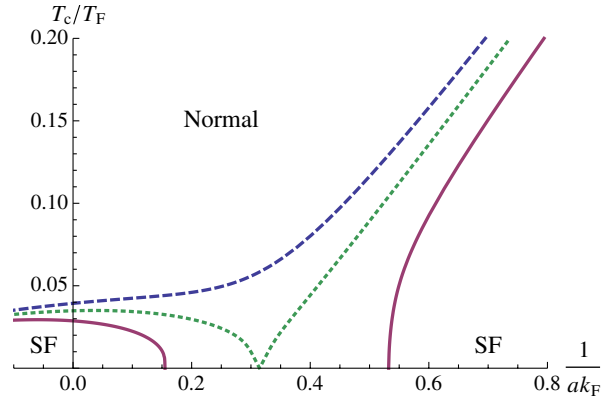

FIG. 3 (color online). Superfluid critical temperatures in units of $T_F \equiv k_F^2/(2mk_B)$ for $R_* k_F = 0.40$ (dashed curve), 0.381 739 (dotted curve), and 0.36 (solid curve). These values of $R_* k_F$ are above, at, and below the critical resonance range for opening the complete depletion of the superfluid by the trimer Fermi gas, which gives rise to a pair of quantum critical points.

disappears at $\mu = 0$, which correspond to the left and right edges of the shaded region in Fig. 2, respectively. While they define sharp boundaries in our simple treatment of trimers as noninteracting particles, such sharp boundaries need not actually exist. Indeed, a more elaborate model with interaction terms $\sim t^\dagger \psi_i \phi_i + \text{H.c.}$ added to the mean-field plus trimer model can incorporate the atom-trimer continuity by hybridizing an unpaired atom and a trimer (represented by t) by a condensed dimer [52]. The shaded region is thus meant to indicate where the smooth crossover from the Fermi surface of atoms to trimers takes place with increasing $1/ak_F$.

We also extend the mean-field plus trimer model (11) to nonzero temperature with a caveat that the mean-field approximation overestimates the critical temperature in the strong coupling region $1/ak_F \gtrsim 0$ [53]. Figure 3 shows the obtained superfluid critical temperature as a function of $1/ak_F$ for three different values of $R_* k_F$. With decreasing $R_* k_F$ from above, the superfluid is gradually suppressed by the emergence of trimers. Eventually, below the critical value $R_* k_F = 0.381739$, there appears a finite window in $1/ak_F$ where the superfluid is completely depleted by the trimer Fermi gas, which gives rise to a pair of quantum critical points. These are the same type of quantum critical point as that studied in the context of Bose-Fermi mixtures [52,54]. Also, the same role of the trimer in the dilute limit was discussed in Ref. [26] based on approximate three-body calculations.

Summary and discussion.—A narrow Feshbach resonance sets the stage to investigate universal aspects of a three-component Fermi gas. We explored its universal phase diagram with equal masses, densities, and interactions and found the minimal but rich phase structure shown in Fig. 1. Our main discovery is a new type of crossover physics: In addition to the ordinary BCS-BEC crossover from loosely bound Cooper pairs to tightly bound dimers, unpaired fermions coexisting with the superfluid can change smoothly from atoms to trimers, corresponding to the quark-hadron continuity in a dense nuclear matter [50,51]. This new feature, termed an atom-trimer continuity, provides a novel analogy between ultracold atoms and quantum chromodynamics and should be investigated further.

At zero temperature, even identical fermions can form Cooper pairs in some partial wave channel [55]. An interaction between unpaired atoms in the weak coupling region $1/ak_F \ll -1$ is induced by a density fluctuation of the other components of fermions, while an interaction between unpaired trimers in the strong coupling region $1/ak_F \gg 1$ is induced by a density fluctuation of the superfluid dimers which dominates over a direct trimer-trimer interaction. Because both induced interactions cause an instability in the p-wave channel [56], the previously unpaired Fermi surface exhibits the p-wave superfluidity regardless of whether it was of atoms or trimers. Therefore, the atom-trimer continuity remains intact. Here the p-wave pairing of atoms or trimers breaks the $\mathrm{SU}(2) \times \mathrm{U}(1)$ symmetry of the s-wave superfluid phase down to $\mathrm{SU}(2) \times \mathrm{Z}_2$ besides broken rotational symmetries. On the other hand, because the s-wave superfluid phase with no unpaired Fermi surface retains the $\mathrm{SU}(2) \times \mathrm{U}(1)$ symmetry, the previous Lifshitz transition (dashed curve in Fig. 1) becomes the p-wave superfluid transition. The trimer Fermi gas phase is still distinct from the other two phases because its symmetry breaking pattern is different, $\mathrm{SU}(3) \times \mathrm{U}(1) \to \mathrm{SU}(3) \times \mathrm{Z}_6$ by any pairing of trimers.

Thus far we focused on the maximally symmetric case, while our approach can be easily generalized to unequal masses, densities, and interactions. Furthermore, the ideas developed here can be used to investigate universal aspects of other systems, such as a two-component Fermi gas with a large mass ratio >13.6 [57], which also lacks a universal ground state in the vicinity of a broad Feshbach resonance.

The author thanks J. Carlson, S. Reddy, and N. Yamamoto for valuable discussions and a LANL Oppenheimer Fellowship for the support of this work.

[1] D. M. Eagles, Phys. Rev. **186**, 456 (1969).
[2] A. J. Leggett, *Modern Trends in the Theory of Condensed Matter*, edited by A. Pekalski and R. Przystawa (Springer-Verlag, Berlin, 1980); J. Phys. (Paris), Colloq. **41**, C7 (1980).
[3] P. Nozières and S. Schmitt-Rink, J. Low Temp. Phys. **59**, 195 (1985).
[4] I. Bloch, J. Dalibard, and W. Zwerger, Rev. Mod. Phys. **80**, 885 (2008).
[5] S. Giorgini, L. P. Pitaevskii, and S. Stringari, Rev. Mod. Phys. **80**, 1215 (2008).
[6] *Ultracold Fermi Gases, Proceedings of the International School of Physics "Enrico Fermi," Course CLXIV*, edited by M. Inguscio, W. Ketterle, and C. Salomon (IOS Press, Amsterdam, 2008).
[7] *The BCS-BEC Crossover and the Unitary Fermi Gas*, Lecture Notes in Physics, Vol. 836, edited by Z. Wilhelm (Springer-Verlag, Berlin, 2012).
[8] Á. Rapp, G. Zaránd, C. Honerkamp, and W. Hofstetter, Phys. Rev. Lett. **98**, 160405 (2007).
[9] F. Wilczek, Nat. Phys. **3**, 375 (2007).
[10] K. M. O'Hara, New J. Phys. **13**, 065011 (2011).
[11] T. B. Ottenstein, T. Lompe, M. Kohnen, A. N. Wenz, and S. Jochim, Phys. Rev. Lett. **101**, 203202 (2008).
[12] A. N. Wenz, T. Lompe, T. B. Ottenstein, F. Serwane, G. Zürn, and S. Jochim, Phys. Rev. A **80**, 040702(R) (2009).
[13] T. Lompe, T. B. Ottenstein, F. Serwane, K. Viering, A. N. Wenz, G. Zürn, and S. Jochim, Phys. Rev. Lett. **105**, 103201 (2010).
[14] T. Lompe, T. B. Ottenstein, F. Serwane, A. N. Wenz, G. Zürn, and S. Jochim, Science **330**, 940 (2010).
[15] J. H. Huckans, J. R. Williams, E. L. Hazlett, R. W. Stites, and K. M. O'Hara, Phys. Rev. Lett. **102**, 165302 (2009).
[16] J. R. Williams, E. L. Hazlett, J. H. Huckans, R. W. Stites, Y. Zhang, and K. M. O'Hara, Phys. Rev. Lett. **103**, 130404 (2009).
[17] S. Nakajima, M. Horikoshi, T. Mukaiyama, P. Naidon, and M. Ueda, Phys. Rev. Lett. **105**, 023201 (2010); **106**, 143201 (2011).
[18] P. F. Bedaque and J. P. D'Incao, Ann. Phys. (Amsterdam, Neth.) **324**, 1763 (2009).
[19] T. Paananen, J.-P. Martikainen, and P. Törmä, Phys. Rev. A **73**, 053606 (2006).
[20] L. He, M. Jin, and P. Zhuang, Phys. Rev. A **74**, 033604 (2006).
[21] S. Y. Chang and V. R. Pandharipande, arXiv:physics/0607008.
[22] H. Zhai, Phys. Rev. A **75**, 031603(R) (2007).
[23] T. Paananen, P. Törmä, and J.-P. Martikainen, Phys. Rev. A **75**, 023622 (2007).
[24] R. W. Cherng, G. Refael, and E. Demler, Phys. Rev. Lett. **99**, 130406 (2007).
[25] G. Catelani and E. A. Yuzbashyan, Phys. Rev. A **78**, 033615 (2008).
[26] S. Floerchinger, R. Schmidt, S. Moroz, and C. Wetterich, Phys. Rev. A **79**, 013603 (2009).
[27] J.-P. Martikainen, J. J. Kinnunen, P. Törmä, and C. J. Pethick, Phys. Rev. Lett. **103**, 260403 (2009).
[28] T. Ozawa and G. Baym, Phys. Rev. A **82**, 063615 (2010).
[29] O. H. T. Nummi, J. J. Kinnunen, and P. Törmä, New J. Phys. **13**, 055013 (2011).
[30] L. Salasnich, Phys. Rev. A **83**, 033630 (2011); arXiv:1108.0076 [Laser Phys. (to be published)].
[31] M. Kanász-Nagy and G. Zaránd, Phys. Rev. B **86**, 064519 (2012).
[32] L. H. Thomas, Phys. Rev. **47**, 903 (1935).
[33] R_* is related to the effective range by $r_{\rm eff} = -2R_* < 0$, D. S. Petrov, Phys. Rev. Lett. **93**, 143201 (2004).
[34] E. L. Hazlett, Y. Zhang, R. W. Stites, and K. M. O'Hara, Phys. Rev. Lett. **108**, 045304 (2012).
[35] C. Kohstall, M. Zaccanti, M. Jag, A. Trenkwalder, P. Massignan, G. M. Bruun, F. Schreck, and R. Grimm, Nature (London) **485**, 615 (2012).
[36] T.-L. Ho, X. Cui, and W. Li, Phys. Rev. Lett. **108**, 250401 (2012).
[37] P. Massignan, Europhys. Lett. **98**, 10 012 (2012).
[38] R. Qi and H. Zhai, Phys. Rev. A **85**, 041603(R) (2012).
[39] C. Trefzger and Y. Castin, Phys. Rev. A **85**, 053612 (2012).
[40] S. Taie, Y. Takasu, S. Sugawa, R. Yamazaki, T. Tsujimoto, R. Murakami, and Y. Takahashi, Phys. Rev. Lett. **105**, 190401 (2010); see also C. Wu, Physics **3**, 92 (2010).
[41] P. Zhang, P. Naidon, and M. Ueda, Phys. Rev. Lett. **103**, 133202 (2009).
[42] Y. Nishida, Phys. Rev. A **82**, 011605(R) (2010); see also P. F. Bedaque, Physics **3**, 58 (2010).
[43] V. Gurarie and L. Radzihovsky, Ann. Phys. (Amsterdam, Neth.) **322**, 2 (2007).
[44] I. Titvinidze, A. Privitera, S.-Y. Chang, S. Diehl, M. A. Baranov, A. Daley, and W. Hofstetter, New J. Phys. **13**, 035013 (2011).
[45] J. Carlson, S. Gandolfi, and Y. Nishida (unpublished).
[46] P. Naidon and M. Ueda, C.R. Physique **12**, 13 (2011).
[47] A. O. Gogolin, C. Mora, and R. Egger, Phys. Rev. Lett. **100**, 140404 (2008).
[48] For a plot of the binding energy, see Fig. 2 in Y. Nishida, arXiv:1207.6971.
[49] V. Efimov, Phys. Lett. B **33**, 563 (1970).
[50] T. Schäfer and F. Wilczek, Phys. Rev. Lett. **82**, 3956 (1999).
[51] T. Hatsuda, M. Tachibana, N. Yamamoto, and G. Baym, Phys. Rev. Lett. **97**, 122001 (2006); N. Yamamoto, M. Tachibana, T. Hatsuda, and G. Baym, Phys. Rev. D **76**, 074001 (2007).
[52] A similar crossover between Fermi surfaces of atoms and composites was observed in Bose-Fermi mixtures, S. Powell, S. Sachdev, and H. P. Büchler, Phys. Rev. B **72**, 024534 (2005).
[53] C. A. R. Sá de Melo, M. Randeria, and J. R. Engelbrecht, Phys. Rev. Lett. **71**, 3202 (1993).
[54] E. Fratini and P. Pieri, Phys. Rev. A **81**, 051605(R) (2010).
[55] W. Kohn and J. M. Luttinger, Phys. Rev. Lett. **15**, 524 (1965).
[56] A. Bulgac, M. M. Forbes, and A. Schwenk, Phys. Rev. Lett. **97**, 020402 (2006).
[57] D. S. Petrov, Phys. Rev. A **67**, 010703(R) (2003).