Fluctuation-Induced Magnetization Dynamics and Criticality at the Interface of a Topological Insulator with a Magnetically Ordered Layer

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We consider a theory for a two-dimensional interacting conduction electron system with strong spin-orbit coupling on the interface between a topological insulator and the magnetic (ferromagnetic or antiferromagnetic) layer. For the ferromagnetic case we derive the Landau-Lifshitz equation, which features a contribution proportional to a fluctuation-induced electric field obtained by computing the topological (Chern-Simons) contribution from the vacuum polarization. We also show that fermionic quantum fluctuations reduce the critical temperature $\tilde{T}_c$ at the interface relative to the critical temperature T_c of the bulk, so that in the interval $\tilde{T}_c \leq T < T_c$ it is possible to have a coexistence of gapless Dirac fermions at the interface with a ferromagnetically ordered layer. For the case of an antiferromagnetic layer on a topological insulator substrate, we show that a second-order quantum phase transition occurs at the interface, and compute the corresponding critical exponents. In particular, we show that the electrons at the interface acquire an anomalous dimension at criticality. The critical behavior of the Néel order parameter is anisotropic and features large anomalous dimensions for both the longitudinal and transversal fluctuations.

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A spin current may exhibit interesting topological properties in systems where a Berry curvature in Bloch momentum space is induced by the underlying band structure [1,2], as for example in the case of some hole-doped semiconductors described by a Luttinger Hamiltonian [3] or systems featuring a Rashba spin-orbit coupling [4]. More recent prominent examples involve the quantum spin Hall insulators or topological insulators (TIs) [5,6], where a Berry curvature in momentum space also arises. Depending on the physical situation the Berry curvature may be Abelian or non-Abelian, and determines a magnetic monopole in momentum space.

The surface of a TI, when in contact with a material exhibiting magnetic order, offers a framework for many topological effects. For instance, the two-dimensional system represented by the surface of a topological insulator can be used as the substrate for a magnetic layer, which can be either ferromagnetic (FM) or antiferromagnetic (AF). For a FM layer having a TI as the substrate, a theoretical study of the magnetization dynamics was carried out recently [7]. In a similar context, the electric charging of magnetic textures has also been discussed [8]. Other interesting electromagnetic topological effects with a similar setup were studied [9–12] and have been shown to exhibit properties similar to those of axion electrodynamics [13]. In axion electrodynamics a topological term of the form $(8\pi^2)^{-1}\alpha\theta\mathbf{E}\cdot\mathbf{B}$ is present [10,13] in the action, where α is the fine structure constant and θ is the so called axion field. For the case where θ is uniform, time-reversal invariant TIs require $\theta = \pi$ [10]. Such a term should play a very important role at the interfaces of TIs with other insulators. For a magnetic insulating layer on the surface of a TI, a modification of the magnetization dynamics occurs, due to a direct coupling of the magnetization to the electric field. Indeed, we have $\mathbf{E}\cdot\mathbf{B} = \mathbf{E}\cdot(\mathbf{H} + 4\pi\mathbf{M})$, giving rise to a magnetoelectric effect, which influences the precession of the magnetization [9].

At the same time, the experimental situation is far from being clear. For instance, from a theory perspective one would expect that the coupling of a TI to a FM layer would make the surface states gapped. However, in a very recent experiment [14] where Fe impurities were deposited on Bi_2Se_3, no sign of a gap was found, in apparent conflict with theoretical expectations. Therefore, further theoretical studies on the coupling of a TI substrate to a magnetic system are necessary.

In this Letter we consider the effects of quantum fluctuations stemming from the proximity-induced magnetism on the surface of a TI. We assume that the electrons on the surface of the TI interact via a long-range Coulomb interaction. For the case of a FM layer in contact with the TI, we will derive a Landau-Lifshitz (LL) equation which accounts for these interaction effects. In our calculation an axionlike term emerges due to quantum fluctuations. At the interface, it manifests itself as a Chern-Simons (CS) term [15], which breaks time-reversal symmetry, as a consequence of the coupling of the surface of the TI to the magnetic layer. Furthermore, the electronic quantum fluctuations make the stiffness anisotropic, even if the bulk of the FM layer features an isotropic stiffness. We also show that due to the quantum fluctuations of the electrons, the critical temperature $\tilde{T}_c$ at the interface is reduced relative to the critical temperature T_c of the FM layer. This allows the existence of gapless fermions at the interface in the temperature range $\tilde{T}_c \leq T < T_c$ where the bulk magnetic layer is still magnetically ordered.

It has been shown in a recent study [16] that the best candidate material to gap the topological surface states of a TI is MnSe, which is an AF insulator. For this case we will show that a second-order quantum phase transition occurs at the interface, and that it defines a new universality class. One consequence of this interface quantum criticality is that the surface electrons become gapless at the quantum critical point (QCP). This does not happen in the FM case we study. Hence, the topologically protected gapless modes can be restored at zero temperature by disordering the AF long-range order at the interface. A further important feature of this interface quantum criticality is the emergence of a large anomalous dimension for the Néel order parameter. Interestingly, at the QCP the fermions will also acquire an anomalous dimension.

Our starting point is the Lagrangian for conduction electrons interacting via a Coulomb interaction on the surface of an insulator either in contact with a bulk ferromagnet composed of several layers, similarly to Ref. [7], or with an AF bulk system. Thus, if $\mathbf{n}$ is the induced magnetization at the interface and $\mathbf{L}$ the angular momentum, the spin of the conduction electrons, $\mathbf{S} = (1/2)c^\dagger \vec{\sigma} c$, is coupled to the total magnetization $(\mu_B/2)\mathbf{L} + \mathbf{n}$ via an exchange term $-2J\mathbf{S} \cdot [(\mu_B/2)\mathbf{L} + \mathbf{n}]$, where $c^\dagger = [c_\uparrow^\dagger c_\downarrow^\dagger]$, with $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ being the Pauli matrices. The lack of inversion symmetry in the direction perpendicular to the interface leads to a spin-orbit coupling of the Rashba type. Thus, the Lagrangian for the conduction electrons at the interface reads (we are assuming units where $\hbar = c = 1$),

$$\mathcal{L}_c = c^\dagger[i\partial_t + e\varphi - iv(\sigma_y\partial_x - \sigma_x\partial_y) + J\vec{\sigma}\cdot\mathbf{n}]c - \frac{\epsilon}{4\pi}\varphi|\nabla|\varphi, \tag{1}$$

where $v \propto J$. We will give further details on the exchange part of the Lagrangian shortly. In writing the above Lagrangian we have assumed that the spin-orbit coupling is much stronger than the usual kinetic term of the conduction electrons, which has been neglected. The auxiliary (Hubbard-Stratonovich) field φ accounts for the Coulomb interaction. Upon integrating out φ the usual Coulomb interaction between the electrons is obtained. The nonlocal Gaussian term for φ reflects the three-dimensional character of the Coulomb interaction in a two-dimensional problem, similarly to graphene [17]. In this term ∇ is the two-dimensional gradient and ϵ represents the dielectric constant.

The full Lagrangian of the systems includes the Lagrangian describing the magnetization dynamics of the bulk ferromagnet, which includes a Landau-Ginzburg type functional and is given by

$$\mathcal{L}_{\rm FM} = \mathbf{b}\cdot\partial_t\mathbf{n} - \frac{\kappa}{2}[(\nabla\mathbf{n})^2 + (\partial_z\mathbf{n})^2] - \frac{m^2}{2}\mathbf{n}^2 - \frac{u}{4!}(\mathbf{n}^2)^2, \tag{2}$$

where κ, $u > 0$ and $m^2 = a_0(T - T_0)$, with T_0 being the (mean-field) critical temperature to disorder the ferromagnet. $\mathbf{b}$ is the Berry connection, which fulfills the usual monopole condition, $\partial b_i/\partial n_j - \partial b_j/\partial n_i = \epsilon_{ijk}n_k/\mathbf{n}^2$. For $m^2 < 0$ (or $T < T_0$) the bulk ferromagnet is in a ferromagnetically ordered state.

Before considering the magnetization dynamics, let us first consider a fluctuation-corrected mean-field theory where the only fluctuation effects that are taken into account are the fermionic ones, i.e., σ is assumed to be uniform, and the transverse fluctuations of the magnetization vanish. We also neglect the fluctuation effects of the Coulomb interaction. The calculations are done in imaginary time and at finite temperature.

In this case, after integrating out the fermions, we obtain the free energy density,

$$\mathcal{F} = -T\sum_{n=-\infty}^{\infty}\int\frac{d^2p}{(2\pi)^2}\ln(\omega_n^2 + v^2\mathbf{p}^2 + J^2\sigma^2) + \frac{m^2}{2}\sigma^2 + \frac{u}{4!}\sigma^4, \tag{3}$$

where $\omega_n = (2n+1)\pi T$ is the fermionic Matsubara frequency. After performing the Matsubara summation, the remaining integral over momenta contains a zero temperature contribution which is divergent, requiring regularization and renormalization. Using an ultraviolet cutoff $\Lambda \sim a^{-1}$, where a is the lattice constant, we can cancel the dependence on the cutoff by minimally absorbing it in a redefinition of the Curie temperature of the bulk ferromagnet precisely at the interface. The physical requirement (or renormalization condition) is that the zero temperature fermionic gap, $m_\psi \equiv J\sigma_0$, is finite in the long-wavelength limit.

The saddle-point approximation yields,

$$a_0(T_c - T) = \frac{u}{6}\sigma^2 + \frac{J^2T}{\pi v^2}\ln\left[2\cosh\left(\frac{J\sigma}{2T}\right)\right], \tag{4}$$

where T_c is the renormalized Curie temperature of the bulk ferromagnet at the interface. The critical temperature, $\tilde{T}_c$, at the interface is obtained by demanding that σ vanishes at $T = \tilde{T}_c$. This yields $\tilde{T}_c = T_c[1 + J^2\ln 2/(\pi a_0 v^2)]^{-1}$. On the other hand, by setting $T = 0$ in Eq. (4), we obtain that at the interface $T_c = um_\psi^2/(6J^2) + J^2 m_\psi/(2\pi a_0 v^2)$. Note that this expression only makes sense at the interface and does not correspond to the physical critical temperature there, which is actually given by $\tilde{T}_c$. Furthermore, since $v \propto J$ and at leading order $\sigma_0^2 \approx 6a_0T_0/u$, we obtain that $T_c \to T_0$ as $v \to 0$. In order to estimate $\tilde{T}_c$, we assume that $T_c \gg um_\psi^2/(6J^2 a_0)$, such that we have approximately, $\tilde{T}_c \approx m_\psi T_c[m_\psi + (2\ln 2)T_c]^{-1}$. If we use the estimates

$m_\psi \approx 28.2$ meV and $T_c \approx 70$ K [16,18], we obtain $\tilde{T}_c \approx 54$ K. Thus, our fluctuation-corrected mean-field theory implies that the critical temperature at the interface is smaller than the Curie temperature of the bulk ferromagnet. Therefore, it is possible to destroy the proximity-induced magnetization at the interface while the bulk ferromagnet is still ordered. This occurs typically in a temperature window $\tilde{T}_c \leq T < T_c$, where we are assuming that T_c does not differ appreciably from T_0. The reduction of the critical temperature at the interface with respect to the bulk one is an important consequence of the interplay between the ferromagnetic proximity effect and the spin-orbit coupling.

The next step is to compute the fluctuations of the order parameter around the mean-field theory. Since we are interested in deriving a differential equation for the magnetization dynamics, we will return to real time in the following and consider a zero temperature calculation. In order to facilitate our analysis of the problem, it is convenient to rewrite the Lagrangian for the conduction electrons in a QED-like form, which is achieved by the rescalings $\varphi \to (J/e)\varphi$, $x_i \to v x_i$ ($i = 1, 2$), in the action, to obtain

$$\mathcal{L}_c = \bar{\psi}(i\not{\partial} - J\not{a})\psi + J\sigma\bar{\psi}\psi - (\zeta/2)a_0|\nabla|a_0, \quad (5)$$

where the Dirac matrices are defined by $\gamma^0 = \sigma_z$, $\gamma^1 = -i\sigma_x$, and $\gamma^2 = i\sigma_y$, $\psi = vc$, such that the usual relativistic notations for spinors hold with $\bar{\psi} = \psi^\dagger\gamma^0$; also the usual Dirac slash notation, $\not{Q} = \gamma^\mu Q_\mu$, is being used. The gauge field is given by $a^\mu = (\varphi, n_y, -n_x)$ and $\sigma = n_z$, and the dielectric constant, $\zeta \equiv \epsilon v J^2/(2\pi e^2)$. We will assume that there are N fermionic orbital degrees of freedom. Thus, integrating out the fermions yields the gauge-invariant contribution to the effective action, $S_{\mathrm{gauge}} = iN\mathrm{Tr}\ln(i\not{\partial} - J\not{a} + J\sigma)$.

The lowest order diagrams associated with the fluctuating fields are shown in Fig. 1. The approximate evaluation of S_{gauge} at long wavelengths yields the leading fluctuation contribution, $S_{\mathrm{gauge}} \approx S_{\mathrm{eff}}^{\mathrm{MF}} + \delta S_{\mathrm{gauge}}$, where

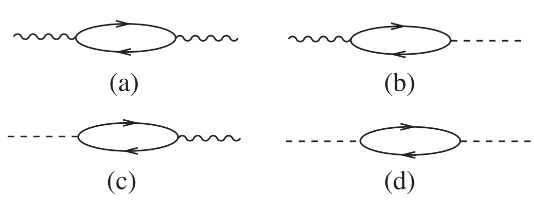

FIG. 1. Diagrams contributing to $\delta S_{\mathrm{gauge}}$, Eq. (6). The wiggled line represents the vector field a_μ, the solid line is a Dirac fermion, and the dashed line represents the fluctuating part of the σ field. Diagram (a) yields the vacuum polarization, while diagram (d) corresponds to the polarization correction to the z-component of the magnetization. The diagrams (b) and (c) cancel out.

$$\delta S_{\mathrm{gauge}} = \frac{NJ^2}{8\pi}\int dt\int d^2r\Big\{-\frac{1}{3m_\psi}(\epsilon_{\mu\nu\lambda}\partial^\nu a^\lambda)^2 + \epsilon_{\mu\nu\lambda}a^\mu\partial^\nu a^\lambda + \frac{1}{m_\psi}[(\partial_t\tilde{\sigma})^2 - (\nabla\tilde{\sigma})^2]\Big\}, \quad (6)$$

with the fluctuation $\tilde{\sigma}$ arising from the decomposition $\sigma = \sigma_0 + \tilde{\sigma}$. The quadratic fluctuation term in $\tilde{\sigma}$ will generate an anisotropy in the magnetic system, which is isotropic in the bulk. The first term in Eq. (6) corresponds to a Maxwell term in (2 + 1)-dimensional electrodynamics. The second term is a CS term [15] generated by the quantum fluctuations. This CS term reflects the breaking of time-reversal symmetry due to the coupling to a magnetic layer. In order to better appreciate the effect of the CS term, it is useful to rewrite the CS contribution to S_{gauge} in the form,

$$S_{\mathrm{CS}} = \frac{\sigma_{xy}}{4\pi}\int dt\int d^2r(n_y\partial_t n_x - n_x\partial_t n_y + 2\mathbf{n}\cdot\nabla\varphi), \quad (7)$$

where $\sigma_{xy} = \sigma_{xy}^0 NJ^2/e^2$, with $\sigma_{xy}^0 = e^2/2$ (in units where $\hbar = 1$), is the induced Hall conductivity. It is readily seen that the contribution proportional to $n_x\partial_t n_y - n_y\partial_t n_x$ yields an additional Berry phase, as discussed previously in Ref. [7]. The term proportional to $\mathbf{n}\cdot\nabla\varphi$ is a crucial contribution stemming from the Coulomb interaction between the fermions at the interface. Indeed, since φ is a fluctuating scalar potential associated to the Coulomb interaction, this term yields a contribution proportional to $\mathbf{M}\cdot\mathbf{E}$, where $\mathbf{E} = -\nabla\varphi$ is a fluctuation-induced electric field, and $\mathbf{M} \sim \mathbf{n}$. Thus, this term corresponds to an emergent axionlike term.

The Landau-Lifshitz equation for the magnetization dynamics at the interface (i.e., at $z = 0$) can now be obtained from the Euler-Lagrange equation for the effective action. We have,

$$\partial_t\mathbf{n} = \mathbf{n}\times\Big\{\rho_s^{ij}(\nabla^2\mathbf{n})_j\mathbf{e}_i + \frac{Z\sigma_{xy}}{2\pi v^2 m_\psi}[(\partial_t^2\mathbf{n})_z\mathbf{e}_z - \nabla(\nabla\cdot\mathbf{n})]\Big\} + \frac{Z\sigma_{xy}}{2\pi v^2}\Big[\mathbf{n}\times\mathbf{E} + \frac{1}{3m_\psi}(\mathbf{n}\cdot\mathbf{e}_z)\partial_t\mathbf{E}\Big], \quad (8)$$

where the stiffness matrix elements are given by $\rho_s^{ij} = (Z/v^2)[\kappa(\delta_{ix}\delta_{jx} + \delta_{iy}\delta_{jy}) + (\kappa + \sigma_{xy}/(2\pi m_\psi))\delta_{iz}\delta_{jz}]$, with $Z = [1 - m_\psi\sigma_{xy}/(2\pi v^2 J)]^{-1}$. For the LL equation in the bulk one has to supply the boundary conditions for the bulk magnetization, which must reflect the influence of the surface states of the TI over some penetration depth into the bulk ferromagnet, assumed to be semi-infinite, having a surface at $z = 0$ coinciding with the interface with the TI. The relevant boundary conditions at $t = 0$ are, $\partial_z\mathbf{n}|_{z=0} = -J\langle c^\dagger\vec{\sigma}c\rangle$, $\partial_z\mathbf{n}|_{z=\infty} = 0$, and $\lim_{z\to\infty}\mathbf{n}(\mathbf{r}, z) = \mathbf{n}_b(\mathbf{r})$, where $\mathbf{n}_b(\mathbf{r})$ is the bulk magnetization far away from the interface. Let us see how these boundary conditions work within a simple mean-field approximation at $T = 0$ and consider the following ansatz for the magnetization precession in the bulk ferromagnet, $\mathbf{n}(\mathbf{r}, z, t) = 2^{-1/2}\sigma(z)[\cos(\mathbf{k}\cdot\mathbf{r} - \omega t)\mathbf{e}_x + \sin(\mathbf{k}\cdot\mathbf{r} - \omega t)\mathbf{e}_y + \mathbf{e}_z]$, where $\omega \propto k^2$. Thus, we have $\mathbf{n}^2 = \sigma^2(z)$. The boundary conditions at the interface are $\sigma(\infty) = \sigma_b$, $\partial_z\sigma|_{z=0} = -J^3\sigma_0^2/(2\pi v^2)$, where $\sigma_0 = \sigma(0) = m_\psi/J$. We will define the k-dependent length characterizing the longitudinal magnetization in the bulk, $\xi_b(k) = (a_0T_0/\kappa - k^2)^{-1/2}$, where $k^2 < a_0T_0/\kappa$. The magnetization σ can now be determined [19] by solving exactly the equation $\partial^2\sigma/\partial z^2 + \xi_b^{-2}\sigma - (u/6\kappa)\sigma^3 = 0$. We obtain $\sigma(z) = \sigma_b(1 - \Delta\sigma e^{-\sqrt{2}z/\xi_b})^{-1}(1 + \Delta\sigma e^{-\sqrt{2}z/\xi_b})$, where $\Delta\sigma = (\sigma_0 - \sigma_b)/(\sigma_0 + \sigma_b)$. The boundary condition at $z = 0$ yields $8\pi v^2\sigma_b\Delta\sigma = 2^{-1/2}J^3\xi_b(1 - \Delta\sigma)^2\sigma_0^2$, which determines ξ_b in terms of σ_0 and σ_b. Note that this condition yields $\sigma_0 = \sigma_b$ for $J = 0$, as it should. This calculation shows that electrons on the surface of the TI influence the magnetization dynamics of the bulk over a characteristic length $\sim\xi_b$ which is uniquely determined by the boundary conditions.

Equation (8) is one of the main results of this Letter. It leads to a fluctuation-induced magnetoelectric effect. One important consequence of Eq. (8) is that due to the fluctuation-induced electric field, the magnitude of the magnetization is not constant, as it would be in the case of the absence of a Coulomb interaction or for a constant electric field. In particular, if the electric field is only due to external effects, this result implies that we can use a time-dependent electric field to control the magnitude of the magnetization.

Part of the coupling to the electric field, discussed previously by Garate and Franz [9], is reproduced here as a fluctuation effect due to the Coulomb interaction between the spin-orbit coupled electrons lying on the surface of a TI. We have obtained in addition a contribution involving $\partial_t\mathbf{E}$ that accounts for the time dependence of the electric field. Note that the term involving $(\partial_t^2\mathbf{n})_z$ is typically small at low energy and can be safely neglected in most calculations.

Next we discuss the case of an AF layer on a TI substrate at zero temperature, which, as we will see, differs fundamentally from the case of a FM layer. In the AF case a quantum phase transition occurs at the interface. In order to study the phase structure of the theory in this case, we will work only in imaginary time from now on. Specifically, we consider a Euclidean effective field theory whose Lagrangian has the form,

$$\mathcal{L} = \bar{\psi}(\not{\partial} - ig_1\not{a} + g_2\sigma)\psi + \frac{\zeta}{2}a_0|\nabla|a_0 + \frac{1}{2}[(\partial_\mu\sigma)^2 + (\partial_\mu\mathbf{a})^2] + \frac{M^2}{2}(\sigma^2 + \mathbf{a}^2) + \frac{\lambda}{4!}(\sigma^2 + \mathbf{a}^2)^2, \quad (9)$$

where we are not making any longer a distinction between the upper and lower covariant indices, since the metric of the theory has now a Euclidean signature. Note that we are also assuming that $g_1 \neq g_2$, as quantum fluctuations induce an anisotropy. In the spirit of effective field theories, the coupling constants are understood as effective parameters to be determined by a renormalization group flow. Thus, the phase structure of the theory is completely determined by the renormalization group equations for the coupling constants.

At low energies and one-loop order (see below) the fixed point structure will be governed by the dimensionless couplings $\hat{g}_i^2 = g_{i,r}^2/M_r^\epsilon$ $(i = 1,\ 2)$ and $\hat{\lambda} = \lambda_r/M_r^\epsilon$, where λ_r and $g_{i,r}$ are corresponding renormalized couplings and we are using the renormalized mass M_r as the renormalization scale [20]. Here $\epsilon = 4 - d$, where $d = D + 1$, with D being the spatial dimension. Our analysis is done in the framework of the ϵ expansion, which is carried out up to one-loop order. As usual, in such a renormalization scheme the renormalized mass gives the inverse of the correlation length, i.e., $M_r = \xi^{-1}$. Due to the coupling between σ and the fermions, a mass anisotropy will be generated, defining in this way two correlation lengths, related to longitudinal and transversal fluctuations. We will assume that ξ refers to the longitudinal correlation length, giving the fluctuations of the σ field. The correlation length due to transversal fluctuations will be denoted by $\xi_\perp$. If ν and $\nu_\perp$ are respectively the critical exponents of the longitudinal and transversal correlation lengths, we easily obtain that $\xi \sim \xi^{\nu_\perp/\nu}$, which determines the crossover exponent $\phi = \nu/\nu_\perp$. The quantum critical behavior can be derived from a generalization of the extended Gross-Neveu model [21] discussed in Ref. [22]. We obtain in this way the one-loop β functions $\beta_{\hat{g}_1^2} \equiv M_r\partial\hat{g}_1^2/\partial M_r$, $\beta_{\hat{g}_2^2} \equiv M_r\partial\hat{g}_2^2/\partial M_r$ and $\beta_{\hat{\lambda}} \equiv M_r\partial\hat{\lambda}/\partial M_r$ in the form, $\beta_{\hat{g}_1^2} = -\epsilon\hat{g}_1^2 + N\hat{g}_1^4/(12\pi^2)$, $\beta_{\hat{g}_2^2} = -\epsilon\hat{g}_2^2 + (N + 3)\hat{g}_2^4/(8\pi^2)$, and $\beta_{\hat{\lambda}} = -\epsilon\hat{u} + (8\pi^2)^{-1}[(11/2)\hat{\lambda}^2 + 2N\hat{\lambda}\hat{g}_2^2 - 12N\hat{g}_2^4]$. The β function for $\hat{\zeta} = \zeta_r/M_r$ follows from the nonlocality of the quadratic term in a_0. Since counterterms are local, this term does not renormalize, which implies simply $\beta_{\hat{\zeta}} = (N\hat{g}_1^2/12\pi^2 - \epsilon)\hat{\zeta}$.

The quantum critical point is determined by demanding that the β functions vanish, which yields the infrared stable fixed points, $\hat{g}_{1*}^2 = 12\pi^2\epsilon/N$, $\hat{g}_{2*}^2 = 8\pi^2\epsilon/(N + 3)$, and $\hat{\lambda}_* = 8\pi^2\epsilon(3 - N + \sqrt{N^2 + 258N + 9})/[11(N + 3)]$. The anomalous dimension η_N of the Néel order parameter at the interface can be defined via the scaling behavior $\langle\sigma\rangle \sim M_r^{(2-\epsilon+\eta_N)/2}$, and is given at one loop by $\eta_N = N\hat{g}_{2*}^2/(8\pi^2) = N\epsilon/(N + 3)$. For $N = 1$ and two spatial dimensions (corresponding to $\epsilon = 1$), we obtain $\eta_N = 1/4$. This large value of the anomalous dimension, as compared to the value obtained from the $O(3)$ universality class, reflects the fact that $\langle\sigma\rangle$ receives contributions from the composite operator $\bar{\psi}\psi$. The scaling behavior of

$\mathbf{n} = (n_x, n_y, \sigma)$ at the interface is anisotropic, and the transversal fluctuations have a different anomalous dimension, which is dominantly determined by the vacuum polarization diagrams, $\eta_N^{\perp} = \epsilon$, yielding $\eta_N^{\perp} = 1$ for $D = 2$. It is worth mentioning that two-loop corrections will be small, but positive (typically $\sim$0.03). Therefore, we expect that a more accurate value for the anomalous dimension $\eta_N^{\perp}$ is slightly above unity.

The electrons at the interface also have an anomalous scaling at the quantum critical point. This is in contrast with the FM case, where the fermionic spectrum is always gapped at zero temperature. Thus, we obtain the low-energy behavior, $\langle \bar{\psi}(p)\psi(p)\rangle \sim -i\not{p}/p^{2-\eta_\psi}$, where $\eta_\psi = \hat{g}_{2*}^2/(16\pi^2) = \epsilon/[2(N+3)]$. For $D = 2$ and $N = 1$, we obtain $\eta_\psi = 1/8$. Note that η_ψ does not receive any contribution from the fixed point $\hat{g}_{1,*}^2$ at one-loop order. This is due to the fact that the vector field propagator takes here the same form as the one in QED where the Feynman gauge has been fixed.

It remains to compute the critical exponents of the correlation lengths. The longitudinal correlation length exponent is given by $\nu = (2 + \eta_M)^{-1}$, where at one loop, $\eta_M = -5\hat{\lambda}_*/(48\pi^2) - \eta_N$. Thus, by expanding up to first order in ϵ, we obtain, $\nu \approx 1/2 + \epsilon[4(N+3)]^{-1}[(5/66)(3 - N + \sqrt{N^2 + 258N + 9}) + N]$. Setting once more $D = 2$ and $N = 1$, we obtain $\nu \approx 0.649$. The transversal correlation length exponent, on the other hand, is given by $\nu_\perp = (2 + \eta_M^{\perp})$, where $\eta_M^{\perp} = \eta_M - \eta_N^{\perp} + \eta_N$. Thus, we obtain $\nu_\perp \approx \nu + 3\epsilon/[4(N+3)]$, which for $N = 1$ and $D = 2$ yields $\nu_\perp \approx 0.83$. Note that the values of the correlation length exponents differ appreciably from the one-loop value of the $O(3)$ universality class, $\nu_{O(3)}^{\text{one-loop}} \approx 0.614$.

In conclusion, we have shown in the FM case that an axionlike term is generated in the form of a CS term, which in turn modifies the magnetization dynamics of the LL equation. Furthermore, we have shown that for a specific temperature window it is possible to have gapless fermions at the interface and, at the same time, a ferromagnetically ordered layer.

For the case of an AF layer, we have shown that a quantum phase transition occurs at the interface, and that the fermion spectrum becomes gapless at the QCP. Moreover, large values of the anomalous dimensions for the Néel order parameter were obtained.

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