Microscopic Model of Quasiparticle Wave Packets in Superfluids, Superconductors, and Paired Hall States

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We study the structure of Bogoliubov quasiparticles, bogolons, the fermionic excitations of paired superfluids that arise from fermion (BCS) pairing, including neutral superfluids, superconductors, and paired quantum Hall states. The naive construction of a stationary quasiparticle in which the deformation of the pair field is neglected leads to a contradiction: it carries a net electrical current even though it does not move. However, treating the pair field self-consistently resolves this problem: in a neutral superfluid, a dipolar current pattern is associated with the quasiparticle for which the total current vanishes. When Maxwell electrodynamics is included, as appropriate to a superconductor, this pattern is confined over a penetration depth. For paired quantum Hall states of composite fermions, the Maxwell term is replaced by a Chern-Simons term, which leads to a dipolar *charge* distribution and consequently to a dipolar current pattern.

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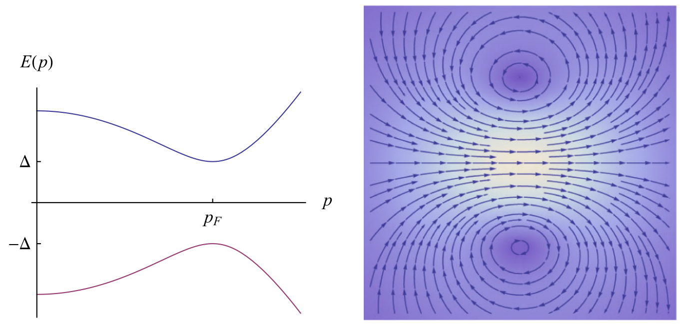
Introduction.—Paired superfluids are among the most ubiquitous of the many ordered phases of interacting fermions in two and three dimensions. In condensed matter settings they include both neutral superfluids such as ^{3}He, charged superconductors, and now also paired quantum Hall (QH) liquids such as the Moore-Read or Pfaffian state that is believed to underlie the quantized Hall plateau at filling factor $\nu = 5/2$ [1]. In all cases paired superfluids exhibit two distinct excitations that dominate much of their physics: vortices and Bogoliubov quasiparticles or bogolons. The former are a generic consequence of superfluidity, but the latter are a particular signature of pairing-they involve breaking apart a Cooper pair into its fermionic constituents.

The structure of vortices is well understood: they are the topological solitons of a complex scalar order parameter in the Landau-Ginzburg description of a superfluid. In a superconductor, additional coupling to a Maxwell gauge field results in an associated quantum of flux, while for a quantum Hall liquid, coupling to a Chern-Simons gauge field associates a quantized charge with each vortex. The structure of bogolons is less well understood as they are, by comparison, much more quantum mechanical particles. We will address that gap by providing a theoretical analysis of their structure for all three examples alluded to above. For superfluids and superconductors we will be able to recover the heuristic description advanced by Kivelson and Rokhsar [2]. For paired quantum Hall liquids our results are new and add to a recent burst of interest in the properties of bogolons [3,4], including work by four of the present authors [5].

In the weak pairing (BCS) limit the momentum (or Bloch) eigenstates of the bogolon exhibit the well known dispersion relation sketched in Fig. 1(a), with a characteristic minimum at the underlying Fermi surface. In terms of these, one can make a localized wave packet state with a spatial extent large compared to the coherence length, ξ, and a well-defined momentum. Unlike a wave packet in the normal state, this bogolon wave packet has a group velocity which is different than the Fermi velocity v_F, and which vanishes on the Fermi surface. It has spin $\frac{1}{2}$, but its (average) charge is smaller than the charge of electron e. Both quantities vanish as the momentum of quasiparticles p approach the Fermi momentum p_F. On the other hand, since the wave packet has a net momentum, it carries a net current [6] equal to ev_F. This indicates that our construction of a localized bogolon is fundamentally inadequate. The problem becomes especially clear in the limit $p = p_F$, where the group velocity of the wave packet is zero. In this case the current density is finite inside the wave packet and zero outside of it. The resolution of this puzzle will lead us to a bogolon structure that involves an algebraically falling, dipolar, return current flow via the condensate for neutral superfluids, a version of this screened on the scale of the London length for superconductors, and a version exhibiting a charge dipole as well as a locally dipolar backflow for two dimensional quantum Hall fluids. Altogether, bogolons are fairly complicated objects!

Bogolon wave packet.—We begin with the mean-field BCS Hamiltonian for a neutral fully gapped paired superfluid which also serves to fix notation,



(a) (b)

FIG. 1 (color online). (a) Quasiparticle dispersion. (b) Current flow around neutral superfluid bogolon (window size $\sim 2\lambda$.)

$$H_{\rm BCS} = \sum_{\substack{\mathbf{k}\\ s=\uparrow,\downarrow}} \xi_{\mathbf{k}} c^\dagger_{\mathbf{k}s} c_{\mathbf{k}s} + \sum_{\mathbf{k}} [\Delta_{\mathbf{k}} c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} + \text{H.c.}], \tag{1}$$

where $\xi_{\mathbf{k}} = \frac{k^2}{2m} - \mu$, $\Delta_{\mathbf{k}}$ is the gap function and we work in units where $\hbar = e/c = 1$. It is a simple matter to diagonalize, $H_{\rm BCS} = \sum_{\mathbf{k},s} E_{\mathbf{k}} \gamma^\dagger_{\mathbf{k},s}\gamma_{\mathbf{k},s}$, with $E_{\mathbf{k}} = \sqrt{\xi^2_{\mathbf{k}} + |\Delta_{\mathbf{k}}|^2}$ by means of a Bogoliubov transformation,

$$\gamma_{\mathbf{k},\uparrow} = u_{\mathbf{k}} c_{\mathbf{k},\uparrow} - v_{\mathbf{k}} c^\dagger_{-\mathbf{k},\downarrow}, \qquad \gamma_{\mathbf{k},\downarrow} = v_{\mathbf{k}} c^\dagger_{\mathbf{k},\uparrow} + u_{\mathbf{k}} c_{-\mathbf{k},\downarrow},$$
$$|u_{\mathbf{k}}|^2 = \frac{1}{2}\left(1 + \frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}}\right), \qquad |v_{\mathbf{k}}|^2 = \frac{1}{2}\left(1 - \frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}}\right). \tag{2}$$

The ($T = 0$) BCS ground state is then the state annihilated by all the $\gamma_{\mathbf{k}s}$, $|\Omega\rangle = \prod_{\mathbf{k}\geq 0}(u_{\mathbf{k}} + v_{\mathbf{k}} c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow})|0\rangle$. Single-quasiparticle states with momentum $\mathbf{k}$ and spin s are given by $|\mathbf{k}s\rangle = \gamma^\dagger_{\mathbf{k}s}|\Omega\rangle$ and it is readily verified that their energy $E_{\mathbf{k}}$ is minimal at $|\mathbf{k}| = k_F$. We will work in $d = 2$ as that naturally includes the case of the paired QH state, but the results are readily generalized to $d = 3$.

A quasiparticle wave packet with average momentum $\hbar\mathbf{k}_0 = \hbar k_F \hat{\mathbf{k}}_0$, spin s, and spatial extent $\sim\lambda$ is obtained by superposing the states $|\mathbf{k}s\rangle$ with momenta near $\mathbf{k}_0$

$$|\Psi^\lambda_{\mathbf{k}_0,s}\rangle = \left(\frac{\lambda}{\sqrt{\pi}}\right)^{d/2} \int d^d k e^{-(1/2)\lambda^2(\mathbf{k}-\mathbf{k}_0)^2}|\mathbf{k}s\rangle. \tag{3}$$

In order that the energy uncertainty of the wave packet be smaller than its average energy, we need to choose $\lambda \gg \xi = \frac{v_F}{\Delta_0}$ as can be deduced from the low lying dispersion relation $E(\mathbf{k}) \approx \Delta_0 + \frac{[v_F \hat{\mathbf{k}}_0 \cdot (\mathbf{k}-\mathbf{k}_0)]^2}{2\Delta_0}$ where $\Delta_0 \equiv |\Delta_{\mathbf{k}_0}|$ and $v_F = k_F/m$ is the Fermi velocity.

Our primary concern is the structure of quasiparticle wave packets centered at momenta close to p_F, so that their group velocity is much smaller than v_F. Clearly, the packet has vanishing group velocity at $p = p_F$. However, a tedious but straightforward computation of the expectation value of the quasiparticle current operator $\mathbf{j}^{\rm qp}_{\mathbf{q}} = \sum_{\mathbf{k},s} \frac{\mathbf{k}}{m} c^\dagger_{\mathbf{k}+\frac{\mathbf{q}}{2}s} c_{\mathbf{k}-(\mathbf{q}/2)s}$ in the state yields [7]

$$\langle \mathbf{j}^{\rm qp}_{\mathbf{q}}\rangle_\Psi = v_F \hat{\mathbf{k}}_0 e^{-(\lambda^2\mathbf{q}^2)4}. \tag{4}$$

We are thus presented with a contradiction: a *stationary* quasiparticle wave packet is associated with a current that has nonzero divergence—violating the continuity equation.

A first step in resolving this puzzle is to observe that we have taken a slippery step in passing from momentum space to real space. In real space, the wave packet state (3) is now inhomogeneous and hence a homogeneous "pair potential" Δ no longer yields a self-consistent mean field theory of the wave packet [8]. It is possible to prove that any state that satisfies the self-consistency conditions respects the equation of continuity. Recomputing the pair potential in the wave packet state and then iterating the construction of the wave packet and the computation of the pair potential should yield a state that does obey current conservation [9]. In the Supplemental Material [10], we show that the first iteration of this process produces a change in the pair potential that already partially cancels the quasiparticle current.

However, implementing this approach requires detailed numerical work. Instead, we construct an effective action which correctly treats the low-energy, long-wavelength physics in the weak coupling limit, $\Delta_0 \ll E_F = k_F^2/2m$. While portions of this work may be reconstructed from existing literature, in particular the "conserving approximations" [11–15] to superconducting response, to our knowledge an explicit quantitative treatment of a bogolon wave packet has not been previously presented.

Neutral superfluids.—As we are interested in a wave packet constructed from momenta very close to the (parent) Fermi surface, it is sufficient that we work with the effective dynamics for this set of degrees of freedom. Formally, we begin with a Hubbard-Stratonovich (HS) decoupling of an attractive four-fermion interaction in the particle-particle channel, and integrate out fermions above a cutoff thus generating an effective action for the HS field, $\Delta(\mathbf{r}, t) = \Delta_0 e^{i\theta(\mathbf{r},t)}$. As we are in the broken-symmetry phase, fluctuations of the amplitude can be neglected. The result is an effective theory of dynamical fermions coupled to a dynamical phase field $\theta(\mathbf{r}, t)$ [16].

To be explicit, we consider the case of s-wave pairing, where the most important terms in this (well known) theory are represented by the action $S = \int dt d^2 r(\mathcal{L}_\psi + \mathcal{L}_p + \mathcal{L}_\theta)$, with

$$\mathcal{L}_\psi = \sum_s \psi_s^\dagger(\mathbf{r},t)\left[i\partial_t - \mu - \frac{\nabla^2}{2m}\right]\psi_s(\mathbf{r},t), \tag{5}$$

$$\mathcal{L}_p = -\Delta_0 e^{i\theta(\mathbf{r},t)}\psi_\uparrow^\dagger(\mathbf{r},t)\psi_\downarrow^\dagger(\mathbf{r},t) + \text{H.c.}, \tag{6}$$

$$\mathcal{L}_\theta = -\frac{\chi_0}{2}(\partial_t\theta)^2 + \frac{n_s}{2m}(\nabla\theta)^2, \tag{7}$$

where χ_0 is the static compressibility (equal to the density of states at the Fermi surface), and n_s is the superfluid density. At $T = 0$, $n_s = \rho$, the total electronic density, for the Galilean invariant systems considered here.

Note that the conserved charge is no longer carried solely by the fermions, but also by the superfluid component via twists in the order parameter. A straightforward application of the Noether procedure allows us to write, for the density and current

$$\rho = \rho^{\text{qp}} - \chi_0\partial_t\theta, \qquad \mathbf{j} = \mathbf{j}^{\text{qp}} + \frac{n_s}{2m}\boldsymbol{\nabla}\theta, \tag{8}$$

where $\rho^{\text{qp}} = \sum_s \psi_s^\dagger\psi_s$ and $\mathbf{j}^{\text{qp}} = \sum_s \frac{1}{m}\,\text{Im}[\psi_s^\dagger\boldsymbol{\nabla}\psi_s]$. From S we then obtain the equations of motion

$$\partial_t\rho^{\text{qp}} = -\boldsymbol{\nabla}\cdot\mathbf{j}^{\text{qp}} + \mathcal{B}_p, \tag{9}$$

$$\chi_0\partial_t^2\theta = \frac{n_s}{2m}\nabla^2\theta + \mathcal{B}_p, \tag{10}$$

where $\mathcal{B}_p \equiv 2i\Delta_0(e^{i\theta}\psi_\uparrow^\dagger\psi_\downarrow^\dagger - e^{-i\theta}\psi_\downarrow\psi_\uparrow)$ is the term that couples the quasiparticles and the superfluid. From Eqs. (8)–(10), it is evident that $\partial_t\rho + \boldsymbol{\nabla}\cdot\mathbf{j} = 0$, i.e., the properly defined density and current obey the continuity equation; it is equally clear that the quasiparticle density is *not* independently conserved.

Let us now specialize to the treatment of a stationary bogolon wave packet in the approximation where we ignore the quantum fluctuations of θ. This implies that the left-hand side of Eqs. (9) and (10) vanish so that

$$\langle\boldsymbol{\nabla}\cdot\mathbf{j}^{\text{qp}}\rangle = \langle\mathcal{B}_p\rangle = -\frac{n_s}{2m}\langle\nabla^2\theta\rangle. \tag{11}$$

Thus, in the wave packet state for which $\langle\mathbf{j}_\mathbf{q}^{\text{qp}}\rangle_\Psi$ is given by Eq. (4), the resulting phase texture is

$$\langle\theta_\mathbf{q}\rangle_\Psi = \frac{i(\mathbf{q}\cdot\hat{\mathbf{k}}_0)}{q^2}\left(\frac{2k_F}{n_s}\right)e^{-(\lambda^2q^2)/4}, \tag{12}$$

which permits us to write for the total current

$$\langle\mathbf{j}_\mathbf{q}\rangle_\Psi = v_F\left[\frac{q^2\hat{\mathbf{k}}_0 - (\mathbf{q}\cdot\hat{\mathbf{k}}_0)\mathbf{q}}{q^2}\right]e^{-(\lambda^2q^2)/4}. \tag{13}$$

Equation (13) corresponds to a real space current $\langle\mathbf{j}(\mathbf{r})\rangle_\Psi = \hat{\mathbf{z}}\times\boldsymbol{\nabla}\varphi_\lambda(\mathbf{r})$, where $\varphi_\lambda(\mathbf{r}) \equiv 2\pi v_F\frac{(\hat{\mathbf{k}}_0\times\mathbf{r})\cdot\hat{\mathbf{z}}}{r^2}(1 - e^{-r^2/\lambda^2})$.

The flow pattern is solenoidal (clearly $\boldsymbol{\nabla}\cdot\langle\mathbf{j}\rangle_\Psi = 0$), and decays as r^{-2} far from the center of the wave packet. Corrections to this expression at short distances are nonuniversal, and are beyond the reach of the field-theory approach. Finally, we note that at *finite* quasiparticle concentration $\overline{\rho^{\text{qp}}}$, the long-range nature of the distribution of current density in a quasiparticle wave packet leads to the conventional expression $\mathbf{j}^{\text{qp}} = e\mathbf{v}_F\rho^{\text{qp}}$ for the quasiparticle contribution to the current density, in agreement with the Boltzmann approach [17] applicable in this limit.

Superconductors.—We now turn to the case of a charged superfluid which is minimally coupled to a fluctuating $U(1)$ Maxwell gauge field A_μ-i.e., the superconductor with dynamical electromagnetism. The effective action is obtained from that of the neutral superfluid by converting the derivatives to covariant derivatives: $\partial_\mu \to D_\mu = \partial_\mu - iA_\mu$, where the dynamics of A_μ are described by $\mathcal{L}_{\text{Maxwell}} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ in which $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the Maxwell field strength. From $S + S_{\text{Maxwell}}$, we find the equations of motion for the quasiparticle and superfluid currents

$$\begin{aligned}&\rho = \rho^{\text{qp}} - \chi_0(\partial_t\theta - 2A_0), \quad \mathbf{j} = \mathbf{j}^{\text{qp}} + \frac{n_s}{2m}(\boldsymbol{\nabla}\theta - 2\mathbf{A}),\\ &\partial_t\rho^{\text{qp}} = -\boldsymbol{\nabla}\cdot\mathbf{j}^{\text{qp}} + \mathcal{B}_p, \quad \chi_0\partial_t(\partial_t\theta - 2A_0)\\ &\quad = \frac{n_s}{2m}\boldsymbol{\nabla}\cdot(\boldsymbol{\nabla}\theta - 2\mathbf{A}) + \mathcal{B}_p,\end{aligned} \tag{14}$$

supplemented by Maxwell's equations

$$\begin{aligned}&\boldsymbol{\nabla}\cdot\mathbf{E} = 4\pi(\rho - \bar{\rho}), \qquad \boldsymbol{\nabla}\cdot\mathbf{B} = 0,\\ &\boldsymbol{\nabla}\times\mathbf{B} = 4\pi\mathbf{j} + \partial_t\mathbf{E}, \qquad \boldsymbol{\nabla}\times\mathbf{E} = -\partial_t\mathbf{B}.\end{aligned} \tag{15}$$

In (14) and (15) the quasiparticle current and density take their gauge-invariant forms, $\rho^{\text{qp}} = \sum_s\psi_s^\dagger\psi_s$ and $\mathbf{j}^{\text{qp}} = \sum_s\frac{1}{m}\,\text{Im}[\psi_s^\dagger\mathbf{D}\psi_s]$, and $\mathbf{E} = -\partial_t\mathbf{A} - \boldsymbol{\nabla}A_0$ and $\mathbf{B} = \boldsymbol{\nabla}\times\mathbf{A}$ are the electric and magnetic fields in the quasiparticle state; in writing the Poisson equation we have assumed the existence of a neutralizing positive background $\bar{\rho} = \langle\sum_s\psi_s^\dagger\psi_s\rangle_\Omega$ in the BCS ground state.

The first comment to be made here is that now even extended bogolon states of definite momentum do not carry current. This basically reflects the Meissner effect. Specifically, the uniform quasiparticle contribution to the current is exactly canceled by a superfluid backflow, which in unitary gauge $\theta = 0$, corresponds to $\frac{n_s}{m}\mathbf{A} = \langle\mathbf{j}^{\text{qp}}\rangle \propto v_F\hat{\mathbf{k}}_0$ [18]. The correct bogolon state carries no current; they are neutral particles.

Still in unitary gauge, let us turn to the construction of the wave packet. For static wave packets we find that the third equation of (14) yields $\frac{n_s}{m}\boldsymbol{\nabla}\cdot\mathbf{A} = \langle\mathcal{B}_p\rangle = \langle\boldsymbol{\nabla}\cdot\mathbf{j}^{\text{qp}}\rangle$, so that as before the total current $\mathbf{j} = \mathbf{j}^{\text{qp}} - \frac{n_s}{m}\mathbf{A}$ is conserved. Using this, we rewrite the third Maxwell equation as

$$[-\nabla^2 + \lambda_L^{-2}]\mathbf{A} = 4\pi\langle\mathbf{j}^{\text{qp}} - \lambda_L^2\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\mathbf{j}^{\text{qp}})\rangle, \tag{16}$$

where we have defined the penetration depth via $\lambda_L^{-2} = \frac{4\pi n_s}{m}$ and the expectation value is taken in the naive wave packet state with $A_0 = \mathbf{A} = 0$. Using either the expectation value of $\langle \mathcal{B}_p \rangle$ computed in the superfluid case or the form of $\mathbf{j}^{\rm qp}$, we may solve (16) by Fourier analysis:

$$\langle \mathbf{j}_{\mathbf{q}} \rangle_\Psi = v_F \left[\frac{q^2 \hat{\mathbf{k}}_0 - (\mathbf{q} \cdot \hat{\mathbf{k}}_0)\mathbf{q}}{q^2 + \lambda_L^{-2}} \right] e^{-(\lambda^2 q^2/4)}, \tag{17}$$

which coincides with (13) in the limit in which the coupling to electromagnetism vanishes (when $\lambda_L \to \infty$). It is easy to see that the power-law asymptotics of the superfluid case are replaced by exponential behavior at long distances, $\mathbf{j}(\mathbf{r}) \sim e^{-r/\lambda_L}$ for $r \gg \lambda$ and λ_L. This reflects the fact that superconductors screen magnetic fields and thus the current pattern is confined to within a penetration depth of the center of the bogolon [19]. (Note that the short-distance behavior of the wave packet is qualitatively different depending on whether the superconductor is type I (or weakly type II) in which case λ_L completely characterizes the current distribution, or strongly type II, with $\xi_0 \ll \lambda \ll \lambda_L$, in which case the bogolon resembles that in a neutral superfluid for $r \ll \lambda_L$.)

Paired QH states.—Our final example is the case of a bogolon in a paired QH state of composite fermions (CFs) [1]. Here, we start with fermions moving in a *static* uniform background field $\mathbf{A}$ (where $\nabla \times \mathbf{A} = \mathbf{B}$), and perform a "flux attachment" by means of a statistical gauge field $\mathbf{a}$ whose dynamics are governed by a Chern-Simons (CS) term, $\mathcal{L}_{\rm CS} = \frac{1}{4\Phi_0} \epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho$, with Φ_0 the quantum of flux. Qualitatively, the role of the CS gauge field is to attach two quanta of magnetic flux to each electron to convert it into a CF, which sees zero net flux at half-filling, i.e., we have $\mathbf{B} = 2\Phi_0 \bar{\rho}$. In this case, we replace $\partial_\mu \to D_\mu = \partial_\mu - i(a + A)_\mu$, and change the currents and densities accordingly. Although more properly we should consider the example of spinless fermions and p-wave pairing, the distinction is unimportant as we are primarily interested in the interplay of the CS electrodynamics and charge conservation, neither of which depends essentially on the pairing symmetry. The equations of motion now follow as a result of $S + S_{\rm CS}$: the "matter" equations are similar to the previous example,

$$\begin{aligned} \rho &= \rho^{\rm qp} - \chi_0[\partial_t \theta - 2(a_0 + A_0)], \\ \mathbf{j} &= \mathbf{j}^{\rm qp} + \frac{n_s}{2m}[\nabla \theta - 2(\mathbf{a} + \mathbf{A})], \\ \partial_t \rho^{\rm qp} &= -\nabla \cdot \mathbf{j}^{\rm qp} + \mathcal{B}_p, \ \chi_0 \partial_t[\partial_t \theta - 2(a_0 + A_0)] \\ &= \frac{n_s}{2m} \nabla \cdot [\nabla \theta - 2(\mathbf{a} + \mathbf{A})] + \mathcal{B}_p, \end{aligned} \tag{18}$$

but the Maxwell equations are replaced by the CS equations, which are pure constraints:

$$\begin{aligned} b &\equiv \nabla \times \mathbf{a} = -2\Phi_0\{\rho^{\rm qp} - \chi_0[\partial_t \theta - 2(a_0 + A_0)]\}, \\ \mathbf{e} &\equiv -\partial_t \mathbf{a} - \nabla a_0 = 2\Phi_0 \hat{\mathbf{z}} \times \left\{ \mathbf{j}^{\rm qp} + \frac{n_s}{2m}[\nabla \theta - 2(\mathbf{a} + \mathbf{A})] \right\}. \end{aligned} \tag{19}$$

Note that now, $\mathbf{A}$ is not a dynamical field, but rather represents the background magnetic field, $\frac{\nabla \times \mathbf{A}}{2\Phi_0} = \bar{\rho}$, where $\bar{\rho}$ is the mean density, $\langle \rho_F \rangle_0$, in the ground state, and, for the present, we will set the external potential $A_0 = 0$ [20]. Proceeding to the wave packet state, and specializing to unitary gauge and to static configurations as in the previous examples we use the identity $\nabla^2 \mathbf{V} = \hat{\mathbf{z}} \times \nabla[\nabla \times \mathbf{V}] + \nabla[\nabla \cdot \mathbf{V}]$ (valid in $d = 2$) to write

$$[-\nabla^2 + \lambda_{\rm CS}^{-2}](\mathbf{a} + \mathbf{A}) = 8\chi_0 \Phi_0^2 \langle \mathbf{j}^{\rm qp} - \lambda_{\rm CS}^2 \nabla(\nabla \cdot \mathbf{j}^{\rm qp}) \rangle,$$

where $\lambda_{\rm CS}^{-2} = \frac{8\chi_0 \Phi_0^2 n_s}{m}$. The solution for the total field $\mathbf{a} + \mathbf{A}$ is similar to (17); observe that the flux is exponentially screened over a distance $\lambda_{\rm CS}$. Using $\chi_0 \sim \frac{m}{2\pi}$ as appropriate to a 2D Fermi surface, and the QH relation $\bar{\rho} = 1/4\pi\ell_B^2$ for filling factor $\nu = 1/2$ where ℓ_B is the magnetic length, we find $\lambda_{\rm CS} \sim \frac{1}{2}\ell_B(\bar{\rho}/n_s)^{1/2}$, so for a Galilean invariant system at $T = 0$ where $n_s = \bar{\rho}$, we find $\lambda_{\rm CS} \sim \frac{1}{2}\ell_B$. Thus, the characteristic size of a bogolon wave packet is of order the magnetic length. A striking difference from the normal superconductor is that the second CS equation forces the existence of an electric field, which leads to a deviation of charge density from the background. The simplest estimate is $\delta\rho \sim \frac{1}{\ell_B^2} \frac{(\hat{\mathbf{k}}_0 \times \mathbf{r}) \cdot \hat{\mathbf{z}}}{r} e^{-2r/\ell_B}$; while this is not the exact form, the important point is that there is *necessarily* a dipolar charge distribution oriented perpendicular to $\vec{k}_0$, with separation $\sim \ell_B$, accompanying the screened dipolar current pattern. Upon inclusion of the long-range Coulomb interaction (ignored so far) [10] both current and charge densities acquire power-law tails similar to those in Ref. [21].

In the QH case, the bogolon has a natural interpretation as the descendant of the CF in the paired phase. Several authors, including one of us [22–25], have observed that upon projection to the lowest Landau level the CF in the compressible phase goes from being a charged particle to a neutral particle with a dipole moment proportional to its speed and perpendicular to its direction of propagation. The argument for a dipolar charge distribution for the bogolon presented here—the application of CS electrodynamics to a paired superfluid—is rather different from projection to a reduced Hilbert space, and the connection between the two cases is an intriguing question that we hope to address in the future. We note that a recent microscopic study [26] reports an excitonic construction of the quantum Hall bogolon in the Pfaffian state that is also consistent with an associated dipolar charge distribution.

Concluding remarks.—In this Letter, we have given a consistent microscopic description of bogolon wave packets in three broad classes of paired fermion states: superfluids, superconductors, and paired composite Fermi liquids with CS electrodynamics. In all cases, the quasiparticle is associated with a decidedly nontrivial current flow pattern carried in part by the condensate, and manifestly obeys global and/or local conservation laws as appropriate. Although for pedagogical simplicity we focused on the case of stationary wave packets, this restriction is merely a matter of convenience: suitably boosted current configurations are associated with bogolons in motion.

Our results are valid in the limit $\overline{\rho^{\rm qp}}\xi^2 \ll 1$ when the concentration of quasiparticle wave packets is small, or in other words when the distance between quasiparticles is much larger than their size. In the opposite limit of high quasiparticle concentration where quasiparticles overlap, the system can be studied using the kinetic equation approach [17]. In this formalism, the Boltzmann equation for the distribution function of quasiparticles $n_{\mathbf{k}}$ is supplemented by equations of motion for the electrodynamic fields and continuity equations expressing charge conservation. The current and charge densities take the form

$$\begin{aligned} \mathbf{j} &= e\rho\mathbf{v}_{\rm s} + e\int d^dk\frac{\mathbf{k}}{m}n_{\mathbf{k}}; \\ \rho &= e\int d^dk[u_{\mathbf{k}}^2 n_{\mathbf{k}} + v_{\mathbf{k}}^2(1-n_{-\mathbf{k}})], \end{aligned} \tag{20}$$

where $\mathbf{v}_s = \frac{1}{2}(\nabla\theta - 2\mathbf{A})$ is the superfluid velocity. The situation is similar to the microscopic scenario discussed here: in order to describe the distribution of $\mathbf{v}_s$, an additional variable included in the kinetic theory (compared to the case of the normal metal), charge conservation must be treated as an independent equation, rather than following directly from the equations of motion.

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[1] G. Moore and N. Read, Nucl. Phys. **B360**, 362 (1991).

[2] S. A. Kivelson and D. S. Rokhsar, Phys. Rev. B **41**, 11693 (1990).

[3] P. Bonderson, A. E. Feiguin, and C. Nayak, Phys. Rev. Lett. **106**, 186802 (2011).

[4] G. Möller, A. Wójs, and N. R. Cooper, Phys. Rev. Lett. **107**, 036803 (2011).

[5] S. A. Parameswaran, S. A. Kivelson, S. L. Sondhi, and B. Z. Spivak, Phys. Rev. Lett. **106**, 236801 (2011).

[6] C. Nayak, K. Shtengel, D. Orgad, M. P. A. Fisher, and S. M. Girvin, Phys. Rev. B **64**, 235113 (2001).

[7] To derive this result we rewrite $c^\dagger$, c in terms of $\gamma^\dagger$, γ and expand $u_{\mathbf{k}}$, $v_{\mathbf{k}}$ around k_F to linear order.

[8] A. R. Bishop, P. S. Lomdahl, J. R. Schrieffer, and S. A. Trugman, Phys. Rev. Lett. **61**, 2709 (1988).

[9] S. A. Parameswaran, S. A. Kivelson, R. Shankar, S. L. Sondhi, and B. Z. Spivak (unpublished).

[10] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.109.237004 for (i) Explicit computations of BdG wave packet properties (ii) First iteration of self-consistency equations showing texturing (iii) Long-range current flow with Coulomb interactions.

[11] P. W. Anderson, Phys. Rev. **110**, 827 (1958).

[12] D. Pines and J. Schrieffer, Il Nuovo Cimento **10**, 496 (1958).

[13] G. Rickayzen, Phys. Rev. **115**, 795 (1959).

[14] Y. Nambu, Phys. Rev. **117**, 648 (1960).

[15] G. Baym and L. P. Kadanoff, Phys. Rev. **124**, 287 (1961).

[16] A. Altland and B. Simons, *Condensed Matter Field Theory* (Cambridge University Press, Cambridge, England, 2006).

[17] A. Aronov, Y. Gal'perin, V. Gurevich, and V. Kozub, Adv. Phys. **30**, 539 (1981).

[18] Here we have in mind bulk bogolon states. The momentum space description breaks down within a London length of any boundaries.

[19] We remark that the otherwise excellent discussion in Ref. [14] seems to only discuss the case where the current pattern decays with a power law as in the superfluid, rather than the superconducting case where perfect screening leads to an exponential profile as computed here.

[20] $A_0 \neq 0$ corresponds to a trapping potential for the quasiparticle—a case which is more involved, and does not offer significantly more insight.

[21] S. L. Sondhi and S. A. Kivelson, Phys. Rev. B **46**, 13319 (1992).

[22] N. Read, Phys. Rev. B **58**, 16262 (1998).

[23] V. Pasquier and F. D. M. Haldane, Nucl. Phys. **B516**, 719 (1998).

[24] D.-H. Lee, Phys. Rev. Lett. **80**, 4745 (1998).

[25] G. Murthy and R. Shankar, Rev. Mod. Phys. **75**, 1101 (2003).

[26] G. J. Sreejith, A. Wójs, and J. K. Jain, Phys. Rev. Lett. **107**, 136802 (2011).