Chiral Anomaly and Local Polarization Effect from the Quantum Kinetic Approach

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A power expansion scheme is set up to determine the Wigner function that satisfies the quantum kinetic equation for spin-1/2 charged fermions in a background electromagnetic field. Vector and axial-vector current induced by magnetic field and vorticity are obtained simultaneously from the Wigner function. The chiral magnetic and vortical effect and chiral anomaly are shown as natural consequences of the quantum kinetic equation. The axial-vector current induced by vorticity is argued to lead to a local polarization effect along the vorticity direction in heavy-ion collisions.

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Introduction.—Chiral anomaly is an important quantum effect, which is absent at the classical level. Recently, it has been shown that such a microscopic quantum effect can have a macroscopic impact on the dynamics of relativistic fluids, termed as the chiral magnetic and vortical effect (CME and CVE) [1–3] as manifested in currents induced by magnetic field and vorticity. Such effects and related topics have been investigated within a variety of approaches, such as AdS-CFT duality [4–8], relativistic hydrodynamics [9–11], and quantum field theory [2,12–17]. However, it is still not clear how CME and CVE can emerge from a microscopic quantum kinetic theory.

In this Letter, we make a first attempt to derive both the CME and CVE from a quantum kinetic theory. A power expansion in space-time derivatives and weak external fields is used to determine the analytic form of vector and axial-vector components of the Wigner function that satisfies the quantum kinetic equation for spin-1/2 massless fermions. The CME and CVE appear naturally in the induced currents. Chiral anomaly and other conservation laws are also automatically satisfied. The axial-vector current induced by vorticity depends quadratically on the temperature, baryonic and chiral chemical potential. So it should be present in both hot and dense matter, and can lead to a local polarization effect in heavy-ion collisions as proposed in earlier studies [18–20]. This provides another possible future experimental measurement of the CVE in high-energy heavy-ion collisions.

The quantum kinetic approach can provide a bridge between the microscopic and macroscopic description of the CME and CVE and should be more suitable for future simulations of both effects in heavy-ion collisions. The power expansion method can also be applied to the calculation of other transport coefficients.

Quantum kinetic equation.—In a quantum kinetic theory, the classical phase-space distribution $f(x, p)$ is replaced by the Wigner function $W(x, p)$ in space-time x and four-momentum p, defined as the ensemble average of the Wigner operator [21–23] for spin-1/2 fermions,

$$\hat{W}_{\alpha\beta} = \int \frac{d^4y}{(2\pi)^4} e^{-ip\cdot y} \bar{\psi}_\beta(x_+) U(x_+, x_-) \psi_\alpha(x_-), \quad (1)$$

where ψ_α and $\bar{\psi}_\beta$ are Dirac spinor fields, $x_\pm \equiv x \pm \frac{1}{2}y$ are two space-time points centered at x with space-time separation y, and the gauge link U,

$$U(x_+, x_-) \equiv e^{-iQ\int_{x_-}^{x_+} dz^\mu A_\mu(z)}, \quad (2)$$

ensures the gauge invariance of $\hat{W}_{\alpha\beta}$. Here, Q is the electromagnetic charge of the fermions, and A_μ is the electromagnetic vector potential. Note that we use the metric convention $g^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$. To simplify the quantum kinetic equation under a background field we consider a massless and collisionless fermionic system in a constant external electromagnetic field $F_{\mu\nu}$ in the lab frame. Since we only consider a classical background field, we have dropped the path ordering in the gauge link in Eq. (2). The Wigner function is a matrix in Dirac space and satisfies the quantum kinetic equation [21–23],

$$\gamma_\mu\left(p^\mu + \frac{i}{2}\nabla^\mu\right)W(x, p) = 0, \quad (3)$$

where γ^μ's are Dirac matrices and $\nabla^\mu \equiv \partial_x^\mu - QF^\mu{}_\nu \partial_p^\nu$. The Wigner function should contain information about quantum interactions and we will prove that all currents including chiral anomaly can be derived from the above equation. To this end, we decompose the Wigner function in terms of 16 independent generators of the Clifford algebra,

$$W(x,p)=\frac{1}{4}\Big[\mathcal{F}(x,p)+i\gamma^5\mathcal{P}(x,p)+\gamma^\mu\mathcal{V}_\mu(x,p)+\gamma^5\gamma^\mu\mathcal{A}_\mu(x,p)+\frac{1}{2}\sigma^{\mu\nu}\mathcal{S}_{\mu\nu}(x,p)\Big]. \tag{4}$$

Equation (3) then leads to two decoupled sets of equations [21–23], one relevant to our study reads

$$p^\mu\mathcal{V}_\mu=0,\qquad p^\mu\mathcal{A}_\mu=0, \tag{5}$$

$$\nabla^\mu\mathcal{V}_\mu=0,\qquad \nabla^\mu\mathcal{A}_\mu=0, \tag{6}$$

$$\epsilon_{\mu\nu\rho\sigma}\nabla^\rho\mathcal{A}^\sigma=-2(p_\mu\mathcal{V}_\nu-p_\nu\mathcal{V}_\mu), \tag{7}$$

$$\epsilon_{\mu\nu\rho\sigma}\nabla^\rho\mathcal{V}^\sigma=-2(p_\mu\mathcal{A}_\nu-p_\nu\mathcal{A}_\mu), \tag{8}$$

where $\epsilon^{\mu\nu\rho\sigma}$ is the Levi-Civita antisymmetric tensor, $\mathcal{V}_\mu(x,p)$ and $\mathcal{A}_\mu(x,p)$ are the vector and axial-vector component of the Wigner function, which will give rise to the vector and axial-vector current, respectively, after integration over four-momentum.

Power expansion.—We assume a system close to local equilibrium under a constant external field $F^{\mu\nu}$. Therefore, $\mathcal{V}_\mu(x,p)$ and $\mathcal{A}_\mu(x,p)$ will depend on x only through fluid four-velocity $u(x)$, temperature $T(x)$, chemical potential $\mu(x)$, and chiral chemical potential $\mu_5(x)$. We will determine the analytic form of the Wigner function in terms of $\{p,F^{\mu\nu},u,T,\mu,\mu_5\}$ from the kinetic equation.

We further assume that the space-time derivative ∂_x and the field strength $F_{\mu\nu}$ are small variables of the same order and can be used as parameters in the power expansion of $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$ (similar to the Knudsen number expansion in hydrodynamics),

$$\begin{aligned}\mathcal{V}^\mu&=\mathcal{V}_0^\mu+\mathcal{V}_1^\mu+\cdots,\\ \mathcal{A}^\mu&=\mathcal{A}_0^\mu+\mathcal{A}_1^\mu+\cdots,\end{aligned} \tag{9}$$

where the subscripts $0,1,\ldots$ denote orders of the power expansion. Note that $\mathcal{V}_n^\mu$ and $\mathcal{A}_n^\mu$ are related to $\mathcal{A}_{n-1}^\mu$ and $\mathcal{V}_{n-1}^\mu$ via Eqs. (7) and (8) ($n\geq 1$). One can therefore use an iterative scheme to solve $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$ order by order.

Note that the field strengths $F^{\mu\nu}$ are assumed to be constant in the lab frame. Later, we have to define electromagnetic fields in the local comoving frame of a fluid cell, $E_\sigma=u^\rho F_{\sigma\rho}$, $B_\sigma=(1/2)\epsilon_{\sigma\mu\nu\rho}u^\mu F^{\nu\rho}$, which depend on x via the fluid velocity $u(x)$. The space-time derivative ∂_x is then given by

$$\partial_\sigma^x=\partial_\sigma T\frac{\partial}{\partial T}+\partial_\sigma u_\rho\frac{\partial}{\partial u_\rho}+\partial_\sigma\mu\frac{\partial}{\partial\mu}+\partial_\sigma\mu_5\frac{\partial}{\partial\mu_5}. \tag{10}$$

Zeroth-order Wigner function.—In general, $\mathcal{V}_0^\mu$ and $\mathcal{A}_0^\mu$ can only have two terms, each proportional to the zeroth-order four-vectors p^μ or u^μ with a total of four independent coefficients. Since the left-hand sides of Eqs. (7) and (8) are at least of first order, the zeroth-order terms on the right-hand sides must vanish, which set the coefficients of the u^μ terms to be zero. With additional constraints by Eq. (5), $\mathcal{V}_0^\mu$ and $\mathcal{A}_0^\mu$ have to take the following forms:

$$\mathcal{V}_0^\mu=p^\mu\delta(p^2)V_0,\qquad \mathcal{A}_0^\mu=p^\mu\delta(p^2)A_0, \tag{11}$$

where V_0 and A_0 are the phase-space distributions of massless spin-1/2 fermions at the zeroth order and cannot be determined by Eqs. (5)–(8). We assume they take the equilibrium form,

$$\begin{aligned}[V_0,A_0]&=\sum_{s=\pm1}\theta(su\cdot p)[(f_{s,R}+f_{s,L}),(f_{s,R}-f_{s,L})],\\ f_{s,\chi}&=\frac{2}{(2\pi)^3}\frac{1}{e^{s(u\cdot p-\mu_\chi)/T}+1},\qquad (\chi=R,L),\end{aligned} \tag{12}$$

where $R(L)$ denotes the right(left)-handed fermions and $\mu_{R,L}=\mu\pm\mu_5$ [2]. Note that V_0 (A_0) is the sum (difference) of two positive distributions for any values of μ and μ_5. This asymmetry between V_0 and A_0 as inputs to the iterative operation will feed down to the first-order Wigner functions $\mathcal{V}_1^\mu$ and $\mathcal{A}_1^\mu$ and the final vector and axial-vector currents, even though the kinetic equations in Eqs. (5)–(8) are symmetric for $\mathcal{V}^\mu$ and $\mathcal{A}^\mu$.

The zeroth-order Wigner functions should also satisfy Eq. (6), which provides constraints on fluid and thermodynamical variables. Substituting Eqs. (11) and (12) into Eq. (6), we obtain $\nabla_\mu\mathcal{V}_0^\mu$ and $\nabla_\mu\mathcal{A}_0^\mu$ as sums of six independent terms involving the momentum vector $\bar{p}_\sigma\equiv\Delta_{\sigma\rho}p^\rho$ ($\Delta_{\sigma\rho}\equiv g_{\sigma\rho}-u_\sigma u_\rho$), tensor $\bar{p}_\sigma\bar{p}_\rho$, scalars $\bar{p}^2$, and $u_\sigma p^\sigma$. To ensure $\nabla_\mu V_0^\mu=\nabla_\mu\mathcal{A}_0^\mu=0$ for any values of p, these six terms all have to vanish, resulting in the following constraints at the first order:

$$\begin{aligned}&\Delta^{\sigma\alpha}\Delta^{\rho\beta}\Big(\partial_\alpha u_\beta+\partial_\beta u_\alpha-\frac{2}{3}\Delta_{\alpha\beta}\Delta^{\gamma\delta}\partial_\gamma u_\delta\Big)=0,\\ &T\Delta^{\sigma\rho}\partial_\rho\frac{\mu}{T}+QE^\sigma=0,\\ &u_\rho\partial^\rho u^\sigma-\Delta^{\sigma\rho}\partial_\rho\ln T=0,\\ &\partial_\sigma\frac{\mu_5}{T}=0,\qquad u_\rho\partial^\rho\frac{\mu}{T}=0,\\ &u_\rho\partial^\rho T+\frac{1}{3}T\Delta^{\rho\sigma}\partial_\rho u_\sigma=0.\end{aligned} \tag{13}$$

Note that we have dropped $\delta(p_0)$ terms from derivatives of $\theta(p_0)$ and $\theta(-p_0)$, which are irrelevant when carrying out the four-momentum integration due to vanishing phase space at zero momentum. Since we are interested in currents induced by external fields and vorticity, we consider only the static case with a constant temperature. The above constraints are reduced to

$$u_\rho \partial^\rho u^\sigma = 0, \qquad \partial_\sigma u^\sigma = 0, \qquad \partial_\sigma \mu = -QE_\sigma,$$
$$\mu_5 = \mathrm{const}, \qquad (\text{for } T = \mathrm{const}), \tag{14}$$

which has a simple solution $\mu = \mathrm{const} - QEx$ and a solenoidal fluid velocity.

First-order Wigner function.—With the zeroth-order Wigner functions in Eqs. (11) and (12) one can determine the first order $\mathcal{V}_1^\mu$ and $\mathcal{A}_1^\mu$ from Eqs. (5)–(8). A general form linear in the first-order variables $X^\mu = (E^\mu, B^\mu, \omega^\mu)$ and constrained by Eq. (5) can be written as

$$\begin{aligned} \mathcal{Z}_1^\mu = \sum_{X=E,B,\omega} [&u_\nu(g^{\nu\mu} - p^\nu p^\mu/p^2)p^2(\bar{p}_\rho X^\rho)Z_{X1} \\ &+ X_\nu(g^{\nu\mu} - p^\nu p^\mu/p^2)p^2 Z_{X2} \\ &+ X_\nu(g^{\nu\mu} - \bar{p}^\nu \bar{p}^\mu/\bar{p}^2)\bar{p}^2 Z_{X3} \\ &+ \epsilon^{\mu\lambda\rho\sigma}u_\lambda p_\rho X_\sigma Z_{X4}], \end{aligned} \tag{15}$$

where $\mathcal{Z}_1^\mu = (\mathcal{V}_1^\mu, \mathcal{A}_1^\mu)$ and $\omega_\mu = (1/2)\epsilon_{\mu\nu\rho\sigma}u^\nu\partial^\rho u^\sigma$ is the fluid vorticity. Note that $X_\rho u^\rho = 0$. There are 24 independent coefficients $Z_{Xi} = (V_{X_i}, A_{Xi})$ $(i = 1, 2, 3, 4)$ in the above power expansion of the first order. With Eqs. (11) and (15) for the zeroth- and first-order Wigner functions, both Eqs. (7) and (8) at the first order contain 3 different tensor structures, each consisting of terms linear in three independent variables $X^\mu = (E^\mu, B^\mu, \omega^\mu)$. Setting these terms to vanish separately gives 18 equations which leave only 6 of the 24 coefficients in Eq. (15) undetermined. Further requiring Eq. (6) be satisfied by $\mathcal{V}_1^\mu$ and $\mathcal{A}_1^\mu$, we can obtain the unique forms of $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$ to the first order,

$$\begin{aligned} \mathcal{Z}^\mu = {}& p^\mu \delta(p^2)Z_0 + \frac{1}{2}p_\nu[u^\mu\omega^\nu - u^\nu\omega^\mu]\frac{\partial \bar{Z}_0}{\partial(u_\rho p^\rho)}\delta(p^2) \\ &- Qp_\nu[u^\mu B^\nu - u^\nu B^\mu]\bar{Z}_0\delta'(p^2) \\ &+ Q\epsilon^{\mu\lambda\rho\sigma}u_\lambda p_\rho E_\sigma \bar{Z}_0\delta'(p^2), \end{aligned} \tag{16}$$

where $\mathcal{Z} = (\mathcal{V}, \mathcal{A})$, $Z_0 = (V_0, A_0)$ and $\bar{Z}_0 = (A_0, V_0)$.

Induced currents, CME and CVE.—We can derive the vector and axial-vector current from the above Wigner functions up to the first order in power expansion,

$$j^\mu = \int d^4p\, \mathcal{V}^\mu = nu^\mu + \xi\omega^\mu + \xi_B B^\mu, \tag{17}$$

$$j_5^\mu = \int d^4p\, \mathcal{A}^\mu = n_5 u^\mu + \xi_5\omega^\mu + \xi_{B5}B^\mu. \tag{18}$$

The energy-momentum tensor $T^{\mu\nu}$ can also be evaluated,

$$\begin{aligned} T^{\mu\nu} &= \frac{1}{2}\int d^4p(p^\mu\mathcal{V}^\nu + p^\nu\mathcal{V}^\mu) \\ &= (\epsilon + P)u^\mu u^\nu - Pg^{\mu\nu} + n_5(u^\mu\omega^\nu + u^\nu\omega^\mu) \\ &\quad + \frac{1}{2}Q\xi(u^\mu B^\nu + u^\nu B^\mu). \end{aligned} \tag{19}$$

The charge n, n_5 and energy density ϵ in equilibrium,

$$N_0 = 2\pi\int dp_0 p_0^i[\theta(p_0) - \theta(-p_0)]Z_{N0}, \tag{20}$$

are determined from the zeroth-order Wigner functions, where $N_0 = n$, n_5, ϵ corresponding to $i = 2, 2, 3$ and $Z_{N0} = V_0, A_0, V_0$, respectively. The pressure is given by $P = \epsilon/3$. Coefficients ξ, ξ_B, ξ_5, and ξ_{B5} are given by

$$\Xi = c\pi\int dp_0 p_0^j[\theta(p_0) - \theta(-p_0)]Z_{\Xi 0}, \tag{21}$$

where $\Xi = \xi, \xi_B, \xi_5, \xi_{B5}$ corresponding to $j = 1, 0, 1, 0$, $c = 2, Q, 2, Q$, and $Z_{\Xi 0} = A_0, A_0, V_0, V_0$, respectively. It is easy to verify following relations: $\xi = (1/2)\partial n_5/\partial\mu$, $\xi_5 = (1/2)\partial n/\partial\mu$, $\xi_B = (Q/2)\partial\xi/\partial\mu$, and $\xi_{B5} = (Q/2)\partial\xi_5/\partial\mu$.

One can complete the above integrals analytically to obtain coefficients ξ, ξ_B, ξ_5, and ξ_{B5} of the induced currents as functions of μ, μ_5, and T,

$$\xi = \frac{1}{\pi^2}\mu\mu_5, \qquad \xi_B = \frac{Q}{2\pi^2}\mu_5, \tag{22}$$

$$\xi_5 = \frac{1}{6}T^2 + \frac{1}{2\pi^2}(\mu^2 + \mu_5^2), \qquad \xi_{B5} = \frac{Q}{2\pi^2}\mu. \tag{23}$$

Thermodynamical quantities n, n_5, and ϵ can be similarly obtained.

The current in Eq. (17) induced by magnetic field and vorticity with coefficients ξ_B and ξ in Eq. (22), known as the CME and CVE [2,3,9], respectively, is a direct consequence of the quantum kinetic equation for the Wigner function. The axial-vector current in Eq. (18) induced by magnetic field and vorticity corresponds to some sort of reversed CME and CVE, respectively. These results are consistent to those obtained from the second law of thermodynamics in Refs. [10,24] except a quadratic term in temperature in ξ_5 induced by vorticity. It should be noted that Eqs. (22) and (23), including the temperature term in ξ_5, have also been obtained independently in Ref. [17] within the Kubo formalism.

Conservation equations for j^μ and j_5^μ

$$\partial_\mu j^\mu = 0, \qquad \partial_\mu j_5^\mu = -\frac{Q^2}{2\pi^2}E_\rho B^\rho, \tag{24}$$

can be derived from Eqs. (17) and (18) with constraints on fluid and thermodynamical variables in Eq. (14). The electric field in the chiral anomaly appears through $\partial_\sigma\mu = -QE_\sigma$ from Eq. (14). Note that we derived the chiral anomaly here without regularization in contrast to the derivation in quantum field theory. This is because the Wigner function contains two fermionic fields separated in space-time (nonlocal) and therefore free of singularities. One can also verify the energy-momentum conservation equation in the background field,

$$\partial_\mu T^{\mu\nu} = QF^{\nu\rho}j_\rho, \tag{25}$$

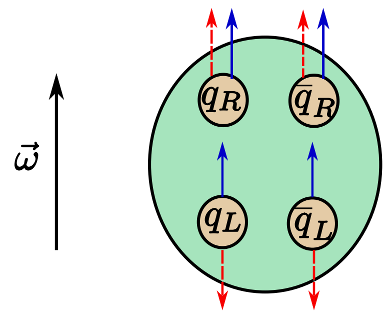

FIG. 1 (color online). The axial current induced by vorticity leads to the local polarization effect. The momentum (spin) direction is in the red-dashed (blue-solid) arrow.

from Eqs. (17) and (19) with constraints in Eq. (14). It is interesting to observe that constraints in Eq. (13) or (14) require $\omega^\mu \| B^\mu \| E^\mu$, which is crucial for the energy-momentum conservation in Eq. (25).

It is remarkable that we have derived from the quantum kinetic equation not only currents in Eqs. (17) and (18) with their coefficients in Eqs. (22) and (23) but also a complete set of conservation equations with the chiral anomaly in Eqs. (24) and (25) for charge, chiral charge and energy-momentum, respectively. In contrast, these conservation equations are used as inputs to obtain the currents in Refs. [9,10] with the requirement of the second law of thermodynamics.

Multi-flavor fluid.—So far we have only considered a fluid with a single type of fermion. An extension to the case of multiflavor quarks is straightforward. We can consider a three-flavor ($N_f = 3$) fluid with u, d, and s quark and their antiquarks. Note that each quark carries N_c fundamental color charges. For the induced electromagnetic and baryonic vector current j^μ,

$$\begin{aligned} \xi^{\mathrm{baryon}} &= \frac{N_c N_f}{3\pi^2}\mu\mu_5, & \xi_B^{\mathrm{baryon}} &= \frac{N_c}{6\pi^2}\mu_5 \sum_f Q_f, \\ \xi^{\mathrm{EM}} &= \frac{N_c}{\pi^2}\mu\mu_5 \sum_f Q_f, & \xi_B^{\mathrm{EM}} &= \frac{N_c}{2\pi^2}\mu_5 \sum_f Q_f^2. \end{aligned} \tag{26}$$

For this three-flavor quark matter we have $\sum_f Q_f = 0$, and $\xi_B^{\mathrm{baryon}} = \xi^{\mathrm{EM}} = 0$. This implies that the CME (CVE) dominates the electromagnetic (baryonic) current [3]. For the induced baryonic axial-vector current j_5^μ,

$$\begin{aligned} \xi_5 &= \frac{N_c N_f}{3}\left[\frac{1}{6}T^2 + \frac{1}{2\pi^2}(\mu^2 + \mu_5^2)\right], \\ \xi_{B5} &= \frac{N_c}{6\pi^2}\mu \sum_f Q_f = 0. \end{aligned} \tag{27}$$

Therefore, magnetic fields cannot induce the axial-vector current in a three-flavor quark matter, which can only be induced by vorticity.

Local polarization effect.—An axial-vector current induced by vorticity implies that the right(left)-handed fermions move parallel (opposite) to the direction of vorticity. Since the momentum of a right(left)-handed massless fermion is parallel (opposite) to its spin, all spins are parallel to the direction of vorticity (see Fig. 1 for illustration). This results in the local polarization effect (LPE) similar to what was proposed in Refs. [18–20] due to spin-orbital coupling. The LPE can be measured via hadron (e.g., hyperon) polarization along the direction of vorticity or the global orbital angular momentum in noncentral heavy-ion collisions [18]. Note that ξ_5 in Eq. (27) has three quadratic terms in T, μ and μ_5. Therefore, the LPE should be present in both high and low energy heavy-ion collisions with either low baryonic chemical potential and high temperature or vice versa.

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