Is Dark Matter with Long-Range Interactions a Solution to All Small-Scale Problems of Λ Cold Dark Matter Cosmology?

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The cold dark matter paradigm describes the large-scale structure of the Universe remarkably well. However, there exists some tension with the observed abundances and internal density structures of both field dwarf galaxies and galactic satellites. Here, we demonstrate that a simple class of dark matter models may offer a viable solution to all of these problems simultaneously. Their key phenomenological properties are velocity-dependent self-interactions mediated by a light vector messenger and thermal production with much later kinetic decoupling than in the standard case.

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Introduction.—Recent advances of cosmological precision tests further consolidate the "cosmological concordance model," indicating that 4.5% of the mass in the Universe is in baryons, 22.6% is nonbaryonic cold dark matter (CDM), and the rest is Einstein's cosmological constant Λ (or behaves like it) [1]. The leading CDM candidates are weakly interacting massive particles (WIMPs) that are thermally produced in the early Universe [2]. While their *chemical* decoupling from the heat bath sets the observed DM relic density today, their *kinetic* decoupling induces a small-scale cutoff in the primordial power spectrum of density perturbations [3]. For neutralino DM, e.g., this cutoff corresponds to a smallest protohalo mass of $M_{\rm cut}/M_\odot \sim 10^{-11}$–$10^{-3}$ [4], but it could be as large as $M_{\rm cut} \gtrsim 10 M_\odot$ if DM couples to new light scalars [5]. After kinetic decoupling, standard WIMP CDM behaves like a collisionless gas. Baryons, on the other hand, can radiate away excess energy and sink to the centers of CDM halos, where they form stars and galaxies. In this picture, structure formation proceeds hierarchically with galaxies to form at sites of constructive interference of small-scale waves in the primordial density fluctuations.

Despite the great success of ΛCDM cosmology, detailed observations of nearby small galaxies pose a number of puzzles to this paradigm. Here, we isolate three distinct classes of problems. (1) The observed galaxy luminosity and HI mass functions show much shallower faint-end slopes than predicted by ΛCDM models [6]; this is locally known as the "missing satellites problem" of the Milky Way (MW), which should contain many more dwarf-sized subhalos than observed [7]. (2) Simulations predict an inner DM *cusp* for the density structure of galaxies, seemingly at odds with the *cored* profiles found in observed low surface brightness galaxies and dwarf satellites [8]. (3) Recently, it was realized that the most massive subhalos in ΛCDM simulations of MW-sized halos have an internal density structure that is too concentrated in comparison to the observed brightest MW satellites: the simulated circular velocity profiles increase more steeply and attain their maximum circular velocity at smaller radii than any of the observed ones. On the other hand, those simulated subhalos should be "too big to fail" in forming stars according to our understanding of galaxy formation (being more massive than the UV photosuppression scale at all redshifts, after formation, for conceivable reionization histories). Thus, it is extremely puzzling why there is no observed analogue to those objects [9].

Astrophysical solutions to (1) invoke suppressing the formation of galaxies within existing dwarf halos or suppressing the star formation in dwarf galaxies. Galaxy formation can be held back by increasing the gas entropy before collapse, e.g., via photoionization [10], blazar heating [11], or active galactic nuclei feedback in the radio-quiet mode [12]. A photoionization-induced lack of HI [13] or intrinsically low metallicities [14] may further suppress the cooling efficiency of collapsing baryons. Numerical simulations with a photoionizing background, however, cannot suppress dwarf galaxy formation at the level implied by observations [15]. In principle, gas may also be removed from dwarfs via photoevaporation [16] and feedback from supernovae [17]. Any such feedback, however, implies remnant stellar populations and HI masses in conflict with the most recent observational constraints [18].

The "cusp-core" problem (2) may be addressed by large velocity anisotropies or reduced central DM densities. There is a degeneracy between cored isotropic and cuspy anisotropic velocity distributions, and the stellar line-of-sight velocity data are still too sparse to dynamically resolve (2) [19]. Reducing central DM densities was proposed as a result of efficient baryonic feedback processes [20], however, in contradiction to cuspy dwarf profiles in other simulations with feedback [21].

The too-big-to-fail problem (3) might be solved by either an increased stochasticity of galaxy formation on these scales or a total MW mass $\lesssim 8 \times 10^{11} M_\odot$ [22]. Abundance matching of stellar and halo masses, which agree with stacking analyses of gravitational lensing signals and satellite dynamics of Sloan Digital Sky Survey galaxies, makes the required large degree of stochasticity implausible [23]. For a $10^{12} M_\odot$ MW, on the other hand, the chance to host two satellites as massive as the Magellanic Clouds is less than 10% [24] and even lower for smaller MW masses (from satellite studies of MW-type Sloan Digital Sky Survey systems [25] and MW and Andromeda orbit timing arguments [26]).

The next logical possibility that could lead to a suppression of small-scale power is a modification of the CDM paradigm itself. The most often discussed options are interacting DM (IDM) [27] and warm DM (WDM) [28], though it should be noted that there exist interesting alternatives such as DM from late decays [29], DM with large annihilation rates [30], extremely light DM particles forming a condensate [31], or inflationary models with broken-scale invariance [32]. As was soon realized, however, IDM with a constant cross section produces spherical cores in conflict with observed ellipticities in clusters [33] and the survivability of satellite halos [34]. While WDM is unlikely to account for some of the large $\sim$1 kpc cores claimed in dwarfs [35] and severely constrained by Lyman-α observations [36–39], it may be able to partially resolve the too-big-to-fail problem by allowing these subhalos to initially form with lower concentrations [40]. Alternatively, DM self-interactions mediated by a Yukawa potential, with the resulting characteristic velocity dependence of the transfer cross section [41,42], avoid constraints on scales of MW-type galaxies and beyond [43] and produce $\sim$1 kpc cores that match the observed velocity profiles of massive MW satellites [44] (see also Ref. [41]).

Most astrophysical and DM solutions have shortcomings or can explain at most two of the three problems, which makes them less attractive on the basis of Occam's razor. Here, we demonstrate that there is a class of IDM models that can *simultaneously* account for all three problems. Encouraged by the results of Refs. [43,44], in particular, we will focus on models with a Yukawa-like interaction between the DM particles that is mediated by a light messenger (see Fig. 1). As we will show, the kinetic decoupling of DM in these models can happen sufficiently late to suppress the power spectrum at scales as large as that of dwarf galaxies, $M_{\rm cut} \gtrsim 10^9 M_\odot$, while at the same time the velocity-dependent self-interaction of DM produces cored density profiles in dwarfs [45].

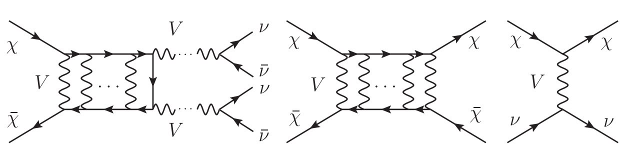

FIG. 1. Interaction processes that set the DM relic density and may lead to observable neutrino annihilation products today (left), change the inner velocity and density profile of dwarf halos (middle), and induce a comparatively large cutoff in the spectrum of primordial density perturbations (right).

Model setup.—In models with new light exchange particles ϕ, kinetic decoupling can happen much later than in standard WIMP scenarios, in particular, for small masses m_ϕ [5]. For *scalar* exchange particles, however, the amplitude for DM scattering with leptons scales like $\sim m_\chi m_\ell / m_\phi^2$, implying that scattering with neutrinos is generally negligible. While a coupling of ϕ to charged leptons also leads to a loop-suppressed effective coupling to photons, $\mathcal{L} \supset g_{\phi\gamma\gamma} \phi F^{\mu\nu} F_{\mu\nu}$, the resulting scattering amplitude does not contribute in the relevant limit of small momentum transfer. Kinetic decoupling therefore never occurs at $T_{\rm kd} \ll 0.1$ MeV, at which point the number density of electrons starts to become strongly Boltzmann-suppressed and there are no lighter (and thus more abundant) particle species left that could keep up kinetic equilibrium instead.

Let us consider instead the situation where DM consists of heavy Dirac fermions χ which only couple to a light *vector* boson V. Due to our interest in late kinetic decoupling, we will require V to also couple to neutrinos:

$$\mathcal{L}_{\rm int} \supset -g_\chi \bar{\chi} \not{V} \chi - g_\nu \bar{\nu} \not{V} \nu. \tag{1}$$

Note that we take a phenomenological approach here and only state couplings that explicitly enter our analysis. In particular, V does not have to be a gauge boson, which leaves couplings to other SM particles unspecified (see, e.g., Ref. [48] for a recent model-independent analysis). DM is then thermally produced in the early Universe via $\bar{\chi}\chi \leftrightarrow VV$. Assuming g_ν is small but large enough to thermalize V at early times, the relic density is given by

$$\Omega_\chi h^2 = \Omega_{\bar{\chi}} h^2 \simeq \frac{0.11}{2} \left(\frac{g_\chi}{0.683} \right)^{-4} \left(\frac{m_\chi}{\rm TeV} \right)^2. \tag{2}$$

This expression receives $\mathcal{O}(1)$ corrections due to the Sommerfeld effect [49], i.e., a multiple exchange of V as shown in Fig. 1, which we fully take into account in our analysis. The kinetic decoupling temperature, on the other hand, will be set by χ-ν scattering. The corresponding amplitude at small momentum transfer reads

$$\sum_{\text{all spins}} |\mathcal{M}|^2_{\chi\nu \leftrightarrow \chi\nu} = 64 g_\chi^2 g_\nu^2 \frac{m_\chi^2 E_\nu^2}{m_V^4}. \tag{3}$$

In the following, we will consider g_ν as an essentially free parameter, while g_χ is fixed by the requirement to obtain the correct relic density (see, e.g., Ref. [50] for a list of possible natural explanations for $g_\nu \ll g_\chi$).

DM self-scattering.—The light vector messenger induces a long-range attractive Yukawa potential between the DM particles, cf. Fig. 1. Concerning elastic DM self-scattering, this is completely analogous to screened

Coulomb scattering in a plasma for which simple parametrizations of the transfer cross section $\sigma_T(v)$ in terms of m_χ, m_V, g_χ, and the relative velocity v of the DM particles exist [41,51]. Using these parametrizations, it was shown that the type of DM model introduced above produces cores rather than cusps [43] and may solve the too-big-to-fail problem [44], without being in conflict with the strong constraints for models with constant σ_T. We also note that σ_T drops with larger v such that for galaxy *clusters* only the very central density profile at $r \lesssim \mathcal{O}(1-10)$ kpc will be smoothed out, matching observational evidence (from improved lensing and stellar kinematic data [52]) for a density cusp in A383 that is slightly shallower than expected for standard CDM.

For our discussion, the astrophysically important quantities are the velocity $v_{\rm max}^2 = g_\chi^2 m_V/(2\pi^2 m_\chi)$ at which $\sigma_T v$ becomes maximal and $\sigma_T^{\rm max} \equiv \sigma_T(v_{\rm max}) = 22.7 m_V^{-2}$. In particular, $v_{\rm max}$ should not be too different from the typical velocity dispersion $\sigma_v \sim \mathcal{O}(10)$ km/s encountered in dwarf galaxies if one wants to make any contact with potential problems with standard structure formation at these scales. On the other hand, the value of $\sigma_T^{\rm max}$ is constrained by various astrophysical measurements; see Ref. [44] for a compilation of current bounds.

Fixing g_χ by the relic density requirement, there is a one-to-one correspondence between the particle physics input (m_χ, m_V) and the astrophysically relevant parameters $(v_{\rm max}, \sigma_T^{\rm max})$. As demonstrated in Fig. 2, a solution to the aforementioned small-scale problems (2)and (3)may then indeed be possible for DM masses of $m_\chi \gtrsim 600$ GeV and a mediator mass in the (sub-)MeV range. We also display the strongest astrophysical bounds on large DM self-interaction rates [43]. For $m_\chi \lesssim 4$ TeV, they arise from collisions with particles from the dwarf parent halo, while at larger m_χ an imminent gravothermal catastrophe is more constraining.

The small-scale cutoff.—For small kinetic decoupling temperatures $T_{\rm kd}$, acoustic oscillations [53] are more efficient than free-streaming effects to suppress the power spectrum [4,54]. The resulting exponential cutoff can be translated into a smallest protohalo mass of

$$M_{\rm cut} \approx \frac{4\pi}{3}\frac{\rho_\chi}{H^3}\bigg|_{T=T_{\rm kd}} = 1.7\times10^8\left(\frac{T_{\rm kd}}{\rm keV}\right)^{-3} M_\odot, \quad (4)$$

where H is the Hubble rate and we assumed late kinetic decoupling such that the effective number of relativistic degrees of freedom $g_{\rm eff} = 3.37$. For scattering with relativistic neutrinos, cf. Eq. (3), the analytic treatment of kinetic decoupling given in Ref. [55] is valid. Extending those expressions to allow for $T_\nu \neq T$, we find

$$T_{\rm kd} = \frac{0.062\,\rm keV}{N_\nu^{1/4}(g_\chi g_\nu)^{1/2}}\left(\frac{T}{T_\nu}\right)_{\rm kd}^{1/2}\left(\frac{m_\chi}{\rm TeV}\right)^{1/4}\left(\frac{m_V}{\rm MeV}\right), \quad (5)$$

where N_ν is the number of neutrino species coupling to V. Combining this with Eq. (2), we therefore expect that $T_{\rm kd}$, and thus $M_{\rm cut}$, are essentially independent of g_χ and m_χ.

Using for definiteness $N_\nu = 3$ and $T_\nu = (4/11)^{1/3} T_\gamma$, we show in Fig. 3 contours of constant $M_{\rm cut}$ in the (g_ν, m_V) plane. We find that the result of the full numerical calculation [4,5] is indeed extremely well described by Eqs. (4) and (5) for $g_\nu \gtrsim 10^{-7}$ (assuming $m_\chi \sim 1$ TeV and $m_V \sim 1$ MeV; this value is even lower for larger m_χ and smaller m_V). For $g_\nu \lesssim 10^{-7}$, DM scattering with the nonrelativistic mediator particles V starts to dominate over scattering with neutrinos, and $M_{\rm cut}$ eventually becomes independent of g_ν. We checked that the new era of DM annihilation generally expected in models with Sommerfeld-enhanced annihilation rates (see Ref. [5] for a consistent treatment) has only a negligible impact on our results for the late decoupling times we focus on here.

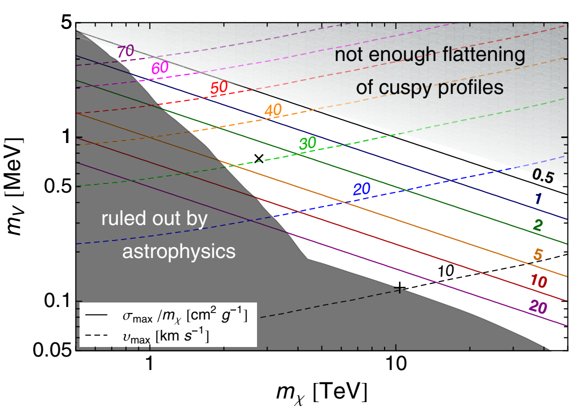

FIG. 2 (color online). The white area corresponds to DM and mediator masses that may solve the cusp-core problem. The crosses indicate two benchmark models for which detailed simulations [44] have found a solution to the too-big-to-fail problem. Dashed and solid lines show contours of the astrophysical relevant quantities $\sigma_{\rm max}^T$ and $v_{\rm max}$. See the text for further details.

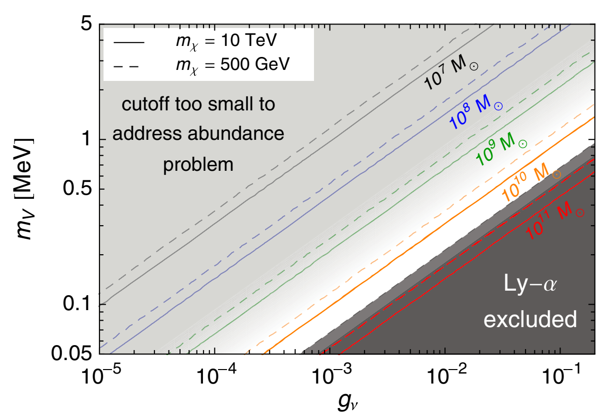

FIG. 3 (color online). This plane shows the mediator mass m_V vs the coupling strength g_ν. Large values of g_ν and small values of m_V lead to late kinetic decoupling and thus a large mass $M_{\rm cut}$ of the smallest protohalos. $M_{\rm cut} \gtrsim 5\times10^{10} M_\odot$ is excluded by Lyman-α data, while $M_{\rm cut} \gtrsim 10^9 M_\odot$ may solve the small-scale abundance problems of ΛCDM cosmology.

Lyman-α forest bounds.—Conventionally, a possible cutoff in the power spectrum is often expressed in terms of the mass m_w of a WDM thermal relic. In this case, it is set by free streaming of the WDM particles, and the comoving free-streaming length R_f is given by $R_f = 0.1(\Omega_m h^2/0.13)^{1/3}(m_w/\mathrm{keV})^{-4/3}$ Mpc [56]. For a characteristic wave number $k_f \equiv 0.46/R_f$, the linear perturbation amplitude is suppressed by a factor of 2 and the characteristic filtering mass can be defined as [56]

$$M_f \equiv \frac{4\pi}{3}\bar{\rho}_m\left(\frac{\lambda_f}{2}\right)^3 = 5.1\times 10^{10}\left(\frac{m_w}{\mathrm{keV}}\right)^{-4} M_\odot, \quad (6)$$

where $\lambda_f = 2\pi/k_f \simeq 13.6R_f$. This choice of M_f is justified by numerical experiments [57] that find the resulting halo statistics for an initial density distribution with a sharp cut in the power spectrum at $k_c = 2\pi/\lambda_f$ to be very similar to the statistics of an initial density field smoothed with a top-hat window of radius $\lambda_f/2$. Cosmological WDM simulations show a deviation of the mass function from the CDM case on scales given by Eq. (6) [6,58].

Combining data of the Lyman-α forest, the cosmic microwave background and galaxy clustering allow us to constrain the cutoff scale in the power spectrum; in terms of the mass of a thermal WDM candidate, a 2σ bound of $m_w > 2$ keV has been claimed [36,37]. This weakens to $m_w > 0.9$ keV when rejecting less reliable data at $z > 3.2$ [36] due to systematic errors [38]. Revisiting Lyman-α data yielded $m_w > 1.7$ keV, which, however, is subject to systematic uncertainties at the $\sim$30% level [39], especially considering that blazar heating was not accounted for in deriving cosmological constraints [59].

Lyman-α data thus firmly exclude $m_w < 1$ keV or $M_f > 5.1\times 10^{10} M_\odot$ (corresponding to a maximal circular velocity $v_{\max} \sim 70$ km s^{-1}). For $m_w \simeq$ 1–2 keV, WDM models are able to alleviate the missing satellites problem (somewhat depending on feedback recipes) [60], bring the faint end of the galaxy luminosity function into agreement with data [61], and match [6] the HI velocity function measured in the ALFALFA survey [62]. For $m_w > 3$ keV, the corresponding mass cutoff $M_f < 6\times 10^8 M_\odot$ is too small to have any impact on the faint end of the galaxy luminosity function. We include these bounds in Fig. 3 to demonstrate that our model can also successfully address the abundance problem (1).

Discussion.—In a phenomenological approach to identify the key properties of DM models that can address all three ΛCDM small-scale problems simultaneously, we found it sufficient to simply postulate the existence of a light vector messenger V that couples to both DM and neutrinos, as in Eq. (1). If V does not couple to quarks or other leptons, the coupling g_ν is essentially unconstrained [48]. While beyond the scope of this Letter, however, we stress that it would be very worthwhile to study possible concrete realizations of our setup.

The greatest challenge for such model building might be to prevent, even at the one-loop level, a kinetic mixing between V and photons, which is severely constrained for $m_V \lesssim$ MeV [64]. On the other hand, limits on tree-level couplings of V to charged leptons (e.g., from contributions to the anomalous magnetic moment of the muon, beam dump experiments, or low-$|q|^2$ ν-e scattering [64,65]) seem less severe and could at least partially be evaded by generation-specific couplings. Another option could be a new $U(1)$ coupling to DM and *sterile* neutrinos ν_s [66]. As long as the ν_s have been in equilibrium in the very early Universe and are relativistic at T_{kd}, this would not change the phenomenology of our model; Eq. (5), in particular, would still apply. It is therefore quite interesting that cosmic microwave background observations seem to favor additional relativistic degrees of freedom, which corresponds to the presence of one light sterile neutrino species [67].

Finally, we note that, for typical galactic velocities $v \sim 10^{-3}$, the type of DM candidate we propose here annihilates with a Sommerfeld-enhanced rate of $\langle\sigma v\rangle \sim 3\times 10^{-24}\,(m_\chi/\mathrm{TeV})^{-2}$ cm^3 s^{-1} into a VV pair that then decays *exclusively* into neutrinos (if $m_V \leq 2m_e$). Such a large annihilation rate will be in reach of future IceCube observations of the Galactic center [68]; for $m_\chi \lesssim 1$ TeV, in fact, a strong Sommerfeld-induced substructure enhancement of the signal [69] may already be constrained.

Conclusions.—We have introduced a class of DM models with the unique property of addressing all three ΛCDM small-scale problems *simultaneously*, which should make them very attractive alternatives to be studied. From a model-building point of view, the only ingredient that is needed is a (sub-)MeV vector messenger particle that weakly couples to neutrinos and even more weakly couples to other standard model particles. While collider and direct searches for DM will be extremely challenging in this scenario, a TeV neutrino signal from the Galactic center could turn out to be a smoking gun signature.

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