Strong Coupling and Long-Range Collective Interactions in Optomechanical Arrays

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We investigate the collective optomechanics of an ensemble of scatterers inside a Fabry-Pérot resonator and identify an optimized configuration where the ensemble is transmissive, in contrast to the usual reflective optomechanics approach. In this configuration, the optomechanical coupling of a specific collective mechanical mode can be several orders of magnitude larger than the single-element case, and long-range interactions can be generated between the different elements since light permeates throughout the array. This new regime should realistically allow for achieving strong single-photon optomechanical coupling with massive resonators, realizing hybrid quantum interfaces, and exploiting collective long-range interactions in arrays of atoms or mechanical oscillators.

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The field of optomechanics has made tremendous progress over the past decades [1], cooling of massive mechanical oscillators to the motional quantum ground state being but one of a series of achievements that demonstrate the power of coupling light to moving scatterers [2,3]. The control of mechanical motion in the quantum regime has many important applications, ranging from precision measurements [4], quantum information processing [5], and fundamental tests of quantum mechanics [6], to the photonics sciences [7]. Despite recent progress the coupling between a single photon and a single phonon remains typically very weak, therefore necessitating the use of many photons to amplify the interaction [1,8]. In this regime, which is useful for cooling and light-motion entanglement generation, a stronger coupling per photon is desirable to limit the negative effects of using large powers, e.g., bulk temperature increases or phase-noise heating [9]. Ultimately, reaching the strong (single-photon) coupling regime, in which a single quantum of light can appreciably affect the motion of the mechanical oscillator, is essential to exploiting fully the quantum nature of the optomechanical interaction, as exhibited by such effects as the optomechanical photon blockade [10] and non-Gaussian mechanical states [11].

Among the various approaches currently followed to couple mechanical oscillators with optical resonators, a successful one involves positioning reflecting objects—dielectric membranes [12,13], atoms [14], or microspheres [15]—inside an optical cavity. With dielectric membranes the optomechanical interaction strength saturates to a fundamental limit g as the reflectivity of the membrane approaches unity [12]. For a highly reflective membrane placed near the center of a Fabry-Pérot (FP) resonator of length L and resonance frequency ω, the single-photon coupling strength is given by the shift in cavity frequency when the mirror moves through a distance equal to the spread x_0 of its zero-point fluctuations, $g = 2\omega x_0/L$, and is typically rather weak for a macroscopic cavity [12]. Several approaches can be followed to improve quantum motional control in single membrane systems, by, e.g., tailoring of the optical and mechanical properties of the individual membranes [16], using photothermal cooling forces [17], active thermal noise compensation [18], optical trapping [19] techniques, or coupling to cold atoms [20].

Another promising approach consists in exploiting *collective* optomechanical interactions using microscopic ensembles of cold atoms [14] or arrays of macroscopic mechanical oscillators [21–24]. In the former the optomechanical coupling strength usually scales as $N^{1/2}$ with the number N of atoms, and the weakly coupled atomic systems are said to demonstrate infinitely long-ranged interactions [14]. In the latter, one can confine the light in periodic structures at the wavelength scale. In this vein, e.g., optomechanical crystals [25] have proven to be very successful at obtaining large coupling strengths by decreasing the length of the effective cavity [2,26]. However, interactions between distant elements in arrays of massive scatterers are believed to be strongly suppressed [24].

Here we provide a unifying formalism that shows these two systems as limiting cases of a more generic model for an array of scatterers in a FP cavity. This allows us to identify an optimized configuration for the scatterers that is transmissive instead of the typical reflective approach. We base our treatment on the observation that around a transmission point of the mechanical system, the cavity response to a certain collective mechanical oscillation can be greatly enhanced. This is to be seen as an alternative to the traditional approach that requires smaller cavities (on the order of a few wavelengths [25]) to increase the field density of modes and thus the coupling between light and mechanics. Our analysis reveals a regime where, regardless of whether the scatterers are atoms or mobile dielectrics, the coupling strength (i) scales superlinearly ($\propto N^{3/2}$) with the number of scatterers and (ii) does not saturate as the reflectivity of the elements approaches unity. This allows, in principle, multielement opto- or electromechanical systems to reach single-photon optomechanical coupling strengths orders of magnitude larger than those currently possible, and does not require wavelength-scale confinement of the light field. Concomitantly, we show that in this configuration (iii) the resonator field couples to a specific collective mechanical mode supporting interelement interactions that are as long ranged as the array itself. Since the model is applicable both to mobile dielectrics, such as membranes [12] or microspheres [15], and to cold atoms in an optical lattice [27,28], this new regime should realistically allow for achieving strong single-photon optomechanical coupling and realizing quantum optomechanical interfaces. It also opens up avenues for the exploitation and engineering of long-ranged cooperative interactions in optomechanical arrays.

Let us again consider a lossless membrane, of thickness smaller than a wavelength, placed inside a FP resonator. This time, we suppose that the membrane has an amplitude reflectivity r, which we parametrize in terms of the polarizability $\zeta \equiv -|r|/\sqrt{1-|r^2|}$. The single-photon optomechanical coupling strength is now $g_0 = g|r|$, which is maximized to g for large $|\zeta|$, i.e., in the *reflective* regime $|r| \to 1$. In order to illustrate the emergence of collective optomechanics, we now consider two identical membranes placed symmetrically in the resonator at a distance d from each other, in the spirit of Fig. 1(a) and Ref. [22]. As we justify below, the effective polarizability of the two-element system is found to be of the form $\chi = 2\zeta[\cos(kd) - \zeta\sin(kd)]$ for light having wave number k (wavelength λ). This effective polarizability, and thereby the reflectivity, vanishes when d is chosen such that $kd = \tan^{-1}(1/\zeta) \bmod \pi$. Assuming this *transmissive* condition, one can linearize the cavity resonance condition for a small variation δd of the mirror spacing. This readily gives an optomechanical coupling strength

$$g_0' = \left|\frac{\delta\omega}{\delta d}\right| x_0' \approx \sqrt{2} g \frac{|r|}{1-|r|}, \tag{1}$$

provided $d|\zeta|^2 \ll L$, where $x_0' = x_0/\sqrt{2}$ is the extent of the zero-point motion for this breathing mode. It is evident that g_0' scales more favorably with $|r|$ than g_0. One can interpret this result by noting that, as the reflectivity of the individual elements is increased, the constructive interference that is responsible for making the array transmissive also strongly

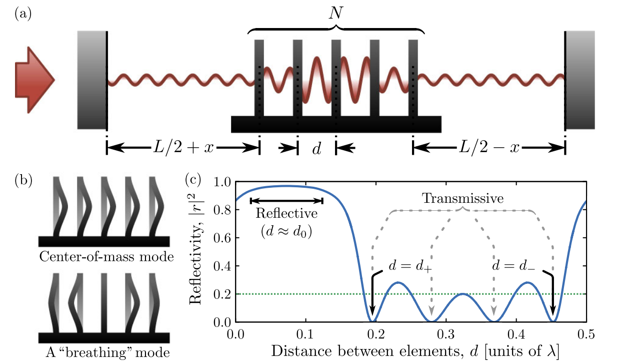

FIG. 1 (color online). Schematic of model, motional modes, and working points. (a) System considered: N equidistant elements positioned in the field of a Fabry-Pérot resonator (we shall consider only the case for which $L \gg Nd$). (b) Two examples of collective motional modes: the center-of-mass mode and an example of a "breathing" mode. (c) Free-space reflectivity of a five-element array as a function of the element spacing d; the curve is periodic with period $\lambda/2$. The intensity reflectivity of each element is 20% (dotted green line).

enhances the dispersive response of the cavity around this working point. In a symmetric situation the displacement of a mirror in one direction will cause the field to adjust so that the other mirror moves in the opposite direction, thereby balancing the power impinging on the two mirrors. In this simple two-element case, the radiation-pressure force thus couples naturally to a breathing mode [Fig. 1(b)].

Optical "superscatterers."—To treat the general case of an array of N equally spaced elements in free space we make use of the transfer matrix formalism [29] for one-dimensional systems of polarizable scatterers, and derive the response of the system to a propagating light field. As is well known from the theory of dielectric mirrors, the reflectivity of the ensemble can be tuned to have markedly different behaviors at a given frequency [Fig. 1(c)]. An array of N equally spaced identical elements, each of polarizability ζ, can be described through a matrix that relates left- and right-propagating fields on either side of the array [30]. For real ζ, N lossless scatterers behave as a collective superscatterer having effective polarizability $\chi = \zeta\sin[N\cos^{-1}(a)]/\sqrt{1-a^2}$, with $a = \cos(kd) - \zeta\sin(kd)$, together with a phase shift μ, which is the phase accrued on reflection from the stack. The ensemble attains its largest reflectivity for $kd = kd_0 \equiv -\tan^{-1}(\zeta)$, $\chi = \chi_0 \equiv -i\sin[N\cos^{-1}(\sqrt{1+\zeta^2})]$, and becomes fully transmissive ($\chi = 0$) for $kd = kd_\pm \equiv -\tan^{-1}(\zeta) \pm \cos^{-1}[(1+\zeta^2)^{-1/2}\cos(\pi/N)]$. For absorbing scatterers, setting $d = d_-$ (modulo $\lambda/2$) helps minimize the effects of absorption [30]. These working points are illustrated in Fig. 1(c). From this point onwards, the array can be treated as a single scatterer, keeping in mind the dependence of χ on the interelement spacing.

Ensemble coupling strength.—When placed inside a cavity, at first neglecting any motion, this array of scatterers modifies the resonance condition, such that the resonances of the system are given by the solutions to [30]

$$e^{ikL} = \frac{e^{-i\mu}}{1 + i\chi}[i\chi\cos(2kx) \pm \sqrt{1 + \chi^2\sin^2(2kx)}], \quad (2)$$

where x is the displacement of the ensemble with respect to the cavity center, and $\mu \equiv \mu(x_1, x_2, \ldots)$ and $\chi \equiv \chi(x_1, x_2, \ldots)$ depend on the positions x_j of the individual elements. For a particular configuration, Eq. (2) is solved numerically to find the resonance frequency $\omega = kc$. A small shift δx_j in the position of the jth element in the array shifts this resonance: $\omega \to \omega - g_j\delta x_j$. The vector (g_j) defines the profile of the collective motional mode that is coupled to the cavity field. In the case of a transmissive ensemble, the intensity profile peaks at the center of the array [Fig. 1(a)]. The optomechanical coupling strength g_j for the jth membrane is strongest where the difference in amplitudes across the membrane is greatest, $j \approx (N+2)/4$ or $(3N+2)/4$, resulting in $g_j \propto \sin[\pi(2j-1)/N]$ and a mechanical mode whose profile varies sinusoidally along the array. In Fig. 2 we plot the transmission of the cavity ($\mathcal{T}_{\text{cav}}$) [30] as a function of frequency and the displacement of this sinusoidal mode. The dashed lines represent solutions to Eq. (2), i.e., in the absence of membrane motion, and are one free-spectral range apart. The gradient of the bright curves at any point is a direct measure of the optomechanical coupling strength for the sinusoidal mode at that point. The center of the plot corresponds to our working point; the adjacent optical resonances are to a good approximation one bare-cavity free-spectral range apart. In the situations we consider here, we have checked that the linear coupling largely dominates over the quadratic coupling [30].

Generically, one obtains the linear optomechanical coupling strength by linearizing Eq. (2) about one of its solutions. For a center-of-mass motion [cf. Fig. 1(b)] in the reflective regime, $d = d_0$, we thus obtain $g_{\text{c.m.}} = g\sqrt{\mathcal{R}/N}$, where $\mathcal{R} = \chi_0^2/(1+\chi_0^2)$ is the maximal intensity reflectivity of the ensemble. As N or ζ increase, $\mathcal{R}$ saturates to 1 and $g_{\text{c.m.}}$ scales as $N^{-1/2}$. This scaling can be explained simply by noting that the motional mass m_N of N elements is N times that of a single one; the single-photon coupling strength, which is proportional to $1/\sqrt{m_N}$, therefore *decreases* with N.

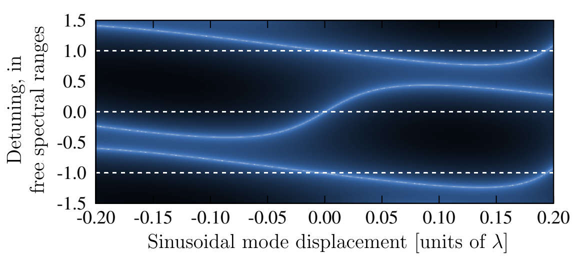

FIG. 2 (color online). Transmission through a cavity with five immobile elements. The dashed lines denote the unperturbed resonances, which are shifted when the displacement of the sinusoidal mode, a type of breathing mode [Fig. 1(b)] that is defined in the text, is nonzero. ($\zeta = -0.5$, $L \approx 6.3 \times 10^4\lambda$, $d = d_-$, bare cavity finesse $\approx 3 \times 10^4$).

In the transmissive regime, $d = d_-$ (modulo $\lambda/2$), then, the cavity field couples to the sinusoidal mode with a collective coupling strength (for large N [30])

$$g_{\text{sin}} = \frac{\frac{\sqrt{2}}{\pi} g\zeta^2 N^{3/2}}{1 + \frac{2}{\pi^2}\frac{d}{L}\zeta^2 N^3} \approx \frac{\sqrt{2}}{\pi} g\zeta^2 N^{3/2}, \quad (3)$$

the last expression being valid for $L/d \gg 2\zeta^2N^3/\pi^2$. Optimizing over N for arbitrary L/d, we obtain $g_{\text{sin}}^{\text{opt}} = \frac{1}{2}g\sqrt{L/d}|\zeta|$ [31]. This favorable scaling with both N and $|r|$, as shown in Fig. 3, is a significant improvement over the state of the art. Close inspection reveals that $g_{\text{sin}}^{\text{opt}}$ is proportional to $1/\sqrt{Ld}$ and therefore can be improved either by making the main cavity smaller (i.e., decreasing L) or, independently, by positioning the elements closer together (decreasing d). The effect we describe is therefore qualitatively different from constructing a smaller cavity having dimensions on the order of λ [26], and also provides a practical route towards integrating strongly coupled optomechanical systems with, e.g., ensembles of atoms in the same cavity [20].

An interesting effect arises in the regime where g_{sin} saturates and eventually starts decreasing as a function of N; the scatterers then act to narrow the cavity resonance substantially. This arises from an effective lengthening of the cavity, due to the presence of the array, to a length $L_{\text{eff}} \equiv L + \frac{2}{\pi^2}d\zeta^2N^3$. Since the cavity finesse in the transmissive regime is fixed by the end mirrors, it follows that the linewidth of the cavity is $\kappa_{\text{eff}} \propto 1/L_{\text{eff}}$ (bare cavity linewidth $\kappa_c \propto 1/L$), which has possible applications in

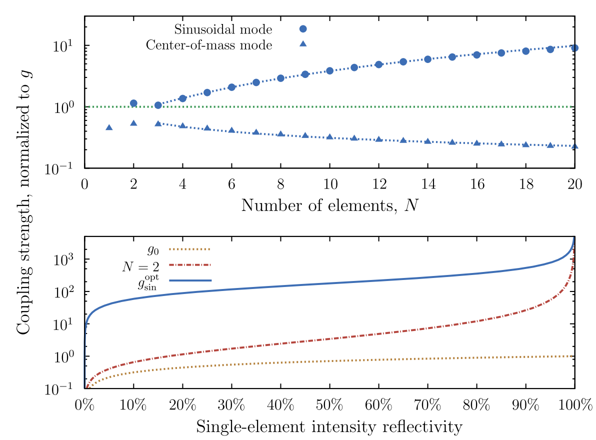

FIG. 3 (color online). Coupling strengths for multielement arrays. These curves are scaled to $g \approx 2\pi \times 15$ Hz (dotted green line), which is the upper bound for the reflective case. Top: Scaling with N of the normalized coupling strength for the sinusoidal ($\propto N^{3/2}$) and center-of-mass ($\propto N^{-1/2}$) modes, as illustrated by the dotted curves. ($\zeta = -0.5$, $L \approx 6.3 \times 10^4\lambda$, $d = d_- + 20\lambda$.) Bottom: Optimized sinusoidal coupling $g_{\text{sin}}^{\text{opt}}$ compared to the coupling for $N = 2$, demonstrating the collectively enhanced coupling strength, and to g_0. ($d = d_-$).

hybrid systems along the same lines as those of electromagnetically induced transparency in Ref. [32]. When using low-finesse cavities and low mechanical oscillation frequencies, this effect could be used to place the system well within the sideband-resolved regime. As shown in Fig. 4, $g_{\sin}$ and κ_{eff} compete to give rise to a constant cooperativity $g_{\sin}^2/(\kappa_{\text{eff}}\Gamma_{\text{dec}})$ for large N ($1/\Gamma_{\text{dec}}$ is the mechanical decoherence time scale, assumed independent of N [30]). In the presence of absorption, which ultimately limits the linewidth narrowing, there exists an optimum number of elements which maximizes the cooperativity to a value that can still be several orders of magnitude larger than the single-element cooperativity.

Long-range collective interactions.—The collective nature of the interaction that is responsible for these large coupling strengths also gives rise to an effective "nonlocal" interaction between the scatterers, where the motion of any particular element greatly influences elements farther away, and not just its nearest neighbors. In the simplest picture of a weak linearized optomechanical interaction [1] in which the field is adiabatically eliminated, the interaction Hamiltonian is proportional to $\sum_{l,j} g_l g_j \hat{x}_l \hat{x}_j$, and mediates a *macroscopically long-ranged* effective interaction between pairs of elements (position operators $\hat{x}_l$ and $\hat{x}_j$). By contrast, in the reflective regime the light does not permeate through the ensemble, and the interelement mechanical interactions would therefore be correspondingly short ranged (see, for example, Ref. [24]). Transmissive arrays with well-designed spacings and polarizabilities could be used to engineer specific optomechanical interactions and gain insight into collective optomechanics phenomena [23,24].

Numerical example, tolerance to imperfections.—The power of this approach to optomechanics is best seen through a numerical illustration. If we take commercial silicon nitride membranes [12] with an intensity reflectivity of 20% ($\zeta = -0.5$) and $x_0 = 1.8$ fm, and a cavity with $L = 6.7$ cm and a wavelength of 1064 nm, we can estimate $g_{\text{c.m.}} \approx 2\pi \times (12.8 \times N^{-1/2}$ Hz) for $N \gtrsim 3$. For the sinusoidal mode, and with the same parameters, $g_{\sin} \approx 2\pi \times (1.3 \times N^{3/2}$ Hz) for large N, an improvement by over an order of magnitude when $N = 10$ (cf. Fig. 3). A transparent ensemble potentially provides a much stronger optomechanical coupling than a reflective one; indeed $g_{\sin}/g_{\text{c.m.}} \propto N^2$. Let us now consider highly reflective membranes [16] having 99.4% intensity reflectivity ($\zeta = -12.9$), $x_0 = 2.7$ fm, and $\omega_m = 2\pi \times 211$ kHz. For a 0.25 cm-long cavity with finesse $F = 1.2 \times 10^5$, $d = d_-$, and $N = 5$ membranes, one obtains $g_{\text{c.m.}} \approx 2\pi \times 600$ Hz and $g_{\sin} \approx 2\pi \times 270$ kHz, which is larger than both ω_m and $\kappa_c = 2\pi \times 250$ kHz. At a temperature of 1 K and with a mechanical quality factor of 10^6 the *single-photon* cooperativity is ca. 14 for this system; strong coupling between a single photon and a single phonon is already within reach with only a few elements. A thorough numerical investigation [30] reveals that our results are robust with respect to various experimentally relevant deviations from the idealized system considered here, such as the errors in the positioning of the individual membranes and nonuniform membrane reflectivity or absorption. For example, for $N = 5$ and $\zeta = -0.5$, the numerically calculated coupling strength typically lies within 12% of the above value for position fluctuations of ± 10 nm, inhomogeneities in ζ of $\pm 10\%$, and absorption per element of $\gtrsim 10^{-3}$.

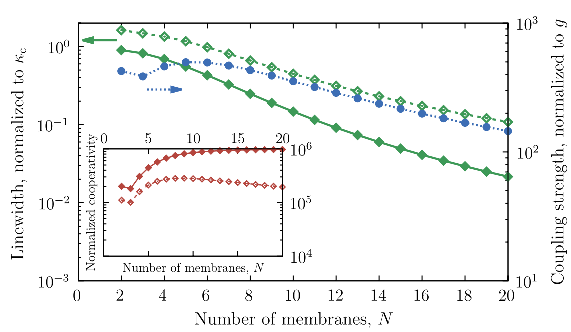

FIG. 4 (color online). Effective cavity linewidth κ_{eff} (diamonds) and optomechanical coupling strength $g_{\sin}$ (cf. Fig. 3, circles) as a function of the number of membranes. Inset: Cooperativity $g_{\sin}^2/(\kappa_{\text{eff}}\Gamma_{\text{dec}})$ normalized to the single-element cooperativity $g^2/(\kappa_c\Gamma_{\text{dec}})$; the non-normalized cooperativity can reach values >10 (see text). Closed symbols represent absorption-free membranes, open symbols an absorption of 10^{-5} per membrane. (99.4% reflectivity, $d = d_- + 10\lambda$, $L \approx 6.3 \times 10^4 \lambda$, bare cavity finesse $\approx 3 \times 10^4$.)

Moving away from highly reflective scatterers, we can apply our results to systems of very low reflectivity, such as atoms, molecules, dielectric microspheres, etc. It should first be noted that all these systems have a reflectivity on the order of 10^{-6}, which means that $N|\zeta| \ll 1$ in typical experiments (for example, with cold atoms in cavities [14]); i.e., the particles do not significantly modify the mode structure of the cavity resonance. In this case $g_{\sin}$ reduces to the expected $N^{1/2}$ scaling that arises from the independent coupling of well-localized scatterers interacting with an unmodified cavity field [14]. We note, however, that recent experiments [28] using "pancake-shaped" clouds of cold atoms in an optical lattice have shown intensity reflectivities as high as 80% and are approaching a regime where the effects discussed previously may be observed.

Conclusions.—We have made use of a fully analytical theory to explore novel interactions between the collective mechanical dynamics of an array of equidistant scatterers inside a cavity and the cavity field itself. Our ideas apply generically across a wide range of systems; any system that can be modeled as a one-dimensional chain of scatterers (e.g., membranes, atoms [28], optomechanical crystals [2], or dielectric microspheres [15]) is amenable to a similar analysis and shows the same rich physics. Similar methods would allow the extension of these ideas to more complicated systems where the polarizability is a function of frequency or of position along the array, or systems involving the interaction of arrays of refractive elements with multiple optical modes.

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