Spatial Coherence and Optical Beam Shifts

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A beam of light, reflected at a planar interface, does not follow perfectly the ray optics prediction. Diffractive corrections lead to beam shifts; the reflected beam is displaced (spatial Goos-Hänchen type shifts) and/or travels in a different direction (angular Imbert-Fedorov type shifts), as compared to geometric optics. How does the degree of spatial coherence of light influence these shifts? We investigate this issue first experimentally and find that the degree of spatial coherence influences the angular beam shifts, while the spatial beam shifts are unaffected.

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A collimated optical beam is the best experimental approximation of a ray in geometrical optics. However, due to the wave nature of light, beams do not behave exactly as rays, and already in the case of refraction and reflection at planar interfaces, deviations from geometric optics occur. Goos and Hänchen [1] found the first experimental proof of this—an optical beam undergoes a small parallel in-plane (longitudinal) displacement upon total reflection. Since then, multiple variants have been found: out-of-plane shifts such as the Imbert-Fedorov shift [2,3] and the spin Hall effect of light [4,5], angular shifts [6], shifts for higher-order modes [7], shifts for photonic crystals [8], shifts for waveguides [9,10], shifts for resonators [11], connection between beam shifts and weak values [12,13], and shifts for matter waves [14–16].

Surprisingly, in spite of this large body of work, the role of the (transverse) spatial coherence of the beam has hardly been addressed. In the original experiment by Goos and Hänchen [1] a Hg lamp was used and some degree of coherence was created in a two-aperture setup, but this was not analyzed [17]. So, it was not clear whether this coherence was essential or not. Almost all modern beam shift experiments have been performed with a single-mode laser source that has near-perfect spatial coherence, or with an extended source filtered by a single mode fiber, which also has very good spatial coherence [6]. An exception is a recent experiment [18], which used a light-emitting diode (without spatial filter); the authors speculate that some nonunderstood aspects of their results could be due to the lack of spatial coherence of their source. The only theoretical papers, as far as we know, that address these issues are Refs. [19–23], where Refs. [19,21,23] lead to diametrically opposed results as compared to Refs. [20,22]. We aim in this Letter to experimentally clarify the role of spatial coherence in beam shift experiments. In short, we find that the theoretical analysis in Ref. [19] and, in more general form, that in Refs. [21,23] does correctly describe our results.

We start by briefly reviewing the theory. We consider a monochromatic partially coherent beam with a Gaussian envelope [24–26], a so-called Gaussian Schell-model beam, where both the intensity distribution $I(\boldsymbol{\rho})$ and the spatial degree of coherence $g(\boldsymbol{\Delta\rho})$ are Gaussian [27] ($\boldsymbol{\rho}$ and $\boldsymbol{\Delta\rho} = \boldsymbol{\rho}_2 - \boldsymbol{\rho}_1$ are the transverse position, and the relative transverse position, respectively). We obtain for the Gaussian Schell-model beam

$$I(\boldsymbol{\rho}) \propto \exp\left(-\frac{\rho^2}{2\sigma_S^2}\right), \qquad g(\boldsymbol{\Delta\rho}) = \exp\left(-\frac{\Delta\rho^2}{2\sigma_g^2}\right). \quad (1)$$

In the source plane, σ_S is the coherent (Gaussian) mode waist, and σ_g determines the correlation length. The latter approaches infinity for a fully coherent mode, and is a measure of the speckle size in case of partial spatial coherence. After propagating over a distance z, these quantities evolve into $\sigma_g(z)$ and $\sigma_S(z)$; however, it turns out that their ratio σ_g/σ_S is independent of propagation [29]: $\sigma_g(z)/\sigma_S(z) = \sigma_g/\sigma_S$; therefore, we use this ratio to quantify transverse coherence. Figure 1(a) shows three examples of such beams, from fully coherent (i) to the case where the coherence length is below one tenth of the beam size (iii). By calculating the intensity autocorrelation, the two length scales which are involved become visible: The Gaussian envelope leads to a wide background, while the emerging speckles in case (ii) and (iii) add a short-range correlation as is easily visible in the cross-section curves in Fig. 1(b). The number of participating modes can be estimated from the number of speckles in Fig. 1(a), or be approximated by $[1 + (\sigma_g/\sigma_S)^{-1}]^2$, which is (1, ≈ 50, ≈ 200) for the cases (i,ii,ii) respectively. The three beams shown in Fig. 1 have been used in the experiments reported below.

To be able to discuss such a Gaussian Schell-model beam within the unifying beam shift framework developed by Aiello and Woerdman [30], we consider a paraxial, monochromatic, and homogeneously polarized

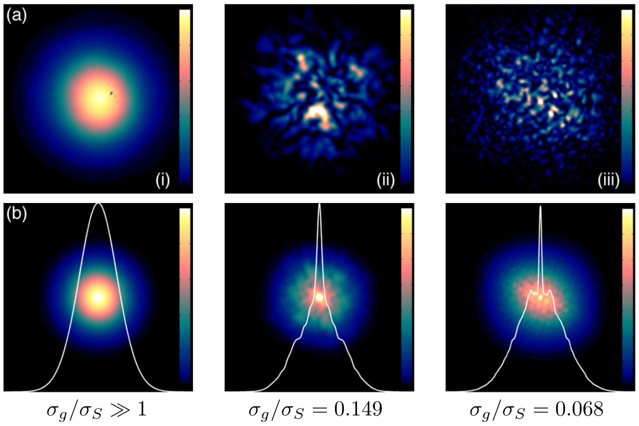

FIG. 1 (color online). Study of the statistical properties of the Gaussian Schell-model beams for three different degrees of spatial coherence, which are used in the experiments ($\lambda = 675$ nm). The first row (a) shows exemplary intensity profiles out of which the statistical ensemble beam consists; these are obtained by stopping the rotation of the diffusor plate. The second row (b) shows the intensity autocorrelation of the corresponding fields in (a) as false-color plots and cross-sectional curves. This shows clearly the two scales involved, i.e., the Gaussian beam width and the speckle size. We have determined the ratio σ_g/σ_S by measuring the beam waist together with the far field divergence angle [24].

($\lambda = 1, 2 \equiv p, s$), but otherwise arbitrary, incoming optical field $\mathbf{U}^i(x, y) = \sum_\lambda U(x, y) a_\lambda \hat{\mathbf{x}}^i_\lambda$. It propagates along $\hat{\mathbf{x}}^i_3$ (z coordinate), and (a_1, a_2) is its polarization Jones vector. We use dimensionless quantities in units of $1/k$, where k is the wave vector. The coordinate systems and their unit vectors $\hat{\mathbf{x}}^{i,r}_\lambda$ are attached to the incoming (i) and reflected (r) beam, respectively [see Fig. 2(a)]. After reflection at a dielectric interface, the polarization and spatial degree of freedom are coupled by the Fresnel coefficients $r_{p,s}$ as [7]

$$\mathbf{U}^r(x, y, z) = \sum_\lambda a_\lambda r_\lambda U(-x + X_\lambda, y - Y_\lambda, z) \hat{\mathbf{x}}^r_\lambda. \quad (2)$$

$X_{1,2}$ and $Y_{1,2}$ are the polarization-dependent dimensionless beam shifts:

$$X_1 = -i\partial_\theta[\ln r_1(\theta)], \qquad Y_1 = i\frac{a_2}{a_1}\left(1 + \frac{r_2}{r_1}\right)\cot\theta, \quad (3a)$$

$$X_2 = -i\partial_\theta[\ln r_2(\theta)], \qquad Y_2 = -i\frac{a_1}{a_2}\left(1 + \frac{r_1}{r_2}\right)\cot\theta. \quad (3b)$$

Their real parts correspond to spatial beam shifts, and their imaginary parts to angular beam shifts. For either variant, beam displacements X_λ (along the $\hat{x}$ coordinate) correspond to longitudinal Goos-Hänchen (GH) type shifts [1], while transverse displacements Y_λ along $\hat{y}$ have Imbert-Fedorov character [3,4]. We observe that the transverse shifts Y_λ require simultaneously finite a_1 and a_2, such as is present in circularly polarized light; this is not necessary for the longitudinal shifts X_λ. This explains why

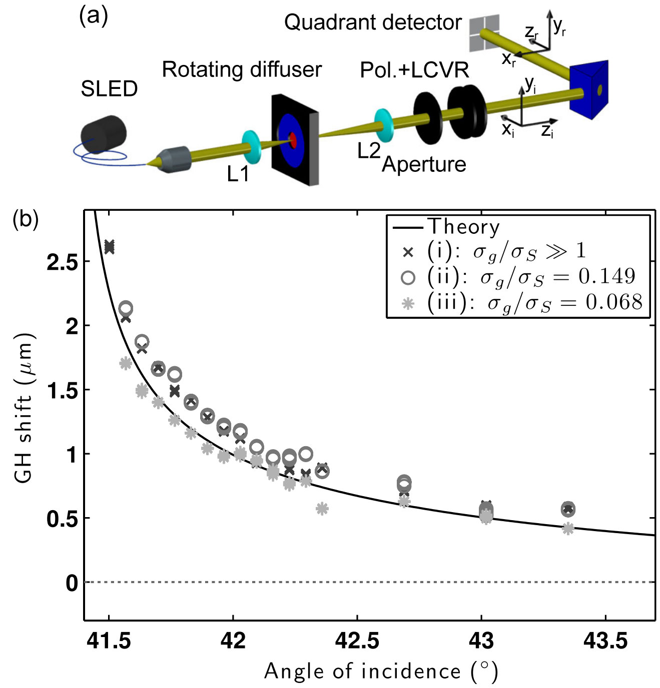

FIG. 2 (color online). Setup. (a) An optical beam with variable spatial coherence is prepared by collimating the light scattered from a holographic diffuser plate. A combination of a polarizer and a liquid-crystal variable retarder (LCVR) is used to modulate between s and p polarization. After total internal reflection from the prism, the displacement is determined with a quadrant detector. Spatial Goos-Hänchen shift measurements. (b) Experimentally obtained spatial GH beam shifts for different degrees of spatial coherence. Shown is the observed polarization-differential shift (symbols); the black curve corresponds to the theoretical prediction.

the spatial shifts depend in the GH case only on one reflection phase, ϕ_1 or ϕ_2 with $\phi_\lambda = \arg(r_\lambda)$, while in the Imbert-Fedorov case, the spatial shift depends on the phase difference (e.g., ϕ_1-ϕ_2).

In the lab, beam shifts are usually measured via the centroid of the reflected beam

$$\langle \mathbf{R} \rangle(z) = \sum_\lambda w_\lambda \frac{\int \rho \langle |U(-x + X_\lambda, y - Y_\lambda, z)|^2 \rangle dxdy}{\int \langle |U(-x + X_\lambda, y - Y_\lambda, z)|^2 \rangle dxdy}, \quad (4)$$

where $w_\lambda = |r_\lambda a_\lambda|^2 / \sum_\nu |r_\nu a_\nu|^2$ is the fraction of the reflected intensity with polarization λ. Equation (4) can be calculated by Taylor expansion around zero shift ($X_\lambda = Y_\lambda = 0$). With the spatial $\mathbf{\Delta}_\lambda = \mathrm{Re}(X_\lambda, Y_\lambda)$ and angular $\mathbf{\Theta}_\lambda = \mathrm{Im}(X_\lambda, Y_\lambda)$ shift vectors we obtain for the centroid $\langle \mathbf{R} \rangle(z) = \sum_\lambda w_\lambda (\mathbf{\Delta}_\lambda + M(z)\mathbf{\Theta}_\lambda)$, where $M(z)$ is a polarization-independent 2×2 matrix which couples longitudinal and transverse beam shifts depending on the transverse mode of the field [7].

For a spatially incoherent beam, the incoming field $\mathbf{U}^i$ corresponds to one realization of the ensemble of random fields with equal statistical properties. For our case of a

Gaussian Schell-model beam, $M(z)$ turns out to be diagonal [21], and we finally obtain

$$\langle \mathbf{R} \rangle (z) = \sum_{\lambda=1}^{2} w_\lambda [\boldsymbol{\Delta}_\lambda + \boldsymbol{\Theta}_\lambda \theta_S^2 z]. \tag{5}$$

The first term is independent of z, it therefore describes shifts of purely spatial nature. Since spatial coherence enters the discussion only via the parameter θ_S (which we discuss below), and the first term is independent thereof, we conclude that spatial shifts are expected to be independent of the degree of transverse coherence.

We test this in our first experiment [Fig. 2(a)], where we examine the *spatial* Goos-Hänchen shift (extension to the spatial Imbert-Fedorov case is straightforward). A single Gaussian mode from a fiber-coupled 675 nm superluminescent diode (FWHM spectral width 20 nm) is focused loosely ($f_{L1} = 20$ cm, beam waist at focus 50 μm) close to the outer edge of a holographic diffuser (Edmund Optics NT47-988 light shaping diffuser, 25 mm diameter, scattering angle 0.5°) [25]. This imprints a random phase on the beam, which in turn leads to speckle pattern formation by random interference. To average over many different realizations of this field, the diffuser is rotated at 70 Hz, which leads to a modulation in the speckle pattern at ~30 kHz (this is related to the microscopic structure of the diffuser). This frequency is much higher than the polarization modulation frequency, see below. We collimate the far field ($f_{L2} = 10$ cm) from the plate and use an adjustable diaphragm [see Fig. 2(a)] to gain full control over the key parameter σ_g/σ_S. We implement polarization modulation (10 Hz) using a polarizer in combination with a liquid-crystal variable retarder to generate an s or p polarized beam. This beam is reflected under total internal reflection in a $45° - 90° - 45°$ prism (BK7, $n = 1.514$ at 675 nm), and refraction at the side faces of the prism is taken into account for determination of the angle of incidence θ. A quadrant detector in combination with a lock-in amplifier (locked to the polarization modulation) is used to measure the relative beam displacement (the quadrant detector is binned so that it effectively acts as a binary split detector). Figure 2(b) shows the measured spatial GH shifts for the three beams with different spatial coherence shown in Fig. 1. We present exclusively polarization-differential shifts $D_{ps} = D_p - D_s$, where $D_{p,s}$ are the displacements of p and s polarized reflected beams from the geometrical-optics position. For $\sigma_g/\sigma_S \gg 1$ we recover the well-known result that the spatial GH shift appears only for $\theta > \theta_c$ [31], where θ_c is the critical angle of 41.35°. However, the essential point of Fig. 2(b) is, that we find that the spatial beam shift is in fact *independent* from the degree of spatial coherence. This demonstrates that the theoretical result in Refs. [19,21,23] is correct, contrary to competing claims [20,22].

We turn now to the *angular* shifts, i.e., to the second term in Eq. (5). The parameter θ_S is simply the effective beam divergence half-angle for a Gaussian Schell-model beam [24]:

$$\theta_S^2 = \frac{2}{k^2}\left[\left(\frac{1}{2\sigma_S}\right)^2 + \left(\frac{1}{\sigma_g}\right)^2\right]. \tag{6}$$

We see that reduced spatial coherence, i.e., reduced σ_g, leads to increased beam divergence, and this in turn leads to increased angular beam shifts.

We test this in our second experiment, where we investigate the case of the in-plane (Goos-Hänchen type) *angular* beam shift. For this we use, as shown in Fig. 3(a), an additional lens L3 ($f_{L3} = 10$ cm) to focus the beam, which is now reflected externally at the hypotenuse plane of the same prism as used before. The angular shift implies that s and p polarized beams follow slightly different paths which both originate at the beam waist [6]. For our experimental parameters and for beam propagation of a few centimeters, this angular shift is expected to lead to many tens of λ displacements of the centroid. We can then use simply a CCD camera to determine the difference in centroid position for p and s polarization. From two of such measurements at different propagation distance

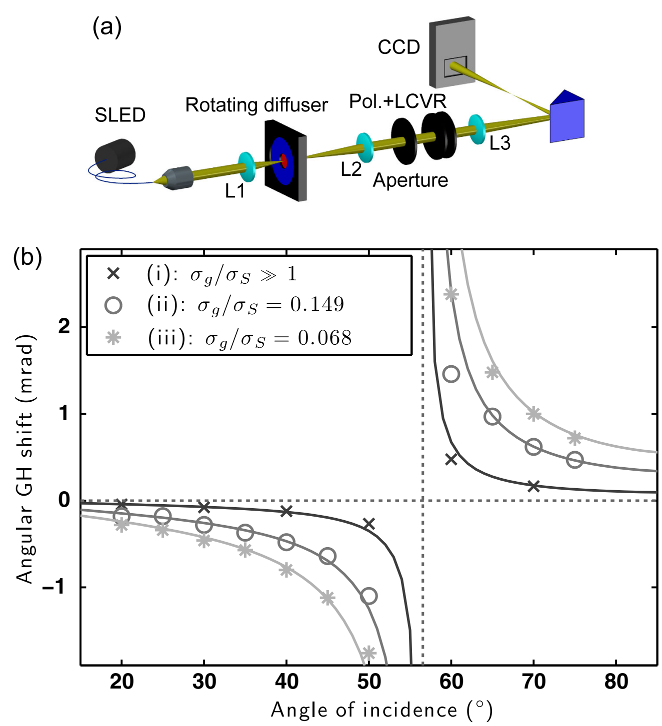

FIG. 3 (color online). Setup to measure angular shifts. (a) Compared to the experiment in Fig. 2(a), we introduce lens $L3$ to give the beam a sizable angular spread, and we use external reflection from the prism hypotenuse face. Further, we use a CCD camera and centroid determination by a computer to measure the relative beam position for s and p polarization. (b) Angular beam shifts for different degrees of coherence. The experimental data (symbols) agree with theory (curves); the vertical line indicates the Brewster angle (56.55°).

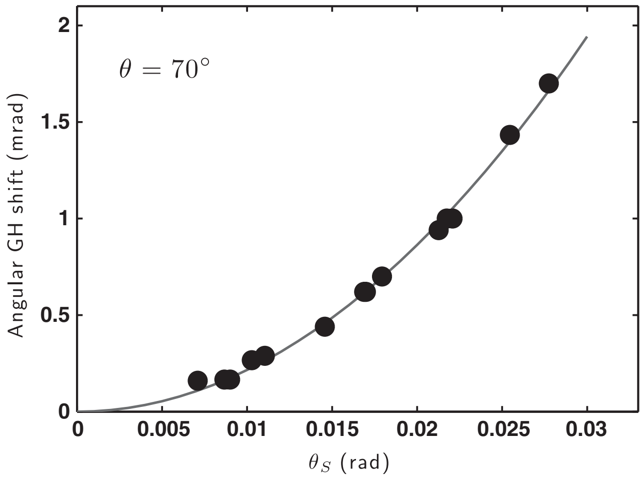

FIG. 4. Demonstration of the particular nature of the decoherence-enhanced angular beam shifts: The shifts scale with θ_S^2. Dots are experimental data. The measurement uncertainty is of the order of the size of the dots. The curve shows the theoretical prediction (there is no fit parameter involved).

(5 cm apart) we determine the angular Goos-Hänchen shift; see Fig. 3(b). The angular shift shows a dispersive shape around the Brewster angle θ_B. This in itself is well known [6]; it is a consequence of the fact that the amplitude reflectivity flips sign at its zero crossing at θ_B. New is that we find a strong influence of the degree of coherence on the shift, in perfect agreement with the theoretical curves shown. We conclude that also in this case the theoretical predictions in Refs. [19,21,23] are correct.

Further, we note that if we replace the coherent-mode opening angle θ_0 in the angular GH shift formulas by the effective beam opening angle θ_S [see Eq. (6)], partially coherent beams are well described. We therefore expect that, for a constant angle of incidence θ, the angular beam shift is proportional to θ_S^2. We have demonstrated this experimentally for $\theta = 70°$; see Fig. 4.

In conclusion, we have found experimentally that partial spatial coherence of a beam does not affect spatial beam shifts, while angular beam shifts are enhanced. Basically, reduced spatial coherence increases the effective angular spread of the beam, and therefore, angular shifts are increased. Our data are in good agreement with the theoretical study of Simon and Tamir [19], as well as later work [21,23]. We can conclude that the dispute in literature [19–23] is now definitively resolved.

We note that partially coherent beams have several advantages: they are less vulnerable to speckle formation and also less susceptible to atmospheric turbulences [32]. Although our results have been obtained for a single dielectric interface, this can be easily extended to the case of multilayer dielectric mirrors and metal mirrors. Also, despite that our experimental results are for longitudinal Goos-Hänchen type shifts only, it is clear from theory that the spatial and angular transverse Imbert-Fedorov shifts depend in the same way on the degree of spatial coherence as the spatial and angular GH shifts do. For completeness we mention that in our work we simulate a truly stochastic beam, namely by using a rotating diffusor; this has a much wider range of applications than when one uses a stationary diffusor [33]. Our findings demonstrate that transverse-incoherent sources, such as light-emitting diodes, can be used in applications which use beam shifts as a sensitive meter, such as in biosensing [34] or position detection [35]; as well as that the use of Goos-Hänchen shifts for beam position control [36,37] is applicable to incoherent beams. Beam shifts may also be relevant in photo lithography where partial spatial coherence plays a role [38]. Finally, our findings are relevant for beam shifts of particle beams (such as electron beams [16] or other matter beams [15]). Such beams are extremely difficult to prepare in a single mode (contrary to light beams) due to the smallness of the de Broglie wavelength; however, we know now that this should not diminish their (spatial) beam shifts.

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