

Fast and Robust Spin Manipulation in a Quantum Dot by Electric Fields

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(Received 27 April 2012; published 13 November 2012)

We apply an invariant-based inverse engineering method to control, by time-dependent electric fields, the spin dynamics in a quantum dot with spin-orbit coupling in a weak magnetic field. The designed electric fields provide a shortcut to adiabatic processes that flip the spin rapidly, thus avoiding decoherence effects. This approach, being robust with respect to the device-dependent noise, can open new possibilities for spin-based quantum information processing.

DOI: [10.1103/PhysRevLett.109.206602](https://doi.org/10.1103/PhysRevLett.109.206602)

PACS numbers: 72.25.Dc, 72.25.Pn, 73.63.Kv

Coherent spin manipulation in quantum dots (QDs) [1–9] is the key element in state-of-the-art spintronics and solid-state quantum information [10,11]. Accurate spin manipulation can be achieved by several techniques. One of them is the conventional electron spin resonance induced by a magnetic field oscillating at the Zeeman transition frequency [1]. A more robust technique is the spin manipulation with geometric Berry phases during adiabatic motion [2,3]. Nowadays, there is also a growing interest in the electric control of spin using spin-orbit (SO) coupling [12]. It has been applied to high-fidelity spin manipulation on the 100 ns time scale [7–9]. This highly efficient all-electrical method has several advantages. For example, it is easy to generate time-dependent electric fields on the nanoscale by adding local electrodes and produce spin manipulation by making them Zeeman resonant [7]. As a result, Rabi spin oscillations appear at a frequency much smaller than the Zeeman frequency, making the flip relatively slow and prone to decoherence. We shall propose here another all-electrical technique to flip spin with high-fidelity via “shortcuts to adiabaticity” in a time that can be much shorter than any decoherence time.

Recently, several shortcuts to adiabaticity have been put forward to speed up the adiabatic passage of quantum systems and achieve a robust and fast adiabaticlike control [13–26]. The transitionless or counterdiabatic control algorithms proposed by Demirplak and Rice [13], and Berry [14], are designed to add supplementary time-dependent interactions that cancel the diabatic couplings of a reference process. The system then follows exactly the adiabatic trajectory of the original unperturbed process, in principle in an arbitrarily short time. Transitionless quantum drivings have been implemented in two-level systems: spins in a magnetic field [14], atoms [15], and Bose-Einstein condensates in optical lattices [16]. A different shortcut is provided by inverse engineering the transient Hamiltonian [17,18] using Lewis-Riesenfeld invariants [27]. This method has been used for time-dependent traps [17–21], atomic transport [22], and other applications

[23,24]. Although these two methods are potentially equivalent [25], their implementations and results can be quite different. Here we choose the invariant-based inverse engineering approach, since it is better suited than the transitionless driving to be produced by the desired all-electrical means.

Model.— We consider the electric control of electron spin in a QD formed by gating in the x - y plane of a two-dimensional electron gas confined in the z direction by the coordinate-dependent material composition, under a weak magnetic field $\mathbf{B}_0 \parallel \mathbf{z}$, as shown in Fig. 1. Here the total Hamiltonian H of the electron interacting with the external electric field $\mathcal{E}(t) = -\partial\mathbf{A}/c\partial t$ (with zero scalar potential) is $H = H_0 + H_{\text{so}} + H_{\text{int}}$, with [12]

$$H_0 = \frac{p_x^2 + p_y^2}{2m} + U(x, y) + \frac{\Delta_z}{2} \sigma_z, \quad (1)$$

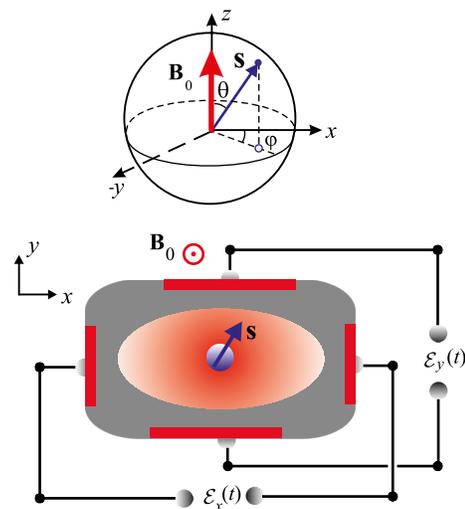


FIG. 1 (color online). Schematic diagram of the spin dynamics of an electron in a gate-formed QD in the presence of electric fields $\mathcal{E}_i(t)$ and perpendicular magnetic field $\mathbf{B}_0 \parallel \mathbf{z}$ axis.

$$H_{\text{so}} = (-\alpha\sigma_y + \beta\sigma_z)p_x + \alpha\sigma_x p_y, \quad (2)$$

$$H_{\text{int}} = -\frac{e}{c}\mathbf{A}(t) \cdot \mathbf{v}, \quad (3)$$

where m is the electron effective mass and σ_i ($i = x, y, z$) are the Pauli matrices. H_0 represents the kinetic energy, the potential $U(x, y)$ formed by lateral gates, and the Zeeman splitting $\Delta_z = g\mu_B B_0$, where μ_B is the Bohr magneton, and g is the Landé factor. The eigenfunctions of H_0 are $\psi_j(x, y)|\sigma\rangle$, where $|\sigma\rangle = |\pm 1\rangle$ is the eigenstate of σ_z , and the spectrum is given by $E_j \pm \Delta_z/2$, where E_j are the orbital eigenenergies in the confinement potential.

The SO coupling is the sum of structure-related Rashba (α) and bulk-originated Dresselhaus (β) terms for the [110] growth axis [28]. The vector potential $\mathbf{A}(t)$ is in the (x, y) plane, and the corresponding spin-dependent velocity operators are

$$v_x = \frac{i}{\hbar}[H_0 + H_{\text{so}}, x] = p_x/m + \beta\sigma_z - \alpha\sigma_y, \quad (4)$$

$$v_y = \frac{i}{\hbar}[H_0 + H_{\text{so}}, y] = p_y/m + \alpha\sigma_x. \quad (5)$$

We focus on the doublet $\Psi_1 = \psi_1|1\rangle$, $\Psi_2 = \psi_1|-1\rangle$, include the higher orbitals by Löwdin partition [29,30], and reduce the full Hamiltonian into an effective 2×2 one [31]:

$$H^{\text{eff}} = \frac{g\mu_B}{2} \begin{pmatrix} Z & X + iY \\ X - iY & -Z \end{pmatrix}, \quad (6)$$

where $X = B_2(1 + \xi_y)$, $Y = (\alpha/\beta)(1 + \xi_x)B_1$, and $Z = B_0 + (1 + \xi_x)B_1$, with the effects of higher states characterized by ξ_x and ξ_y , and the components of the designed electric field are renormalized by the factors of $1/(1 + \xi_i)$. The effective magnetic fields are expressed with the SO coupling parameters as $B_1 = -2e\beta A_x/cg\mu_B$ and $B_2 = -2e\alpha A_y/cg\mu_B$. The resulting electric fields are

$$\mathcal{E}_x(t) = \frac{g\mu_B}{2e\beta} \frac{\partial B_1}{\partial t}, \quad \mathcal{E}_y(t) = \frac{g\mu_B}{2e\alpha} \frac{\partial B_2}{\partial t}. \quad (7)$$

In practice, some slowly varying electric fields can be applied to drive the state from Ψ_1 to Ψ_2 adiabatically along an instantaneous eigenstate of the Hamiltonian in Eq. (6). To accelerate the driving using the transitionless algorithm, counterdiabatic fields should be provided [25]. However, the common dependence of Y and Z on B_1 precludes the implementation of the fast driving terms only by electric fields. In contrast, invariant-based inverse engineering naturally leads to an all-electrical driving.

Dynamical invariant and spin-flip example.—We shall design the time dependence of the external electric fields to guarantee the state transfer in some fixed time t_f by using the dynamical 2×2 invariant $I(t)$ satisfying the condition $dI(t)/dt \equiv \partial I(t)/\partial t - [H^{\text{eff}}(t), I(t)]/i\hbar = 0$. Parametrizing the Bloch sphere (Fig. 1), by the angles θ and

φ , we construct yet unknown orthogonal eigenstates $|\chi_{\pm}(t)\rangle$ of $I(t)$ as

$$|\chi_+(t)\rangle = \begin{pmatrix} \cos\frac{\theta}{2} e^{i\varphi} \\ \sin\frac{\theta}{2} \end{pmatrix}, \quad |\chi_-(t)\rangle = \begin{pmatrix} \sin\frac{\theta}{2} \\ -\cos\frac{\theta}{2} e^{-i\varphi} \end{pmatrix}. \quad (8)$$

They satisfy $I(t)|\chi_{\pm}(t)\rangle = \lambda_{\pm}|\chi_{\pm}(t)\rangle$. Introducing $\lambda_{\pm} = \pm g\mu_B B_c/2$, we construct the invariant as [25]

$$I(t) = \frac{g\mu_B}{2} B_c \begin{pmatrix} \cos\theta & \sin\theta e^{i\varphi} \\ \sin\theta e^{-i\varphi} & -\cos\theta \end{pmatrix}, \quad (9)$$

where B_c is an arbitrary constant magnetic field to keep $I(t)$ with units of energy. According to the Lewis-Riesenfeld theory, the solution of the Schrödinger equation, $i\hbar\partial_t\Psi = H^{\text{eff}}(t)\Psi$, is a superposition of orthonormal ‘‘dynamical modes’’ $\Psi(t) = \sum_n C_n e^{i\alpha_n(t)} |\chi_n(t)\rangle$ [27], where C_n are time-independent amplitudes and $\alpha_n(t)$ is the Lewis-Riesenfeld phase:

$$\alpha_n(t) = \frac{1}{\hbar} \int_0^t \langle \chi_n(t') | i\hbar \frac{\partial}{\partial t'} - H^{\text{eff}}(t') | \chi_n(t') \rangle dt'. \quad (10)$$

From the invariant condition $dI(t)/dt = 0$, the angles θ and φ are related to X , Y , and Z by auxiliary equations

$$\dot{\theta} = \eta(X \sin\varphi - Y \cos\varphi), \quad (11)$$

$$\dot{\varphi} = \eta(X \cos\varphi \cot\theta + Y \sin\varphi \cot\theta - Z), \quad (12)$$

where $\eta = g\mu_B/\hbar$. Since X is a function of B_2 , while Y and Z are functions of B_1 , once θ and φ are fixed, Eqs. (11) and (12) give the effective magnetic fields

$$B_1 = \frac{-\beta\dot{\theta} \cot\theta \cos\varphi + \beta(\dot{\varphi} + \eta B_0) \sin\varphi}{\eta(1 + \xi_x)(\alpha \cot\theta - \beta \sin\varphi)}, \quad (13)$$

$$B_2 = \frac{\alpha\dot{\theta} \cot\theta \sin\varphi + \alpha(\dot{\varphi} + \eta B_0) \cos\varphi - \beta\dot{\theta}}{\eta(1 + \xi_y)(\alpha \cot\theta - \beta \sin\varphi)}, \quad (14)$$

from which the electric fields are calculated by using Eq. (7). During the spin-flip process, there exist some time instants $t = t_s$ which satisfy

$$\alpha \cot\theta(t_s) = \beta \sin\varphi(t_s) \quad (15)$$

and make the denominators of B_1 and B_2 zero. To get rid of such singularities, we impose the conditions

$$\beta \sin\varphi(t_s) [\dot{\varphi}(t_s) + \eta B_0 - (\beta/\alpha)\dot{\theta}(t_s) \cos\varphi(t_s)] = 0, \quad (16)$$

$$\alpha \cos\varphi(t_s) [\dot{\varphi}(t_s) + \eta B_0 - (\beta/\alpha)\dot{\theta}(t_s) \cos\varphi(t_s)] = 0, \quad (17)$$

which make the numerators of B_1 and B_2 zero simultaneously. In the following example, we will show how this works.

In general, the eigenstates of the invariant are not the same as the instantaneous eigenstates of the Hamiltonian, since $I(t)$ and $H^{\text{eff}}(t)$ do not commute. If we impose for θ at $t = 0$ and t_f that

$$\theta(0) = 0, \quad \theta(t_f) = \pi, \quad \dot{\theta}(0) = 0, \quad \dot{\theta}(t_f) = 0, \quad (18)$$

then $[H^{\text{eff}}(0), I(0)] = 0$ and $[H^{\text{eff}}(t_f), I(t_f)] = 0$, which guarantees common eigenstates at initial and final times. Moreover, the state obeying Eq. (18) will flip from $|\Psi_1\rangle$ at $t = 0$ to $|\Psi_2\rangle$ at $t = t_f$, up to phase factors, along the eigenstate $|\chi_+(t)\rangle$. To design the trajectory at intermediate times, we assume the polynomial *Ansatz* $\theta = \sum_{j=0}^3 a_j t^j$, where the a_j can be fixed by solving the system implied by Eq. (18). This leads to $\theta(t_f/2) = \pi/2$, so $\cot\theta$ covers the whole $(-\infty, \infty)$ range passing through zero at $t = t_f/2$. This leads to one or several times satisfying Eq. (15), as we will see below in more detail.

To determine $|\chi_+(t)\rangle$ fully, we also need the trajectory for φ . As the initial and final states are the poles of the Bloch sphere, the phase φ is not well defined there. We may nevertheless specify how the trajectory approaches them and impose limits from the right at $t = 0$ and from the left at $t = t_f$, for example,

$$\varphi(0^+) = \pi/2, \quad \varphi(t_f^-) = \pi/2. \quad (19)$$

These conditions are not sufficient, though, since we still have to deal with the singularities and their cancellation. As $\cot\theta(t_f/2) = 0$, we may satisfy Eq. (15) and impose zeros of the denominators for $B_{1,2}$ at $t_s = t_f/2$, if $\sin\varphi(t_f/2) = 0$. Imposing the two conditions

$$\varphi(t_f/2) = 0, \quad (20)$$

$$\dot{\varphi}(t_f/2) = (\beta/\alpha)\dot{\theta}(t_f/2) - \eta B_0, \quad (21)$$

to satisfy Eqs. (16) and (17) at $t_s = t_f/2$, we cancel the singularity there. With the conditions in Eqs. (19)–(21), we solve the third-order polynomial *Ansatz* $\varphi = \sum_{j=0}^3 b_j t^j$ to determine $\varphi(t)$.

Explicit calculations demonstrate that for the third-order polynomial *Ansatz* and boundary conditions imposed here, there is only one (removable) singularity at $t_s = t_f/2$ when the field B_0 is smaller than a certain upper limit B_0^{max} , shown in Fig. 2 as a function of t_f . For $B_0 > B_0^{\text{max}}$, more solutions of (15) appear [B_0 and $\sin\varphi(t)$ are coupled by Eq. (21)], which cannot be canceled with the third-order polynomial. To satisfy Eqs. (16) and (17) at more than one zero of the denominators of $B_{1,2}$, one may set higher order polynomials for φ and further conditions. This increases the bound B_0^{max} but also complicates the driving fields. In the present examples, we just apply the third-order polynomial *Ansatz* with the boundary conditions in Eqs. (18)–(21), so that the applied magnetic field B_0 should not go beyond the upper limit in Fig. 2. As the upper limit field grows for smaller times t_f , this is not a problem in practice. Figure 3 shows examples of spin flip for different values of B_0 .

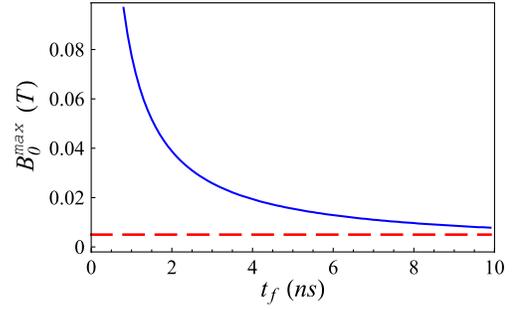


FIG. 2 (color online). Dependence of the maximum of applied magnetic field B_0^{max} (solid blue line) on the time t_f for a third-order polynomial *Ansatz* for θ and φ , with the parameters $\hbar\alpha = 2 \times 10^{-6}$ meV · cm, $\beta = \alpha/2$, and $g = -0.44$ for GaAs. $B_0 = 0.005$ T (dashed red line) corresponds to $\Delta_z = 1.5$ mK.

With the functions θ and φ fixed [see Fig. 3(a)], the designed electric fields $\mathcal{E}_x(t)$ and $\mathcal{E}_y(t)$, corresponding to $B_0 = 0.01$ T and $B_0 = 0.07$ T (close to the upper limit), are depicted in Figs. 3(b) and 3(c). The populations (not shown) of the two spin states, given by $P_1 = \cos^2(\theta/2)$ and $P_{-1} = \sin^2(\theta/2)$, cross each other smoothly as θ goes from 0 to π . The choice of B_0 determines the trajectory on the Bloch sphere for a given t_f . When B_0 approaches the upper limit, the electric fields exhibit sharp peaks; see Fig. 3(c). The smooth time dependence in Fig. 3(b) is well suited for applications, while the complicated dependence in Fig. 3(c) should be avoided. Undesirable excitation of the orbital modes does not occur here, since the spin flip $t_f \sim 1$ ns, while the energy split of the orbital states in typical QDs exceeds 0.1 meV. Therefore, regarding the orbital motion, our perturbation is strongly adiabatic, and no orbital excitation occurs.

To check the stability with respect to initialization errors, we assume now the initial state as $(\sqrt{1-\epsilon}e^{i\varphi_0}, \sqrt{\epsilon})^T$ with an arbitrary phase φ_0 and find θ and φ from Eqs. (11) and (12) for the same designed electric fields (Fig. 4). The final value $\theta(t_f)$ depends on the error ϵ but is insensitive to the initial phase φ_0 , while $\varphi(t_f)$ is sensitive to both initial conditions. Since our goal is to realize the spin flip, the final φ is irrelevant, and the experimental effort should focus on achieving a small error ϵ .

Decoherence and noise effects.— To show the feasibility of our approach, we study the effects of noise and decoherence on the spin-flip fidelity. We begin with a generic approach for coupling to the incoherent environment, based on the conventional Lindblad formalism as can arise, e.g., from interaction with the conduction electron bath under Born and Markov approximations. The master equation reads [32]

$$\dot{\rho} = -\frac{i}{\hbar}[H^{\text{eff}}, \rho] - \frac{\gamma}{2} \sum_i [\sigma_i, [\sigma_i, \rho]], \quad (22)$$

where γ is the dephasing rate. We introduce the Bloch vector with components $u = \rho_{1-1} + \rho_{-11}$,

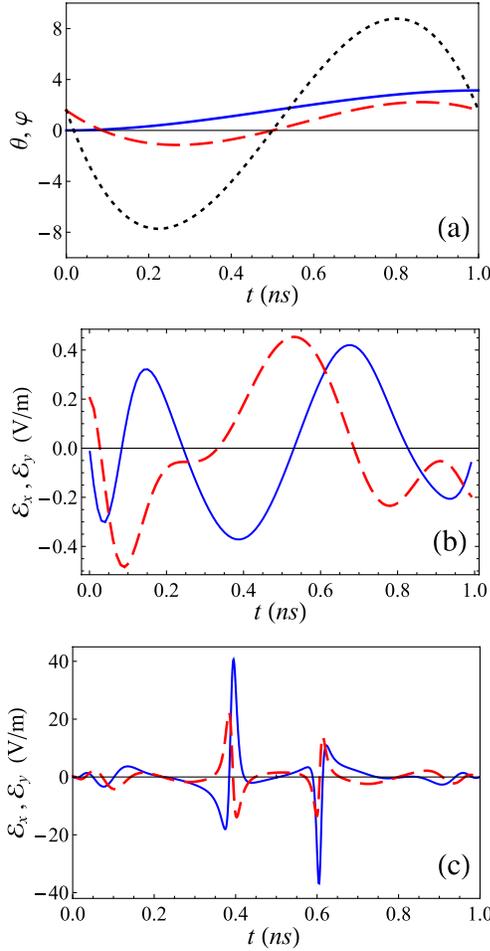


FIG. 3 (color online). For $t_f = 1$ ns, (a) polynomial *Ansätze* of auxiliary angles $\theta = \sum_{j=0}^3 a_j t^j$ (solid blue line), $\varphi = \sum_{j=0}^3 b_j t^j$ with $B_0 = 0.01$ T (dashed red line), and $B_0 = 0.07$ T (dotted black line). The designed electric fields \mathcal{E}_x (solid blue line) and \mathcal{E}_y (dashed red line) by which spin flip can be realized for $B_0 = 0.01$ T (b) and $B_0 = 0.07$ T (c). Other parameters are the same as those in Fig. 2. For simplicity, we put here $\xi_x = \xi_y = 0$ to skip the trivial dependence on these factors.

$v = -i(\rho_{1-1} - \rho_{-11})$, and $w = \rho_{11} - \rho_{-1-1}$ and obtain from Eq. (22)

$$\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -4\gamma & \eta Z & -\eta Y \\ -\eta Z & -4\gamma & \eta X \\ \eta Y & -\eta X & -4\gamma \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix}. \quad (23)$$

We solve Eq. (23) numerically and calculate fidelity $F = |\langle -1 | \Psi(t_f) \rangle|$; see Fig. 5. For $\gamma t_f \ll 1$ the time-dependent perturbation theory [21] yields the bound $F \geq 1 - 2\gamma t_f$. Since the induced flip occurs very fast, it can overcome the main danger for the low-temperature spin manipulation in QDs coming from the hyperfine coupling to the nuclear spins, where the decoherence times exceed 100 ns [7].

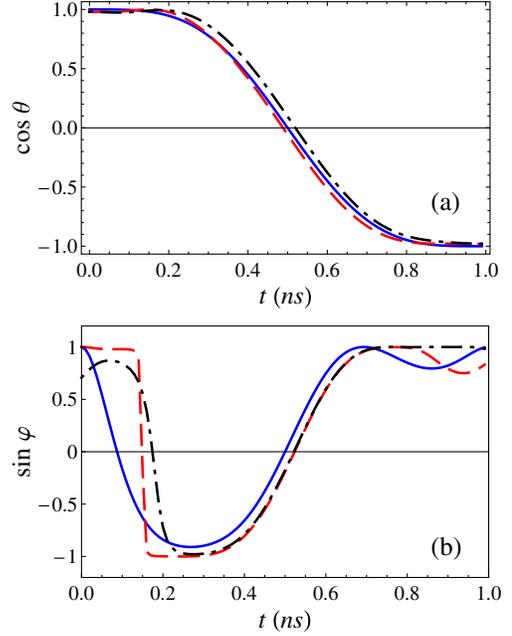


FIG. 4 (color online). Time evolution of $\cos\theta$ (a) and $\sin\varphi$ (b) for the same Hamiltonian with the designed electric fields and $B_0 = 0.01$ T, where $\epsilon = 0$, $\varphi_0 = \pi/2$ (solid blue line); $\epsilon = 0.01$, $\varphi_0 = \pi/2$ (dashed red line); and $\epsilon = 0.01$, $\varphi_0 = \pi/4$ (dot-dashed black line). Other parameters are the same as in Fig. 3.

Another source of decoherence is the device-dependent noise in the electric field acting on the spin. This can be important when the relatively weak electric fields in Fig. 3(b) are applied. We analyze in detail the effect of this noise and find that our method is robust to this randomness in Ref. [33].

Conclusions and outlook.— We have proposed a fast and robust method to flip electron spin in a QD with SO coupling and a weak perpendicular magnetic field. The spin-flip process, designed by Lewis-Riesenfeld invariants,

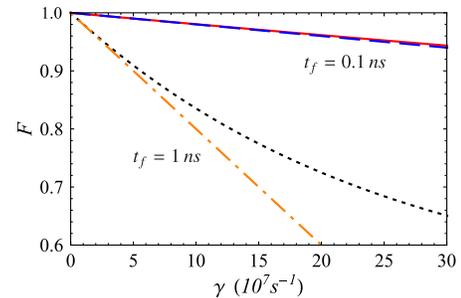


FIG. 5 (color online). Fidelity as a function of γ for different times $t_f = 0.1$ ns (solid red line) and $t_f = 1$ ns (dotted black line). The fidelity estimated from perturbation theory is also compared, for $t_f = 0.1$ ns (dashed blue line, undistinguished) and $t_f = 1$ ns (dot-dashed orange line). $B_0 = 0.01$ T and other parameters are the same as in Fig. 2.

is faster than the decoherence for any known low-temperature dephasing mechanism. This method can be further complemented by optimal control theory for time and energy minimization subjected to different physical constraints [34]. Implementation of this technique will allow for high-fidelity spin manipulation for quantum information processing.

We acknowledge funding by the Basque Country Government (Grants No. IT472-10 and No. BFI-2010-255), Ministerio de Ciencia e Innovacion (Grant No. FIS2009-12773-C02-01), UPV/EHU program UFI 11/55, National Natural Science Foundation of China (Grant No. 61176118), and Shanghai Rising-Star Program (Grant No. 12QH1400800).

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