Nonlinear Anomalous Hall Effect and Negative Magnetoresistance in a System with Random Rashba Field

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We predict two spin-dependent transport phenomena in two-dimensional electron systems, which are induced by a spatially fluctuating Rashba spin-orbit interaction. When the electron gas is magnetized, the random Rashba interaction leads to the anomalous Hall effect. An example of such a system is a narrow-gap magnetic semiconductor-based symmetric quantum well. We show that the anomalous Hall conductivity reveals a strongly nonlinear dependence on the magnetization, decreasing exponentially at large spin density. We also show that electron scattering from a fluctuating Rashba field in a two-dimensional nonmagnetic electron system leads to a negative magnetoresistance arising solely due to spin-dependent effects.

Introduction.—The effects of electron spin on charge transport, being the basic idea of spintronics, have attracted a great deal of interest in fundamental and applied physics as well as in materials science. There are several origins of the spin dependence of charge transport; one of them is the spin-orbit (SO) interaction. This interaction also plays an important role in spin manipulation in spintronics devices [1–4]. Of particular importance is the Rashba SO coupling, usually attributed to quasi two-dimensional (2D) electron systems in metallic or semiconductor nanolayers on a substrate, 2D semiconductor heterostructures with no $z \to -z$ symmetry (with the axis z normal to the layer), or to surface states [5–7]. This interaction enables an electrical control of spin precession of 2D electrons—the phenomenon used in the still hypothetical Datta-Das transistor [8], where the current depends on the angle of spin precession when carriers pass through the device.

Important manifestations of the SO coupling are the anomalous Hall effect (AHE) and spin Hall effect (SHE) [9–11]. In the case of the AHE, a charge current perpendicular to the electric field appears without an external magnetic field when the system has a nonzero spontaneous magnetization [12]. In the SHE, in turn, a static electric field generates a spin current perpendicular to the field orientation. Both effects, however, can be completely suppressed by disorder [13–15].

In symmetrical semiconductor quantum wells, the Rashba SO interaction vanishes on average. However, spatial fluctuations of the Rashba coupling may still appear in the system and can play a qualitatively important role [16–18]. Surprisingly, in the case of spatially fluctuating Rashba field, the SHE in 2D electron gas becomes robust to the effect of impurities; i.e., there is no complete suppression of the spin-Hall conductivity by disorder [19].

In this Letter we further predict two manifestations of spin-dependent transport in 2D electron systems with random Rashba interaction. First, we show that systems with homogeneous magnetization display an AHE. More specifically, we calculate the off-diagonal conductivity of a 2D magnetized electron gas in a symmetric quantum well with spatially correlated fluctuations of SO interaction. We show that the anomalous Hall conductivity reveals a very unusual dependence on the magnetization: it reaches a maximum followed by a fast decrease to zero for magnetization increasing further. Second, we calculate the magnetoresistivity of a nonmagnetic electron gas for a magnetic field parallel to the quantum well and show that the magnetoresistance is negative. This negative magnetoresistance (NMR) is a purely classical effect, unrelated to quantum localization corrections to the conductivity. In both the above mentioned phenomena, the random spin-orbit coupling is crucial and either generates a macroscopic electric current or modifies the macroscopic conductivity.

Model.—The generic model of a 2D electron gas with random Rashba interaction is described by the Hamiltonian $\hat{H} = \hat{H}_0 + \hat{H}^{(\mathrm{so})}$, where (we use the units with $\hbar \equiv 1$)

$$\hat{H}_0 = -\frac{\nabla^2}{2m} + U_{\mathrm{rnd}} - M\sigma_z + \sigma_x \beta B, \tag{1}$$

$$\hat{H}^{(\mathrm{so})} = -\frac{i}{2}\sigma_x\{\nabla_y, \lambda(\mathbf{r})\} + \frac{i}{2}\sigma_y\{\nabla_x, \lambda(\mathbf{r})\}. \tag{2}$$

Here M is half of the spin splitting corresponding to the homogeneous magnetization along the axis z, having purely spin character and not related to any conventional macroscopic magnetic field [15]. The random Rashba coupling parameter $\lambda(\mathbf{r})$ [for $\mathbf{r} = (x, y)$] has zero average, $\langle \lambda(\mathbf{r}) \rangle = 0$, and a Gaussian correlator $\langle \lambda(\mathbf{r}) \lambda(\mathbf{r}') \rangle$, while $\sigma_i (i = x, y, z)$ are the spin Pauli matrices and m is the electron effective mass. The term U_{rnd} describes the spin-independent disorder assumed to be of the white-noise type, B is the in-plane magnetic field along the axis x, and $\beta = g\mu_B/2$, where g is the electron Landé factor. Assuming an in-plane magnetic field, we avoid the diamagnetic orbital effects, and therefore, we take into account only the Zeeman term in Eq. (1). The magnetization leads to splitting of the Fermi surface, and in the limit of small magnetization $\Delta K \equiv k_{F\uparrow} - k_{F\downarrow} = 2Mm/k_F$, as shown in Fig. 1, where k_F is the Fermi wave vector in the corresponding nonmagnetic state.

In the momentum representation Eq. (2) can be rewritten as

$$H_{\mathbf{k}\mathbf{k}'}^{(\mathrm{so})} = \frac{\lambda_{\mathbf{k}\mathbf{k}'}}{2}[\sigma_x(k_y + k_y') - \sigma_y(k_x + k_x')], \quad (3)$$

where $k_i(k_i')$ are the in-plane momentum components $(i = x, y)$. For an external electromagnetic field, $\mathbf{A}(t) = \mathbf{A}_0 e^{-i\omega t}$, one has to make the following replacement in Eqs. (1) and (2): $\nabla \rightarrow \nabla - ie\mathbf{A}/c$. Thus, the matrix elements of the coupling to the electromagnetic field take the form

$$H_{\mathbf{k}\mathbf{k}'}^{(A)} = \left[-\frac{e}{mc}(\mathbf{k} \cdot \mathbf{A}) + \frac{e^2}{2mc^2}A^2 \right]\delta_{\mathbf{k}\mathbf{k}'}$$
$$- \frac{e}{c}\lambda_{\mathbf{k}\mathbf{k}'}(\sigma_x A_y - \sigma_y A_x), \quad (4)$$

and include the term following from the random Rashba interaction. Correspondingly, the matrix elements of the charge current operator $\hat{\mathbf{j}} = -c\partial\hat{H}^{(A)}/\partial\mathbf{A}$ are
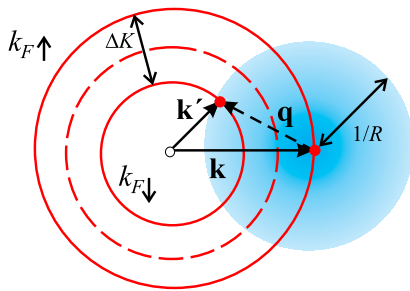

FIG. 1 (color online).   Spin-split Fermi surface. $1/R$ is the possible range of momentum change, $|\mathbf{q}| = |\mathbf{k}' - \mathbf{k}|$ in the spin-flip scattering process between two Fermi surfaces. The dashed circle corresponds to the Fermi momentum k_F. In the limit of small Fermi surface splitting, $\Delta K = 2Mm/k_F$ for $B = 0$ (spin orientation is taken with respect to the z axis), or $|\Delta K| = 2\beta Bm/k_F$ for $M = 0$ (spin orientation is taken with respect to the x axis).

$$(j_{x,y})_{\mathbf{k}\mathbf{k}'} = \frac{e}{m}\left(k_{x,y} - \frac{e}{c}A_{x,y}\right)\delta_{\mathbf{k}\mathbf{k}'} \mp e\lambda_{\mathbf{k}\mathbf{k}'}\sigma_{y,x}. \quad (5)$$

The random spin-orbit coupling leads to spin-flip scattering between states with opposite spin orientations. This, in turn, results in two effects to be analyzed below: the AHE and the NMR.

Anomalous Hall effect.—Let us begin with the AHE induced by a fluctuating Rashba field. Hence, we put here $B = 0$, assume an electric field $\mathbf{E}$ along the axis y, and consider charge current normal to the field. Upon calculating the off-diagonal linear conductivity [20], we find that it can be written as the difference of the contributions from two different spin channels,

$$\sigma_{xy} = \sigma_{xy\uparrow} - \sigma_{xy\downarrow}. \quad (6)$$

In the approximation corresponding to the loop diagrams shown in Fig. 2 one finds $(s \neq s')$

$$\sigma_{xy,s} = \frac{e^2}{m}\sum_{\mathbf{k}\mathbf{q}}|\lambda_q|^2 k_y(2k_y - q_y)\delta(\mu - \varepsilon_{\mathbf{k}-\mathbf{q},s'})G_{k,s}^R G_{k,s}^A,$$
$$(7)$$

where $G_{k,s}^R$ and $G_{k,s}^A$ ($s = \uparrow, \downarrow$) are the retarded and advanced Green functions, respectively. Due to scattering from the fluctuating SO field, electrons with opposite spins are turned to the opposite transverse directions, so $\sigma_{xy\uparrow}$ and $\sigma_{xy\downarrow}$ enter Eq. (6) with opposite signs (they are added in the case of the spin Hall effect). The resulting AHE for $M \neq 0$ is nonzero due to the spin polarization (magnetization) of the electron gas.

Upon calculating the sum over $\mathbf{k}$ in Eq. (7) one obtains

$$\sigma_{xy\uparrow,\downarrow} = \frac{2e^2 m\nu\tau_{\uparrow,\downarrow}}{\pi k_{F\uparrow,\downarrow}} \int_0^\infty dq\left(\mu - \frac{q^2}{8m} \pm \frac{3M}{2}\right)|\lambda_q|^2$$
$$\times \int_0^\pi d\varphi\,\delta\left(\cos\varphi - \frac{q}{2k_{F\uparrow,\downarrow}} \mp \frac{2mM}{k_{F\uparrow,\downarrow}q}\right), \quad (8)$$

where $\tau_{\uparrow,\downarrow}$ is the spin-dependent relaxation time, ν is the density of states per spin, and the upper (lower) sign corresponds to spin-up (spin-down) electrons. The spin-dependence of the relaxation time $\tau_{\uparrow,\downarrow}$ can result from the spin dependence of the corresponding Fermi momentum. In the case of white-noise short range disorder, τ is
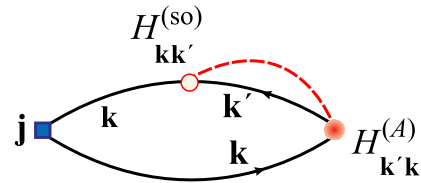

FIG. 2 (color online).   Feynman graph for the anomalous Hall current. Matrix elements in the vertices are determined by Eqs. (3)–(5).

determined solely by the density of states at the Fermi level, and, therefore, does not depend on the spin orientation.

If the magnetization is relatively weak, $\sqrt{2mM} < k_F$, both spin subbands are occupied and the spin-projected conductivity is

$$\sigma_{xy\uparrow,\downarrow} = \frac{2e^2 m \nu \tau_{\uparrow,\downarrow}}{\pi k_{F\uparrow,\downarrow}} \int_{\Delta K}^{k_{F\uparrow}+k_{F\downarrow}} dq |\lambda_q|^2$$
$$\times \frac{\mu - q^2/8m \pm 3M/2}{\sqrt{1 - (q/2k_{F\uparrow,\downarrow} \pm 2mM/k_{F\uparrow,\downarrow}q)^2}}. \quad (9)$$

In turn, when the magnetization is sufficiently strong, $\sqrt{2mM} > k_F$, only one spin subband is occupied. Spin-flip scattering is then absent and σ_{xy} vanishes. As we show below in more detail, dependence of the AHE on the magnetization M is rather unusual.

For further calculations we need a more specific Rashba field correlator, and we assume it in the following generic form: [18]

$$|\lambda_q|^2 = 2\pi \langle \lambda^2 \rangle R^2 e^{-qR}. \quad (10)$$

Here $\langle \lambda^2 \rangle$ characterizes the field variation, and the correlation length R is of the order of the distance between the quantum well and the dopant layers since the in-plane distribution of electric field in the well is controlled by this parameter [21].

Using Eqs. (10) and (6), we have calculated numerically the Hall conductivity as shown in Fig. 3. The conductivity is presented there in the units of $\sigma_0 = \sigma_D \langle \lambda^2 \rangle k_F^2/E_F^2$, where $\sigma_D = ne^2\tau/m$ is the Drude conductivity, and $E_F = k_F^2/2m$. We used the parameters characteristic for InSb: effective mass $m = 0.0134 m_0$, and electron density $n = 5 \times 10^{11}$ cm^{-2} related to the Fermi momentum $k_F = (2\pi n)^{1/2}$. We can estimate the ratio of σ_0/σ_D by using the relation between the electric field $\mathcal{E}$ and the spin-orbit coupling in the form $\langle \lambda^2 \rangle = \xi^2 e^2 \langle \mathcal{E}_r^2 \rangle$, where the variation of the random electric field $\langle \mathcal{E}_r^2 \rangle = 2\pi e^2 n_d/\kappa^2 R^2$. For InSb $\xi \approx 5$ nm^2 [22,23], and for typical parameters of quantum wells we obtain $\sqrt{\langle \lambda^2 \rangle}$ of the order of 10^{-6} meV cm and

FIG. 3 (color online).   Anomalous Hall conductivity as a function of spin splitting parameter M for indicated values of R. Other parameters are defined in the text.

σ_0/σ_D of the order of 10^{-2}–10^{-1}. Thus, the maximum anomalous Hall conductivity can be of an order of 0.01–0.1 of the Drude conductivity.

As follows from Fig. 3, the Hall conductivity has a sharp maximum at a certain value of the parameter M. The physical reason for such a behavior is that the spin-flip scattering from the fluctuating Rashba coupling is effective only for a relatively small change in the electron momentum, $q < 1/R$. Thus, if $\Delta K > 1/R$, these elastic spin-flip processes become suppressed and the AHE vanishes. To have a better physical insight into the problem, we present the Hall conductivity in an approximate form as

$$\frac{\sigma_{xy}}{\sigma_0} \sim k_F^2 R^2 \frac{M}{E_F} \exp\left(-\frac{M}{E_F} k_F R\right). \quad (11)$$

At zero M the contributions from different spins exactly compensate each other, while with the increase in M to the region where $\Delta K R = (M/E_F)k_F R \gg 1$, the spin-flip transitions are suppressed and the Hall conductivity vanishes as well. The maximum of conductivity is achieved for $M = E_F/k_F R$. The resulting maximum anomalous Hall conductivity is then of the order

$$\max\left(\frac{\sigma_{xy}}{\sigma_0}\right) \sim k_F R/e, \quad (12)$$

where $e \approx 2.718\ldots$ is the base of the natural logarithm.

It is instructive to compare the anomalous and the conventional Hall conductivities. Magnetization of the order of $E_F/k_F R$ for InSb with the g-factor close to 50 can be achieved in the fields B_z of the order of 1 T. The ratio of the Drude and Hall conductivity is $\omega_c \tau$, where ω_c is the cyclotron frequency. In the field of 1 T for a sample with the mobility of 10^5 cm^2/V s, one finds $\omega_c \tau \approx 10$. As a result, the anomalous Hall conductivity can be of an order of 0.1–1.0 of the conventional Hall conductivity. It can be extracted from the conventional Hall measurements taking into account an unusual dependence on the external magnetic field.

Negative magnetoresistance and anisotropic conductivity.—We have shown above that the spin-flip scattering from the fluctuating spin-orbit field leads to the AHE in a magnetized electron gas. Now we demonstrate that such scattering also modifies the diagonal conductivity as the electron relaxation rate includes generally, not only the term due to scattering from impurities, but also the contribution from spin-flip scattering due to the fluctuating Rashba coupling. Since the spin-flip scattering is essential at small spin splitting and decreases at large splitting, its contribution to spin relaxation time strongly depends on the spin polarization of the electron gas. This spin polarization can be modified by an external magnetic field, which in turn leads to the magnetoresistance. For this effect it is sufficient to consider the situation when the band splitting is not related to the homogeneous magnetization but is due to the Zeeman field. Therefore we consider now
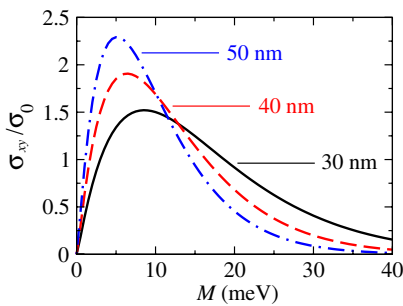
a nonmagnetic electron gas, $M = 0$, and assume a nonzero magnetic field along the x axis. We show below that the spin-orbit fluctuating Rashba field leads then to a NMR.

To calculate the diagonal component of the conductivity tensor we can use the kinetic equation, which takes into account spin-conserving and spin-flip transitions,

$$\frac{e}{m}(\mathbf{E}\cdot\mathbf{k})\frac{\partial f^0_{\mathbf{k}\uparrow}}{\partial \varepsilon_k} = -\sum_{\mathbf{k}'}[W_{\mathbf{k}\mathbf{k}'}(f_{\mathbf{k}\uparrow} - f_{\mathbf{k}'\uparrow}) + W^f_{\mathbf{k}\mathbf{k}'}(f_{\mathbf{k}\uparrow} - f_{\mathbf{k}'\downarrow})],$$

(13)

and a similar equation for the opposite spin orientation. Here we introduced the notation

$$W_{\mathbf{k}\mathbf{k}'} = 2\pi[w_{\mathbf{k}\mathbf{k}'} + |\lambda_{\mathbf{k}\mathbf{k}'}|^2(k_y + k'_y)^2]\delta(\varepsilon_k - \varepsilon_{k'}), \quad (14)$$

$$W^f_{\mathbf{k}\mathbf{k}'} = 2\pi|\lambda_{\mathbf{k}\mathbf{k}'}|^2(k_x + k'_x)^2\delta(\varepsilon_k - \varepsilon_{k'} \pm 2\beta B), \quad (15)$$

for spin-conserving and spin-flip transitions, respectively, where $w_{\mathbf{k}\mathbf{k}'}$ corresponds to the spin-conserving scattering from usual disorder. If the potential of impurities is short range, $w_{\mathbf{k}\mathbf{k}'}$ can be taken independent of the momentum, $w_{\mathbf{k}\mathbf{k}'} \simeq w_0$. Although the solution of Eq. (13) can in general be presented as a spin-dependent sum of cylindrical Fermi surface harmonics, it cannot be solved in the general form due to anisotropy of matrix elements in Eqs. (14) and (15) and the presence of two Fermi surfaces. However, for $k_F R \gg 1$, where one can with a high accuracy put $k_x = k'_x$ and $k_y = k'_y$, they can be greatly simplified for the electric field $\mathbf{E}$ parallel and perpendicular to the magnetic field $\mathbf{B}$.

Let us start with the limit of a weak magnetic field, when the Zeeman splitting is very small, $\Delta K \ll 1/R$. The spin-split Fermi surfaces are then almost identical, and can be considered as a single surface. The corresponding isotropic conductivity, $\sigma = \sigma_D + \delta\sigma^{so}$, can be also presented as $\sigma(B \to 0) = ne^2\tau_{tot}/m$, where $1/\tau_{tot} = 1/\tau + 1/\tau_{so}$ includes the transport scattering rate $1/\tau_{so}$ related to Eqs. (15) (spin-flip processes) and (14) (spin-conserving processes caused by spin-orbit coupling). As a result, $\delta\sigma^{so} = -\tau\sigma_D/\tau_{so}$. The rate $1/\tau_{so}$ can be evaluated as $1/\tau_{so} \sim 4\langle\lambda^2\rangle/v_F R$ and is of the order of 10^{11} s^{-1}, leading to the negative correction of the order of 0.1 σ_D to the Drude conductivity at mobility 10^5 cm^2/V s.

When the magnetic field increases, the spin-flip processes become suppressed, as can be qualitatively seen in Fig. 1, while spin-conserving ones remain almost intact. In the limit of a strong field, $\beta B m R \gg 1$, the spin-flip term, Eq. (15), vanishes and the conductivity is larger than at $B = 0$ for both $\mathbf{E} \parallel \mathbf{B}$ and $\mathbf{E} \perp \mathbf{B}$ geometries. At the same time, as we see from (14), the remaining spin-conserving processes include scattering from the random Rashba field that strongly depends on the orientation of momenta $\mathbf{k}$ and $\mathbf{k}'$. As a result, the total scattering at large field becomes anisotropic. Solving the kinetic equation for this case, we find that the conductivity at large B is also anisotropic with

$|\sigma(\mathbf{E} \parallel \mathbf{B}) - \sigma(\mathbf{E} \perp \mathbf{B})| = |\delta\sigma^{so}/2|$, and the degree of anisotropy is of the order of several percent.

Note that the NMR effect we consider here is qualitatively different from the one of Ref. [24] for systems with long- and short-range disorder. The mechanism proposed by us has a solely spin-related origin and is not related to the orbital motion considered in Ref. [24]. Recent experiments [25] showed that the NMR cannot be fully explained in terms of the model of Ref. [24] and some spin-related effects can be involved in the physics of this phenomenon. Moreover, our effect is purely classical and not related to NMR in the weak localization regime [26,27].

Summary.—We have calculated the anomalous Hall conductivity and the magnetoresistance of 2D electron systems due to scattering from spatial fluctuations of Rashba spin-orbit interaction. The materials where this interaction can be essential are symmetric narrow-gap semiconductor quantum wells. One of the usual characterization methods for these materials is based on the measurements of AHE under the assumption that AHE is proportional to magnetization. This proportionality, however, is completely destroyed when the AHE is related to the fluctuating Rashba field. It should be stressed that the proposed mechanism of AHE can be important when the usual impurity scattering within the quantum well is small. In the case of nonmagnetic semiconductors, it can be easily realized by donor impurities outside the quantum well. We have also demonstrated that the scattering of electrons from the fluctuations of the Rashba field in nonmagnetic symmetric semiconductor quantum wells leads to a negative magnetoresistance. This effect can be realized for example in InSb or other narrow gap semiconductors, in which the Zeeman splitting and spin-orbit coupling are both very strong.

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