# Nonanomalous Discrete *R* Symmetry Decrees Three Generations

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We show that more than two generations of quarks and leptons are required to have an anomaly free discrete $R$ symmetry larger than $R$ parity, provided that the supersymmetric standard model can be minimally embedded into a grand unified theory. This connects an explanation for the number of generations with seemingly unrelated problems such as supersymmetry breaking, proton decay, the $\mu$ problem, and the cosmological constant through a discrete $R$ symmetry. We also show that three generations is uniquely required by a nonanomalous discrete $R$ symmetry in classes of grand unified theories such as the ones based on (semi)simple gauge groups.

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*Introduction.*—Approximately 70 years ago I. I. Rabi famously quipped "who ordered that?" in regards to the discovery of the second electron, i.e., the muon. Since that time the origin of multiple generations of quarks and leptons has been a mystery. A partial answer to this question can be found in the leptogenesis mechanism [1]. In leptogenesis, at least two generations of right-handed neutrinos are required for $CP$-violation [2], an essential ingredient in baryogenesis. This solution, however, does not explain the existence of the third generation [3].

In this Letter, we show that more than two generations of quarks and leptons are necessary for an anomaly free discrete $R$ symmetry [9], $\mathbb{Z}_{NR}$, of order $N > 2$ [10]. An $R$ symmetry is important when considering model building and phenomenology with supersymmetry: generic [12] and metastable supersymmetry breaking [13], proton decay [14], and the $\mu$ problem (see, for instance, Refs. [11,15]) can all be solved by, or require, an $R$ symmetry. Furthermore, without a discrete $R$ symmetry, a constant term in the superpotential is allowed and expected to be of the order of the Planck scale. A large constant term in the superpotential necessitates Planck scale supersymmetry breaking to cancel the large cosmological constant. Therefore, low scale supersymmetric extensions of the standard model have various difficulties which well motivate an $R$ symmetry. Additionally, $R$ symmetries are motivated by considering string theory, where they arise as "leftover" symmetries from higher dimensional Lorentz groups. By considering a minimal embedding into a grand unified theory (GUT), we show that this discrete $R$ symmetry requires (at least) three generations to be anomaly free.

Furthermore, if the GUT group is semisimple, then considering anomalies with $U(1)_Y$ shows that four and five generations are not consistent with an anomaly-free discrete $R$ symmetry. More than five generations will lead to a Landau pole in the theory; only three generations are viable. Additionally, we will show that the discrete $R$ symmetry forbids a $\mu$ term, successfully suppresses proton decay, and is consistent with the seesaw mechanism for neutrino masses.

*Anomaly free discrete R symmetry.*—Now, let us consider the anomaly free conditions of a discrete $R$ symmetry. (Notice that we are assuming that a discrete $R$ symmetry stems from a gauged $R$ symmetry since no global symmetries are expected in a quantum theory of gravity [16].) In the following, we consider a class of GUT models where each generation of the quark and lepton supermultiplets are unified into a $\mathbf{10}$ and a $\mathbf{5}^*$ representation of $SU(5)_{\rm GUT}$. Furthermore, we also assume that there are no additional light degrees of freedom charged under the supersymmetric standard model (SSM) gauge groups beyond the ones in the SSM. In particular, we expect that the colored Higgs multiplet associated with the SSM Higgs doublets have masses of the order of the GUT scale.

In this class of models, the anomaly free conditions from $SU(3)_c$ and $SU(2)_L$ gauge symmetries with the discrete $R$ symmetry give [17,18],

$$6 + n_g(3r_{10} + r_5 - 4) = 0, \tag{1}$$

$$4 + n_g(3r_{10} + r_5 - 4) + (r_u + r_d - 2) = 0, \tag{2}$$

respectively, where these equations are modulo $N$. Here, $n_g$ denotes the number of generations, $r_{10}$, $r_5$, $r_u$, $r_d$ are the $R$ charges of the superfields $\mathbf{10}$, $\mathbf{5}^*$, $H_u$, $H_d$, respectively. Notice that, in our discussion, the $R$ charges are assumed to be generation independent [19]. The presence of Yukawa interactions constrains the $R$ charges

$$2r_{10} + r_u = 2, \qquad r_{10} + r_5 + r_d = 2, \tag{3}$$

modulo $N$ [20]. By combining Eqs. (1)–(3), the anomaly free conditions reduce to

$$6 - 4n_g = 0 (\mathrm{mod}\, N), \tag{4}$$

$$r_u + r_d = 4 (\mathrm{mod}\, N). \tag{5}$$

 

The condition in Eq. (4) remarkably relates the number of generations of quarks and leptons to the order of the discrete $R$ symmetry. Interestingly, this condition shows that no discrete $R$ symmetry with $N > 2$ is allowed for $n_g = 1, 2$. In fact, three generations or more are needed to allow $R$ symmetries with $N \geq 3$ (see also Table I). Consequently, we find that anomaly free discrete $R$ symmetries with $N > 2$ require more than two generations of quarks and leptons, provided that the SSM is minimally embedded into GUT representations.

Another interesting feature of the above conditions is that the $R$ charges of $H_u$ and $H_d$ in Eq. (5) forbid the so-called $\mu$ term when $N \geq 3$. Therefore, the $\mu$ term is automatically small in this class of models. The same arguments also apply to the infamous dimension five proton decay operator, $\mathbf{10}\,\mathbf{10}\,\mathbf{10}\,\mathbf{5}^*$ [21,22], since the $R$ charge of this operator is $3r_{10} + r_5 = 4 - (r_u + r_d) = 0$ modulo $N$. Thus, the dimension five proton decay operator is also automatically suppressed by the discrete $R$ symmetry (see also Refs. [14,23]).

One may wonder whether other anomaly free conditions such as $\mathbb{Z}_{NR}U(1)_Y^2$, $\mathbb{Z}_{NR}^2U(1)_Y$, $\mathbb{Z}_{NR}^3$, and $\mathbb{Z}_{NR}(\text{gravity})^2$, could give further constraints. The anomalies that are not linear in $\mathbb{Z}_{NR}$ [$\mathbb{Z}_{NR}^2U(1)_Y$, for instance] are sensitive to questions about the UV structure of the theory and are thus not useful here [24]. The $\mathbb{Z}_{NR}(\text{gravity})^2$ anomaly condition is also not useful for constraining the SSM $R$ symmetries since it depends on states not in the low energy SSM spectrum (see, for instance, the discussion in Ref. [23]). The $\mathbb{Z}_{NR}U(1)_Y^2$ condition is also model dependent, and hence, this anomaly is not useful without specifying the models. (We will come back to this point later.)

*R-invariant grand unified theory.*—So far, we have not discussed the mechanism which gives mass to the colored Higgs multiplet, i.e., the infamous doublet-triplet splitting problem. It is, however, known to be difficult to realize doublet-triplet splitting naturally in GUT models with a simple gauge group and only the SSM matter content below the GUT scale. In addition to this naturalness problem, it was recently shown in Ref. [25] that a low scale SSM with discrete $R$ symmetries having $N > 2$ are not consistent with GUT models based on a simple gauge group.

Here, we present an example of a GUT model where the doublet-triplet splitting can be naturally realized [26–31] and is based on a nonsimple group. In fact, an $R$-invariant GUT model of this class has been constructed in Ref. [28] based on a $SU(5)_{\text{GUT}} \times U(3)_H$ gauge symmetry (see also Ref. [18]). In Table II, we show the $\mathbb{Z}_{6R}$-charge assignments, which includes $R$ parity, of this GUT model which satisfies all the anomaly free conditions of the previous section for $n_g = 3$. We also find the model presented in Table II with three generations satisfies the anomaly free conditions, $\mathbb{Z}_{6R}SU(5)_{\text{GUT}}^2$ and $\mathbb{Z}_{6R}SU(3)_H^2$.

Let us briefly review the product group unification model in the Higgs phase (In the Higgs phase, $U(1)_H$ is necessary since the GUT gauge group is broken by the vacuum expectation values of $Q$ and $\bar{Q}$. The $U(1)_H$ gauge group in the present GUT model unfortunately destroys the automatic explanation of the charge quantization in the usual GUT model [32]. In this model, no adjoint of $SU(5)_{\text{GUT}}$ is required and the GUT gauge symmetry is broken by the expectation values of the bifundamental fields $Q$ and $\bar{Q}$ in Table II [26–28],

$$\langle Q_i^\alpha \rangle = v\delta_i^\alpha, \qquad \langle \bar{Q}_\alpha^i \rangle = v\delta_\alpha^i, \tag{6}$$

where $v$ denotes a dimensional parameter at the GUT scale and the indices run $\alpha = 1$–$3$ and $i = 1$–$5$. With the above expectation values, the standard model gauge groups are the unbroken subgroups of $SU(5)_{\text{GUT}} \times U(3)_H$. Specifically, $SU(3)_c$ and $U(1)_Y$ are the diagonal subgroups of $SU(5)_{\text{GUT}} \times U(3)_H$. The above expectation values are obtained as a supersymmetric solution of the superpotential [26–28],

$$\begin{aligned} W = {} & \sqrt{2}\bar{Q}_\alpha^i \Phi^a (t^a)_\beta^\alpha Q_i^\beta + \sqrt{2}\bar{X}_\alpha \Phi^a (t^a)_\beta^\alpha X^\beta \\ & + \sqrt{2}\bar{Q}_\alpha^i \Phi^0 (t^0)_\beta^\alpha Q_i^\beta + \sqrt{2}\bar{X}_\alpha \Phi^0 (t^0)_\beta^\alpha X^\beta \\ & - \sqrt{2}v^2\Phi^0, \end{aligned} \tag{7}$$

where (Here, we used the Gell-Mann matrix $t^a (a = 1\ldots 8)$ with the normalizations, $\text{tr}[t^a t^b] = \delta^{ab}/2$ and $t^0 = \mathbf{1}_{3\times 3}/\sqrt{6}$.) we have distinguished the octet and singlet $\Phi$ of $SU(3)_H$ by $\Phi^a$ and $\Phi^0$, respectively. We have omitted the coupling constants of each term. $i$ denotes the $SU(5)_{\text{GUT}}$ representations, and $\alpha$, $\beta$ the $SU(3)_H$ representations.

With the above expectation value, the colored Higgs multiplets in $H$ and $\bar{H}$ form Dirac mass terms with $\bar{X}$ and $X$ via the superpotential

TABLE I. The relation between the number of generations and the order of the discrete $R$ symmetry.

| $n_g$ | $\mathbb{Z}_{NR}$ |
|---|---|
| 1 | $N = 2, 1$ |
| 2 | $N = 2, 1$ |
| 3 | $N = 6, 3, 2, 1$ |
| 4 | $N = 10, 5, 2, 1$ |
| 5 | $N = 14, 7, 2, 1$ |

TABLE II. The $R$-charge assignments of our model based on $SU(5)_{\text{GUT}} \times U(3)_H$, which is consistent with the see-saw mechanism [see Eq. (10)]. The Higgs doublets are embedded into (anti) fundamental representation of $SU(5)_{\text{GUT}}$ (i.e., $H_u \subset H(\mathbf{5}^*)$ and $H_d \subset \bar{H}(\mathbf{5})$).

| | $\mathbf{10}$ | $\mathbf{5}^*$ | $H(\mathbf{5})$ | $\bar{H}(\mathbf{5}^*)$ | $Q(\mathbf{5})$ | $\bar{Q}(\mathbf{5}^*)$ | $X(\mathbf{1})$ | $\bar{X}(\mathbf{1})$ | $\Phi(\mathbf{1})$ |
|---|---|---|---|---|---|---|---|---|---|
| $U(3)_H$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{3}^*$ | $\mathbf{3}$ | $\mathbf{3}^*$ | $\mathbf{3}$ | $\mathbf{8}+\mathbf{1}$ |
| $\mathbb{Z}_{6R}$ | $-1$ | 3 | 4 | 0 | 0 | 0 | $-2$ | 2 | 2 |

$$W = \bar{H}_i Q^i_\alpha \bar{X}^\alpha + H^i \bar{Q}^\alpha_i X_\alpha. \quad (8)$$

In this way, we can successfully realize doublet-triplet splitting while forbidding the mass term of the Higgs doublets (see Refs. [26–28] for details).

Finally, we comment on the gauge couplings. In the Higgs phase, the low energy coupling $\alpha_{3_c}$ is given by $1/\alpha_{3_c} = 1/\alpha_5 + 1/\alpha_{3H}$ with a similar relationship between hypercharge, $\alpha_Y$, and $\alpha_{1H}$ [28]. Here $\alpha_5$, $\alpha_{3H}$, and $\alpha_{1H}$ are the gauge coupling constants of $SU(5)_{\rm GUT}$, $SU(3)_H$, and $U(1)_H$, respectively. Thus, the unification is not automatically realized in this model. For a strongly coupled $SU(3)_H$ and $U(1)_H$, however, we see that at the GUT scale the approximate GUT relation, i.e., $\alpha_{3_c} \simeq \alpha_2 \simeq \alpha_1$, still holds.

*Use of $U(1)_Y$ anomalies.*—As we have mentioned earlier, the anomaly free conditions involving $\mathbb{Z}_{NR}$ and $U(1)_Y$ are model dependent and not as powerful as the ones in Eqs. (1) and (2) on general grounds. With more specifications of the GUT models (The $\mathbb{Z}_{NR}U(1)_Y^2$ condition is only useful if $U(1)_Y$ is embedded in a non-Abelian part of the GUT group.), however, these anomaly conditions can also play an important role. Moreover, these conditions might be the key to single out $n_g = 3$ out of $n_g > 2$.

For example, if we assume that the GUT group is semisimple and the normalization of the $U(1)_Y$ charges is as in the standard model, then the $\mathbb{Z}_{NR}U(1)_Y^2$ anomaly free condition can be used. As a function of $n_g$, it is given by

$$2(-10n_g + 3) = 0(\mathrm{mod}\, N). \quad (9)$$

Substituting $n_g = 3$ for each $N = 6, 3, 2, 1$, we find that the anomaly free condition for $\mathbb{Z}_{NR}U(1)_Y^2$ is also satisfied. For $n_g = 4$, the allowed $R$ symmetries are $N = 74, 37, 2$, which are not consistent with the other anomaly conditions for $n_g = 4$, with the exception of the (non-$R$) $\mathbb{Z}_2$ (See Table I). For $n_g = 5$, the allowed $R$ symmetries are $N = 94, 47, 2$ and again are not consistent with the other anomaly conditions in Table I. By remembering that $n_g \geq 6$ leads to a Landau pole, we find that three generations are uniquely required by a nonanomalous discrete $R$ symmetry in Eqs. (1), (2), and (9).

It should be noted that the product group unification considered in the previous section is not semisimple, and hence, the uniqueness of three generations does not hold and the model allows $n_g = 3, 4, 5$. For the uniqueness of three generations, $U(3)_H$ is needed to be successfully embedded into a larger semisimple gauge group.

*Discussion.*—Any discrete $R$ symmetry with $N > 2$ should be spontaneously broken down to $R$ parity at some scale much lower than the GUT or Planck scale. Such spontaneous breaking of exact discrete symmetries could cause a cosmological domain wall problem [33]. One option for avoiding this domain wall problem is to assume that the spontaneous breaking of the discrete $R$ symmetry occurs well before inflation. This leads to constraints on the Hubble constant during inflation and the reheating temperature relative to the $R$-symmetry breaking scale. Another interesting possibility is a model where the vacuum expectation value of the inflaton breaks the discrete $R$ symmetry [34]. In this class of models, the flatness of the inflaton potential near the origin is naturally explained by the $R$ symmetry [35], while the domain wall problem is avoided because the radius of the coherent domain of the inflaton field is inflated to $e^{N_e}(N_e \gtrsim 60)$ times larger than the Hubble radius at the end of inflation.

In Ref. [23] (see also Ref. [36]), anomaly cancellation of discrete $R$ symmetries was also studied, but with the addition of the Green-Schwarz mechanism. Our approach is to take the minimal setup and constraints, and as we stated above, we do not include gravitational effects for the anomalies nor extra matter content for cancellation. Including a Green-Schwarz mechanism fundamentally changes the anomaly relations to be satisfied and the resulting analysis. However, taking a minimal approach and requiring the discrete $R$ symmetry to be anomaly free and unbroken (at the GUT scale at least) alleviates proton decay problems without additional assumptions or constraints on $R$ breaking when using mechanisms like Green-Schwarz.

We also comment on the $R$ charge of the right-handed neutrinos, $r_1$, which are essential for the seesaw mechanism [37]. By including right-handed neutrinos, we obtain two additional conditions on the $R$ charges;

$$r_5 + r_1 + r_u = 2, \qquad 2r_1 = 2, (\mathrm{mod}\, N). \quad (10)$$

The first condition is due to the Yukawa interactions of the right-handed neutrinos, and the second condition is due to the Majorana masses of the right-handed neutrinos. By combining the conditions in Eqs. (1)–(3) and (10), we find the $R$ charges in Table II. Thus, we find that the $\mathbb{Z}_{6R}$ symmetry is consistent with the seesaw mechanism. However, we find that these charge assignments are not consistent with $SO(10)$ unification since $r_{10} = r_5 = r_1$ is not satisfied, which was pointed out in Ref. [23].

Lastly, we discuss some possible hints of this discrete $R$ symmetry. The discrete $R$ symmetry discussed above forbids a $\mu$ term. We can then add a singlet with a discrete $R$ charge of 4 to generate the $\mu$ term. Inclusion of this singlet leads to the NMSSM-like extension (but the model without a cubic term for the singlet in the superpotential) discussed in Ref. [38]. Thus, a relatively large Higgs boson mass and other NMSSM-like singlet signals would be quite suggestive of this framework.

In addition to the $\mu$ term naturally leading us to possible future physical signals, we showed in the previous section that proton decay is also addressed by this discrete $R$ symmetry; the phenomenological aspects of a $\mathbb{Z}_{6R}$ symmetry are rich. To conclude, we have shown that a discrete $R$ symmetry (larger than $\mathbb{Z}_2$) requires at least three generations of quarks and leptons to be anomaly free, assuming a minimal embedding in a GUT. This nonanomalous discrete $R$ symmetry is rich phenomenologically, and a useful ingredient in model building.


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