Long-Range Quantum Ising Spin Glasses at $T = 0$: Gapless Collective Excitations and Universality

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We solve the Sherrington-Kirkpatrick model in a transverse field Γ deep in its quantum glass phase at zero temperature. We show that the glass phase is critical everywhere, exhibiting collective excitations with a gapless Ohmic spectral function. Using an effective potential approach, we interpret the latter as arising from disordered collective excitations behaving like weakly coupled, underdamped oscillators. For a small transverse field Γ, the low-frequency spectrum takes a form independent of the fluctuation strength Γ.

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Spin glasses are canonical representatives of a wide class of complex disordered systems in which competing interactions induce frustration, suppressing the emergence of simple ordering patterns. Nevertheless, the interactions induce a phase transition from a disordered paramagnetic state to a glassy "ordered" state at low temperatures. The emerging glass phase features remarkable properties, which have been studied in detail in classical glasses: their free-energy landscape is very rough and has many local minima, separated by high barriers [1]. This entails ergodicity breaking and intriguing long-time out-of-equilibrium phenomena. Classical spin glasses feature critical (power law) spin correlations [2,3], despite the absence of a broken continuous symmetry. In systems with long-range interactions [e.g., in Coulomb glasses or the Sherrington-Kirkpatrick model], criticality is reflected by a pseudogap in the distribution of local fields at low temperatures [4,5].

Quantum Ising glasses appear in many guises: as genuine spin glasses (e.g., $LiHo_xY_{1-x}F_4$, with quantum fluctuations tunable by a magnetic field [6–8] or in lightly doped cuprates [9]), proton glasses (frustrated ferroelectrics) [10], atoms in random laser cavities [11–13], or Coulomb-frustrated semiconductors close to a metal insulator transition [14]. Especially in the latter, collective low-energy excitations play an important role in transport and absorption, since they act as a thermal bath that provides or absorbs energy from single-particle processes. It is thus important to understand how such frustrated systems behave in the presence of quantum fluctuations, and how they influence the dynamics of collective excitations. It is well-known that strong quantum fluctuations suppress the glass and restore ergodicity. The associated quantum phase transition [15] has been studied in detail [10,16–26]. Here, we focus instead on the properties of the scarcely understood bulk glass phase (cf. Fig. 1).

In the past, many theoretical studies on quantum glasses focused on short-range interacting spin glasses and the enhanced relevance of Griffith effects on the low-frequency dynamics [17]. However, several condensed matter realizations of frustrated systems feature longer range interactions and may be better approached, theoretically, from the limit of mean-field models, such as transverse-field Ising spin glass [10,18–22], quantum rotors [23], or the $SU(N)$ Heisenberg spin glass [27]. While the quantum phase transition of the mean-field Ising glass is well understood [22], rather little is known about its deep glass phase except in the vicinity of criticality. Within the lowest-order Landau expansion valid near the transition, the glass phase appears to be critical [24], and exact diagonalization in small systems [20] has corroborated such a trend. In this Letter, we provide proof of criticality within the whole bulk phase and an analytical and physical description of the relevant modes. So far, the exact solutions of quantum glassy models have been found only in the limit of a large number of vector or rotor components, in which the complex multivalley free-energy landscape of realistic Ising glasses [18,19] is absent. Our solution for the Ising case shows that its complex landscape is very soft: the criticality of classical spin glasses carries over to the quantum glass phase, providing it with abundant soft collective excitations. This is in contrast to frustrated quantum systems with closer similarity to structural glasses, which exhibit first-order glass transitions and a noncritical glass phase [25,27].

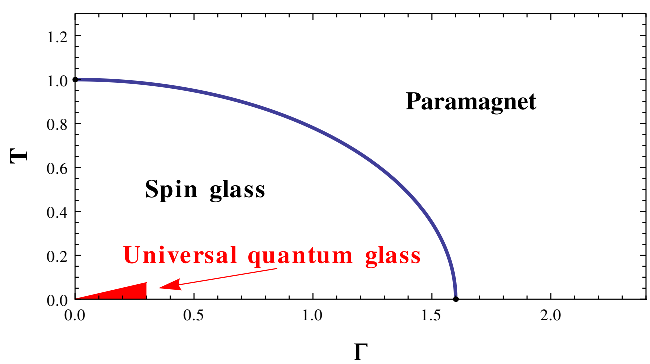

FIG. 1 (color online). Phase diagram of the mean-field quantum spin glass. The whole glass phase is gapless, reflecting permanent criticality. The deep quantum glass (red triangle) has gapless collective excitations with a low-frequency spectrum independent of the transverse field Γ.

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This Letter provides the missing link between quantum glass transition and the deep quantum spin glass phase of Ising systems. The latter combine a continuous glass transition with a critical glass phase and nontrivial ergodicity breaking. The Ising spin glass serves as a prototype for many glasses with long-range interacting, discrete degrees of freedom (e.g., localized electrons, dipoles, etc.). Its mean-field version may become amenable to experimental study as well, as it may be realized rather faithfully in random laser cavities, where a multitude of modes provide random long-range couplings between trapped atoms, as in the Dicke model [12].

We study the quantum glass phase of the mean-field version of the Ising quantum spin glass in a transverse field,

$$\mathcal{H} = -\sum_{i<j} J_{ij}\sigma_i^z\sigma_j^z - \Gamma\sum_i \sigma_i^x, \tag{1}$$

where σ^x and σ^z are Pauli operators. Every spin interacts with all others via random Gaussian couplings J_{ij} of zero mean and variance J^2/N. The quantum fluctuations are tuned by the transverse field Γ. The phase diagram and the deep quantum glass regime of particular interest to us are shown in Fig. 1. In the classical limit ($\Gamma = 0$), a glass transition takes place at $T_c = J$. It is connected by a critical line to the quantum glass transition at $\Gamma_c(T = 0) \approx 1.52J$ [22,26]. Below, we use units with $J = 1$, and restore J occasionally for clarity.

Replica formalism.—We first solve the model by the replica approach, and then interpret its features with the physically more transparent effective potential (Thouless-Anderson-Palmer) method [25,28]. The disorder average of the free energy is carried out using the replica trick following Ref. [15], reducing the problem to an effective, self-consistent single spin model:

$$\begin{aligned}
\beta F &= \frac{(\beta J)^2}{2}\sum_{b\neq 1} Q_{1b}^2 + \frac{1}{4}\iint_0^\beta d\tau d\tau' Q_{aa}^2(\tau,\tau') - \lim_{n\to 0}\left[\frac{1}{n}\log \mathrm{Tr} e^{\mathcal{S}_{\mathrm{eff}}}\right],\\
\mathcal{S}_{\mathrm{eff}} &= \iint_0^\beta d\tau d\tau'\left[\sum_{a<b} Q_{ab}\sigma_a^z(\tau)\sigma_b^z(\tau') + \frac{1}{2}\sum_a Q_{aa}(\tau,\tau')\sigma_a^z(\tau)\sigma_a^z(\tau')\right] + \Gamma\sum_a\int_0^\beta d\tau\sigma_a^x(\tau), \quad Q_{ab} = \langle\sigma_a^z\sigma_b^z\rangle \to q(x);\\
Q_{aa}(\tau,\tau') &= \langle T\sigma_a^z(\tau)\sigma_a^z(\tau')\rangle = C_{\tau-\tau'} = R_{\tau-\tau'} + q(1).
\end{aligned} \tag{2}$$

The saddle point values of the off-diagonal Q_{ab} are time independent [29] and have an ultrametric structure parameterized by the monotonic function $0 \leq q(x) \leq 1$ [19,30], with $x \in [0, 1]$ measuring the distance between the replica in phase space, and thus being a proxy of time in the aging regime. The dynamic spin correlator $R_{\tau-\tau'}$ is the connected part of $Q_{aa}(\tau, \tau')$, which tends to the Edwards-Anderson parameter $q_{\mathrm{EA}} \equiv q(1)$ at large time separations at $T = 0$.

Assuming a continuous function $q(x)$, the self-consistency problem is equivalent to solving the equations [31,32]

$$\begin{aligned}
\dot{m}(y,x) &= -\frac{\dot{q}(x)}{2}[m''(y,x) + 2\beta x m(y,x)m'(y,x)],\\
\dot{P}(y,x) &= \frac{\dot{q}(x)}{2}[P''(y,x) - 2\beta x[m(y,x)P(y,x)]'],\\
q(x) &= \int dy P(y,x)m^2(y,x),
\end{aligned} \tag{3}$$

where the dots and primes denote derivatives with respect to x and y, respectively; $P(y, x)$ is the distribution of frozen exchange fields y, averaged over the time scales corresponding to spin correlations $q(x)$, with $P(y, 0) = \delta(y)$; and $m(y, x)$ is the magnetization of a spin in a frozen field y on that time scale. Short-time observables are described by the local field distribution $P(y, 1)$ at $x = 1$ and the correlator R_τ, which encodes the dynamics within a metastable state. The difference to the classical problem lies in the modified set of boundary conditions, which read as $m(y, x = 1) = \langle\sigma^z\rangle_{\mathcal{S}(y)}$ with the local action

$$\mathcal{S}(y) = \frac{1}{2}\iint_0^\beta d\tau d\tau' \sigma_\tau^z R_{\tau-\tau'}\sigma_{\tau'}^z + \int_0^\beta d\tau(y\sigma_\tau^z + \Gamma\sigma_\tau^x). \tag{4}$$

The equations are closed by the self-consistency requirement

$$R_{\tau-\tau'} = \int_y P(y,1)\langle \mathrm{T}\sigma_\tau^z\sigma_{\tau'}^z\rangle_{\mathcal{S}(y)} - q(1). \tag{5}$$

Solution at $T = 0$.—Like at any quantum critical point, the gap closes at the transition, as reflected in the power-law tail $R_\tau \sim \tau^{-2}$ [22]. However, it was found within replica symmetric Landau theory [26] that, remarkably, the gap remains closed in its vicinity. Similar behavior was found in the analysis of a rotor model, where, in the limit of the $M \to \infty$ components [23], the replica symmetry is not broken. Here, we show that in the deep Ising glass phase (believed to be the $M = 1$ limit of the rotor model), one needs to account for full replica symmetry breaking. This in turn ensures gaplessness in the entire glass phase. We point out that this phenomenology contrasts with that of the exactly solvable model $SU(N \to \infty)$ Heisenberg spin glass [26,27], which exhibits a random first-order transition with distinct dynamic freezing and thermodynamic glass transitions. Its thermodynamically dominant states are gapped and are thus very different from the states obtained in the Ising limit $N \to 1$ analyzed here.

The main objective of our study, spectral function, is encoded in the Fourier transform R_ω of the average spin-spin correlator R_τ, which we analyze following Miller and Huse [22]. Representing the action $\mathcal{S}(y)$ with fermions, we can expand the spin correlator into a formally exact power series in R_ω

$$\langle \sigma^z_{-\omega}\sigma^z_\omega\rangle_{\mathcal{S}(y)} \equiv \chi_\omega(y) = \frac{\Pi_\omega(y)}{1 - R_\omega \Pi_\omega(y)}, \tag{6}$$

where $\Pi_\omega(y)$ is the proper polarizability [33]. It is itself a functional of R_ω, with the important feature that it remains analytic at small ω, $\Pi_\omega \sim \Pi_0 - a\omega^2$, even when R_ω turns nonanalytic in the glass phase [22], which is a direct consequence of the marginal stability

$$\int_0^\beta dy P(y,1)(m'(y,1))^2 = 1, \tag{7}$$

implied by Eq. (3). Indeed, with the small-frequency expansion $R_\omega = R_0 + \delta R_\omega$, and noting that $m'(y,1) = \chi_{\omega\to 0}(y)$ as well as $\chi_\omega(y) = \chi_0(y) + \chi_0^2(y)\delta R_\omega + O(\omega^2)$, Eqs. (5)–(7) require that $\delta R_\omega^2 \sim \omega^2$. This implies the nonanalytic form $R_{\omega\to 0} = R_0 - B|\omega|$ of the low-frequency correlator. Upon analytic continuation, the spin spectral function

$$A(\omega \to 0) \equiv \frac{1}{\pi N}\sum_i \mathrm{Im}\langle s_i^z(\omega)s_i^z(-\omega)\rangle|_{\omega\to\omega - i\delta} = \frac{B\omega}{\pi}, \tag{8}$$

is found to be always gapless and Ohmic, even deep in the Ising glass phase. Remarkably, as we will derive below, B becomes Γ-independent as $\Gamma \to 0$. Thus, the low-energy spectral function is universal in the sense that it is independent of the strength of quantum fluctuations.

Marginal stability Eq. (7) and the related gaplessness Eq. (8) are natural by-products of full replica symmetry breaking in mean-field glasses. They are pendants of the Goldstone modes in systems with a broken continuous symmetry and arise from the particular softness of the free-energy landscape, which results from the competition of many nearly degenerate states.

Deep glass phase.—To obtain quantitative results, we solve the self-consistency problem [Eqs. (3)–(5)] in detail, focusing on the deep quantum glass phase, where $J \gg \Gamma \gg T \to 0$. The limit $T \to 0$ is taken by replacing the variable $x \in [0,1]$ with $\beta_{\rm eff} \equiv \beta x \in [0,\infty]$, which has the interpretation of an inverse effective temperature in the aging dynamics [34]. In the limit $\Gamma \ll J$, the flow of Eq. (3) is attracted to a scaling regime [35], where $dq/d\beta_{\rm eff} = c(\beta_{\rm eff})/\beta_{\rm eff}^3$, $m(x,y) \to \tilde{m}(\beta_{\rm eff} y)$, and $P(x,y) \to \beta_{\rm eff}\tilde{p}(\beta_{\rm eff} y)$, which holds for $1/J \ll \beta_{\rm eff} \ll 1/\Gamma$ and $y \ll J$. Here, $c(\beta_{\rm eff}) \to 0.411$, and $\tilde{m}$ and $\tilde{p}$ are the same fixed-point functions that appear in the classical low-T limit [35]. In particular,

$$\tilde{p}(\beta_{\rm eff} y) \approx \begin{cases} \alpha|\beta_{\rm eff} y| & 1 \ll |\beta_{\rm eff} y|, \\ \text{const} & |\beta_{\rm eff} y| \lesssim 1, \end{cases} \tag{9}$$

which displays a linear pseudogap with slope $\alpha = 0.301$ in the distribution of frozen fields, smeared on the scale $y \sim 1/\beta_{\rm eff}$.

For $\beta_{\rm eff} \gtrsim 1/\Gamma$, the overlap ceases to scale and $dq/d\beta_{\rm eff}$ drops [$c(\beta_{\rm eff}) \to 0$, cf. Fig. 2]. At that point, $q(\beta_{\rm eff})$ reaches the constant value $q_{\rm EA}$, and P and m freeze to the form describing short times (response within the metastable states). From this, it follows that the frozen field distribution $P(y,1)$ has a pseudogap, smeared on the scale Γ, as expected from stability arguments [36]. The evolution of the pseudogap deeper and deeper in the quantum glass is shown in Fig. 3.

In the limit $\Gamma \ll J$, Γ is the only relevant dynamic energy scale, while J merely determines the width of the distribution of frozen fields. We prove this by showing that the following scaling ansatz solves the problem in this limit: (i) for $\beta_{\rm eff} \gg 1/J$, $c(\beta_{\rm eff}) = \hat{c}(u)$ is only a function of $u = \beta_{\rm eff}/(\beta_{\rm eff} + 1/\Gamma)$; (ii) $R(\omega) = \Gamma\hat{r}(\omega/\Gamma)$; (iii) $P(y,1) = \Gamma\hat{p}(y/\Gamma)$ [and $P = (\Gamma/u)\hat{p}(yu/\Gamma, u)$, $m = \hat{m}(yu/\Gamma, u)$]. One then verifies that $\hat{c}$, $\hat{r}$, $\hat{p}$, and $\hat{m}$ satisfy self-consistent equations, which are independent of $\Gamma \ll \Gamma_c$. Figures 2 and 3 illustrate that their solution is indeed the limit of the full solutions of Eqs. (3)–(5) as $\Gamma \to 0$. The scaling of the solution implies a nontrivial scaling of the dynamic properties of the glass. This entails the remarkable result that the coefficient B of the Ohmic spectral function Eq. (8) tends to a constant $B \approx 0.59/J^2$ as $\Gamma \to 0$. Note that the Ohmic regime describes quantum

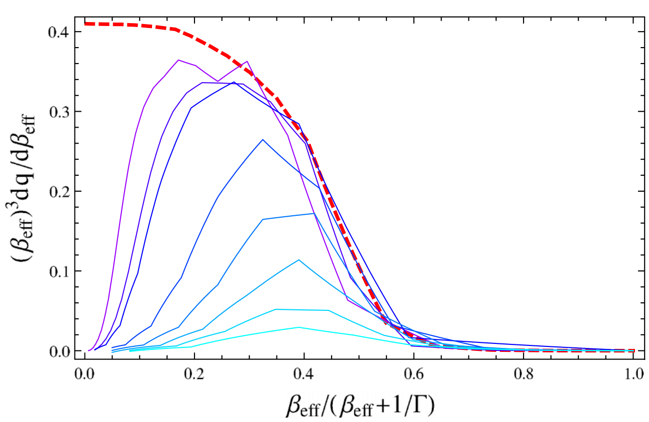

FIG. 2 (color online). Rescaled derivative of the overlap function $c = \beta_{\rm eff}^3 dq/d\beta_{\rm eff}$ as a function of $u \equiv \beta_{\rm eff}/(\beta_{\rm eff} + 1/\Gamma)$, extracted from the full solution for finite Γ (solid curves). The dashed curve is the solution of the asymptotically exact scaling ansatz for $\Gamma/\Gamma_c \to 0$ [40]. Γ takes values (bottom to top) $\Gamma/\Gamma_c = 0.05$, 0.08, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6. Numerical instabilities are more pronounced for lower values of Γ, $\Gamma \lesssim 0.1\Gamma_c$, which is responsible for the rougher curves [35,40]. Scaling obtains for $\beta_{\rm eff} \ll 1/\Gamma$ ($c \to 0.411$), and reflects the ultrametric, self-similar properties of the phase space [35]; $q(x)$ reaches its plateau value $q_{\rm EA}$ at $\beta^c_{\rm eff} = x_c/T \approx 0.5/\Gamma$ where $c \to 0$.

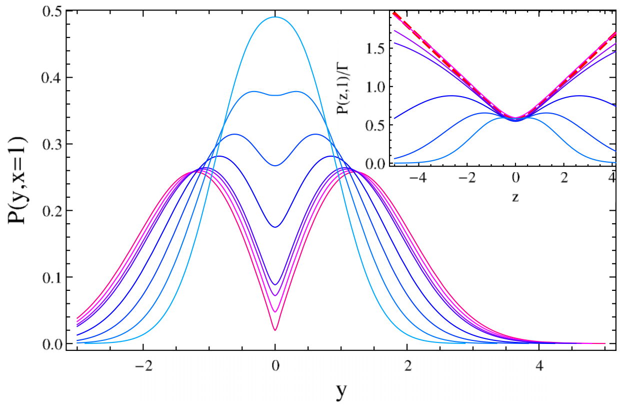

FIG. 3 (color online). Opening of the linear pseudogap in the distribution of frozen fields $P(y, 1)$ at $T = 0$, progressively deeper in the quantum glass ($\Gamma/\Gamma_c \to 0$ from top to bottom; values as in Fig. 2). The inset shows the rescaled distribution $P(y = z\Gamma)/\Gamma$ (solid lines), and the asymptotic scaling function $\hat{p}(z)$ (dashed line).

dynamics at low frequencies $\omega < \Gamma$ and disappears in the classical limit.

Physical interpretation.—We now interpret the exact replica results with an effective potential approach [28]. Using the methods of Refs. [25,37], we construct the Gibbs potential $G[\mathbf{m}, C_\tau]$ describing the free energy of the system constrained to have a magnetization pattern $\{m_i\}$ and global autocorrelation function $C_\tau = 1/N\sum_i\langle\sigma_i^z(\tau)\sigma_i^z(0)\rangle$ (at $T = 0$):

$$G[\mathbf{m}, C_\tau] = \sum_i G_0[m_i, C_\tau] - \sum_{i<j} J_{ij}m_im_j - \frac{1}{4}NJ^2\int_0^\infty d\tau(C_\tau - q_{\mathrm{EA}})^2. \qquad (10)$$

Here $q_{\mathrm{EA}} = 1/N\sum_i m_i^2$, and G_0 is the free energy of a single, constrained spin. The magnetization of the local minima $m_i = \langle\sigma_i^z\rangle$ is computed self-consistently via $\delta G/\delta m_i = 0$:

$$\frac{\partial G_0}{\partial m_i}\bigg|_C - \sum_j J_{ij}m_j + J^2 m_i\chi^0 = 0, \qquad (11)$$

where $\chi^0 = \int_0^\infty d\tau(C_\tau - q_{\mathrm{EA}}) = 1/N\sum_i\chi_i$ is the static susceptibility. However, for the quantum problems, Eq. (11) is not closed, since G_0 depends on the global autocorrelation function $C(\tau)$, which has to be evaluated self-consistently [25]. Since this exact formalism is too involved to yield direct physical insight, we approximate the static susceptibilities χ_i and the local functional G_0 by those of single spins, whose magnetization m_i is constrained by an auxiliary static field:

$$G_0[m_i] = -\Gamma(1 - m_i^2)^{1/2}, \qquad \bar{\chi}_i = \frac{(1 - m_i^2)^{3/2}}{\Gamma}. \qquad (12)$$

This approximation is similar but not identical to the "static approximation" employed in replica approaches to quantum spin glasses [21]. It overestimates the susceptibility to longitudinal fields, enhancing the stability of the glass. However, this reproduces qualitatively the results of the rigorous replica theory, furnishing a useful complementary physical picture.

Collective excitations in a local minimum are governed by the curvature of the energy landscape, i.e., by the Hessian

$$\mathcal{H}_{ij} = \frac{\delta^2 G}{\delta m_i \delta m_j} = -J_{ij} + \left[\frac{1}{\bar{\chi}_i} + J^2\langle\bar{\chi}\rangle\right]\delta_{ij},$$

where $\langle\bar{\chi}^k\rangle = 1/N\sum_i\bar{\chi}_i^k$ The replica theory assures that the glass phase is marginal. Here, this translates into a gapless spectrum of eigenvalues of $\mathcal{H}_{ij}$, which requires that [38]

$$J^2\langle\bar{\chi}^2\rangle = 1. \qquad (13)$$

This is the natural analog of Eq. (7). Under this condition, the density of eigenvalues of the Hessian λ starts as a semicircle

$$\rho(\lambda) = \frac{\sqrt{\lambda}}{\pi J^3\sqrt{\langle\bar{\chi}^3\rangle}} \sim \frac{\sqrt{\lambda\Gamma}}{J^2}, \quad \text{for } \lambda \lesssim \Gamma \ll J.$$

To establish the link with the spin spectral density Eq. (8), we interpret the low-energy normal modes of $\mathcal{H}$ as weakly interacting [39] harmonic oscillators with spring constants λ, and the effective mass scaling as $M \sim 1/\Gamma$; thus, the eigenfrequency $\omega(\lambda) = \sqrt{\lambda/M}$. Hence, the density of modes is

$$\rho(\omega) = \int d\lambda\rho(\lambda)\delta(\omega - \omega(\lambda)) \sim \frac{\omega^2}{\Gamma J^2}, \quad \text{for } \Gamma \ll J.$$

With the mean square displacement $\langle x^2\rangle_\omega = 1/M\omega \sim \Gamma/\omega$, one predicts the spectral function to scale as $A(\omega) \approx \rho(\omega)\langle x^2\rangle_\omega \sim \omega/J^2$ for $\omega \lesssim \Gamma$. Thus, these qualitative arguments are seen to reproduce correctly the Ohmic spectrum, its frequency range, and the Γ-independent coefficient of the replica solution Eq. (8). This agreement is rather nontrivial, given that both the mode density and the kinetic energy of the soft modes do depend on Γ. We are thus confident that the physical picture of a set of gapless, underdamped collective harmonic oscillators is indeed the correct interpretation of the low-energy excitations in this quantum spin glass. It is interesting to note that an analogous reasoning for spin glasses with metallic background leads to a similar picture, although with overdamped oscillators, and the spectral function growing as $A(\omega) \sim |\omega|^{1/2}$, again in agreement with replica theory [24,40]. This insight may serve as a starting point to describe collective excitations in long but finite-range interacting quantum Ising glasses [41].

Implications and conclusion.—The collective excitations act as a bath with which local (single site) excitations exchange energy. Such coupling to a bath of collective modes is of particular importance for conduction or absorption processes, e.g., in quantum electron glasses (localized electrons taking the role of the Ising spins). Because of the discreteness of localized excitations, such processes require a continuum of collective modes, which provides or absorbs energy. Their transition rate depends on the bath's spectral function and determines, e.g., the pre-exponential factor of variable range conduction of glassy electrons close to the metal-insulator transition. Our result for the spectral function in the infinite-range Ising glass suggests that the low energy bath-couplings remain nearly independent of the strength of quantum fluctuations, tuned, e.g., by the degree of localization in electron glasses. This may be a promising route to understand the preexponential factor of variable range hopping conduction, which is experimentally known to remain nearly constant as the distance to the metal-insulator transition is varied.

In realistic models, where the interactions are not infinite ranged, criticality and Ohmic gaplessness might not exist at all length scales. Nevertheless, collective modes are expected to persist to energy scales, which are parametrically small in the interaction range. In quantum glasses of charged bosons or fermions, interesting questions arise regarding the interplay of glassy order with other quantum phenomena such as Bose-Einstein condensation [42] or Anderson localization [13]. The physical picture we provided for the relevant collective low-energy modes of the glass should be helpful in analyzing such disordered quantum phases.

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