Nonadiabatic Holonomic Quantum Computation in Decoherence-Free Subspaces

G. F. Xu,[1] J. Zhang,[1] D. M. Tong,[1,*] Erik Sjöqvist,[2,3] and L. C. Kwek[3]

[1]Department of Physics, Shandong University, Jinan 250100, China

[2]Department of Quantum Chemistry, Uppsala University, Box 518, Se-751 20 Uppsala, Sweden

[3]Center for Quantum Technologies, National University of Singapore, Science Drive 2, Singapore 117543

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Quantum computation that combines the coherence stabilization virtues of decoherence-free subspaces and the fault tolerance of geometric holonomic control is of great practical importance. Some schemes of adiabatic holonomic quantum computation in decoherence-free subspaces have been proposed in the past few years. However, nonadiabatic holonomic quantum computation in decoherence-free subspaces, which avoids a long run-time requirement but with all the robust advantages, remains an open problem. Here, we demonstrate how to realize nonadiabatic holonomic quantum computation in decoherence-free subspaces. By using only three neighboring physical qubits undergoing collective dephasing to encode one logical qubit, we realize a universal set of quantum gates.

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The discovery of geometric phase [1,2] and quantum holonomy [3,4] accompanying evolutions of quantum systems has unveiled important geometric structures in the description of physical states. These structures show that the twisting of subspaces, e.g., eigenspaces of an adiabatically varying Hamiltonian, can be used to manipulate quantum states in a robust manner. This is the initial idea of holonomic quantum computation (HQC), first proposed by Zanardi and Rasetti [5]. HQC has emerged as a key tool to implement quantum gates acting on sets of quantum bits (qubits). As is well known, errors in the control process of a quantum system are one main practical difficulty in building a quantum computer, and propagation of these errors may quickly spoil the whole quantum computational process. Since HQC is fault tolerant with respect to certain types of errors in the control process, it has been used to realize robust quantum computation [6–17].

Besides errors produced in the control process, decoherence is another main practical difficulty in building a quantum computer. Decoherence is caused by the inevitable interaction between the computational system and its environment. It collapses the desired coherence of the system and may thereby be detrimental to the efficiency of quantum computation. Protecting qubits from the effects of decoherence is a vital requirement for any quantum computer implementation. Various strategies have been proposed to protect quantum information against decoherence. Among them, decoherence-free subspaces (DFSs) provide a promising way to avoid quantum decoherence [18]. The basic idea of DFSs is to utilize the symmetry structure of the interaction between the system and its environment. Information is encoded in a subspace of the Hilbert space of a system, over which the dynamics is unitary. DFSs have been experimentally realized in many physical systems [19–23].

To protect quantum information from both errors produced in the control process and decoherence caused by the environment, quantum gates that combine the coherence stabilization virtues of DFSs and the fault tolerance of geometric holonomic control are of great practical importance. To this end, schemes of HQC in DFSs have been proposed recently [10–12]. Wu *et al* [10] proposed the first scheme of adiabatic HQC in DFSs, in which one logical qubit is encoded by four neighboring physical qubits and the quantum holonomies are accumulated by adiabatically changing the couplings between the qubits along dark states. The scheme is robust against collective dephasing and some stochastic errors. Yet, the requirement of adiabatic control of four neighboring physical qubits undergoing collective dephasing is an experimental challenge. All other schemes that can realize a universal set of holonomy quantum gates in DFSs are based on adiabatic evolution too, and they met the same problem of long run-time requirement.

In this Letter, we develop a scheme for nonadiabatic universal holonomic quantum computation in decoherence-free subspaces. Our proposal avoids the long run-time requirement but shares all the robust advantages of its adiabatic counterpart. An additional attractive feature of this nonadiabatic setting is that only three neighboring physical qubits undergoing collective dephasing are needed to encode one logical qubit. We further demonstrate that three neighboring physical qubits is the minimal number for realizing nonadiabatic HQC in DFSs, although two neighboring physical qubits may construct the minimal DFS.

Before proceeding further, we explain how quantum holonomy may arise in nonadiabatic unitary evolution. Consider a quantum system described by an N-dimensional state space and exposed to the Hamiltonian $H(t)$. Assume there is a time-dependent L-dimensional subspace $\mathcal{S}(t)$ spanned by the orthonormal basis vectors $\{|\phi_k(t)\rangle\}_{k=1}^{L}$ at each instant t. Here, $|\phi_k(t)\rangle$ satisfy the Schrödinger equation $i|\dot{\phi}_k(t)\rangle = H(t)|\phi_k(t)\rangle$. That is, $|\phi_k(0)\rangle \rightarrow |\phi_k(t)\rangle = U(t,0)|\phi_k(0)\rangle$ with the time evolution operator $U(t,0) = \mathbf{T}\exp(-i\int_0^t H(t')dt')$, $\mathbf{T}$ being time ordering. One may conclude that the unitary transformation $U(\tau,0)$ is a holonomy matrix acting on the L-dimensional subspace $\mathcal{S}(0)$ spanned by $\{|\phi_k(0)\rangle\}_{k=1}^L$ if $|\phi_k(t)\rangle$ satisfy the following requirements:

$$\text{(i)} \sum_{k=1}^{L} |\phi_k(\tau)\rangle\langle\phi_k(\tau)| = \sum_{k=1}^{L} |\phi_k(0)\rangle\langle\phi_k(0)|, \quad (1)$$

$$\text{(ii)} \langle\phi_k(t)|H(t)|\phi_l(t)\rangle = 0, \qquad k, l = 1, \ldots, L. \quad (2)$$

To verify that $U(\tau,0)$ is a holonomy matrix acting on $\mathcal{S}(0)$, we first note that condition (i) entails that the subspace undergoes cyclic evolution; i.e., we can introduce a set of the auxiliary bases $|\nu_k(t)\rangle$ of $\mathcal{S}(t)$ with the property

$$|\nu_k(\tau)\rangle = |\nu_k(0)\rangle = |\phi_k(0)\rangle, \qquad k = 1, \ldots, L. \quad (3)$$

Note that $|\nu_k(t)\rangle$ need not satisfy the Schrödinger equation, and therefore such bases can always be found [24]. By the aid of $|\nu_k(t)\rangle$, $|\phi_k(t)\rangle$ may be expressed as

$$|\phi_k(t)\rangle = \sum_{l=1}^{L} |\nu_l(t)\rangle C_{lk}(t), \quad (4)$$

where $C_{kl}(t)$ are time dependent coefficients. Substituting Eq. (4) into the Schrödinger equation yields

$$\frac{d}{dt}C_{lk}(t) = i\sum_{m=1}^{L}(A_{lm}(t) - K_{lm}(t))C_{mk}(t), \quad (5)$$

where $A_{kl}(t) = i\langle\nu_k(t)|\frac{d}{dt}|\nu_l(t)\rangle$, and $K_{kl}(t) = \langle\nu_k(t)|H(t)|\nu_l(t)\rangle$. Condition (ii) is equivalent to $K_{kl}(t) = 0$; i.e., the Hamiltonian vanishes on $\mathcal{S}(t)$ and hence $C(t) = \mathbf{T}\exp(i\int_0^t A(t')dt')$. The matrix $A(t)$ transforms as a proper gauge potential under the change $|\nu_k(t)\rangle \rightarrow \sum_{l=1}^{L}|\nu_l\rangle V_{lk}(t)$, where $V(t)$ is any unitary once differentiable $L \times L$ matrix such that $V(\tau) = V(0)$. At time $t = \tau$, there is $C(\tau) = \mathbf{P}e^{i\oint\mathcal{A}}$, where $\mathcal{A} = Adt$ is the connection one form and $\mathbf{P}$ is path ordering. From Eq. (4), we have $|\phi_k(\tau)\rangle = \sum_{l=1}^{L}|\nu_l(\tau)\rangle C_{lk}(\tau) = \sum_{l=1}^{L}|\phi_l(0)\rangle C_{lk}(\tau)$. It indicates that $C(\tau)$ is just the transformation matrix from initial states to final states in the subspace considered. Hence, we finally obtain

$$U(\tau) \equiv C(\tau) = \mathbf{P}e^{i\oint\mathcal{A}}. \quad (6)$$

Equation (6) shows that $U(\tau)$ is a holonomy matrix in the space spanned by $\{|\phi_k(t)\rangle\}_{k=1}^L$.

Let us now elucidate our physical model. The computational system consists of N physical qubits interacting collectively with a dephasing environment. The Hamiltonian of the system reads

$$H = \sum_{k<l}(J_{kl}^x R_{kl}^x + J_{kl}^y R_{kl}^y), \quad (7)$$

where J_{kl}^x and J_{kl}^y are controllable coupling constants, which are driven to enact the quantum computation, and

$$R_{kl}^x = \frac{1}{2}(\sigma_k^x\sigma_l^x + \sigma_k^y\sigma_l^y), \quad R_{kl}^y = \frac{1}{2}(\sigma_k^x\sigma_l^y - \sigma_k^y\sigma_l^x). \quad (8)$$

The operators R_{kl}^x and R_{kl}^y are XY and Dzialoshinski-Moriya [25,26] interaction terms, where σ_k^x (σ_k^y) represents the Pauli X (Y) operator acting on the kth qubit. A variety of quantum systems, including trapped ions and quantum dots, can be described by this Hamiltonian [27–30]. The major source of decoherence in the quantum system is dephasing. The effect of the dephasing environment on the N-qubit system is described by the interaction Hamiltonian,

$$H_I = \left(\sum_k \sigma_k^z\right) \otimes B, \quad (9)$$

where σ_k^z is the Pauli Z operator acting on the kth qubit, and B is an arbitrary environment operator. The symmetry of the interaction implies that there exists a DFS that can be used to protect quantum information against decoherence. Our aim is to find a realization of nonadiabatic HQC in this DFS.

We begin by showing that two physical qubits are not sufficient to realize decoherence-free nonadiabatic HQC in the presence of a dephasing environment. For a two-qubit system, the corresponding DFS is spanned by $\{|01\rangle, |10\rangle\}$. In order to protect the quantum gates from decoherence, logical qubits must be encoded in this DFS, and the state of the system must be kept within the subspace during the whole evolution. Thus, the DFS itself must be an invariant subspace during the system's evolution. In addition, to ensure that the gates are holonomic, condition (ii) must be satisfied, i.e., $\langle k|U(t,0)^\dagger H(t)U(t,0)|l\rangle = 0$ for k, $l = 01, 10$. This is equivalent to $\langle k|H(t)|l\rangle = 0$ since the DFS is an invariant subspace. Thus, $H(t) = 0$ in the subspace and it follows that one cannot realize nonadiabatic HQC in the DFS of two physical qubits since there is no nontrivial Hamiltonian to meet conditions (i) and (ii) above.

For three physical qubits interacting collectively with the dephasing environment, there exists a three-dimensional DFS

$$\mathcal{S}^D = \mathrm{Span}\{|100\rangle, |010\rangle, |001\rangle\}. \quad (10)$$

We encode a logical qubit in the subspace

$$\mathcal{S}^L = \mathrm{Span}\{|010\rangle, |001\rangle\}, \quad (11)$$

and denote the computational basis elements as $|0\rangle_L = |010\rangle$, $|1\rangle_L = |001\rangle$. Clearly, $\mathcal{S}^L$ is a subspace of $\mathcal{S}^D$ and the remaining vector $|100\rangle$ is used as ancillae, denoted as $|a\rangle = |100\rangle$ for convenience. In the following paragraphs, we utilize the DFS of three physical qubits to implement nonadiabatic HQC. To this end, we need to generate two noncommuting single-qubit gates and one nontrivial two-qubit gate.

Firstly, we demonstrate how to realize the one-qubit holonomic gate

$$U_{xz}(\phi_1) = X_L e^{i\phi_1 Z_L}. \tag{12}$$

Here, $X_L = |0\rangle_L\langle 1|_L + |1\rangle_L\langle 0|_L$, $Z_L = |0\rangle_L\langle 0|_L - |1\rangle_L\langle 1|_L$ are the Pauli operators of the logical qubit and ϕ_1 is an arbitrary phase. In the computational basis $\{|0\rangle_L, |1\rangle_L\}$, the gate reads

$$U_{xz}(\phi_1) = \begin{pmatrix} 0 & e^{-i\phi_1} \\ e^{i\phi_1} & 0 \end{pmatrix}. \tag{13}$$

In order to realize $U_{xz}(\phi_1)$, we set $J_{12}^x = J_1 \cos\frac{\phi_1}{2}$, $J_{12}^y = -J_1 \sin\frac{\phi_1}{2}$, $J_{13}^x = -J_1 \cos\frac{\phi_1}{2}$, $J_{13}^y = -J_1 \sin\frac{\phi_1}{2}$, and all other $J_{kl}^{x(y)}$ to zero, where J_1 is a time-independent parameter [31]. The Hamiltonian then reads

$$H_1 = J_1\left[(R_{12}^x - R_{13}^x)\cos\frac{\phi_1}{2} - (R_{12}^y + R_{13}^y)\sin\frac{\phi_1}{2}\right]. \tag{14}$$

$\mathcal{S}^D$ itself is an invariant subspace of the evolution operator $U_1(t) = e^{-iH_1 t}$. In the basis $\{|a\rangle, |0\rangle_L, |1\rangle_L\}$, we have

$$H_1 = J_1\begin{pmatrix} 0 & e^{i(\phi_1/2)} & -e^{-i(\phi_1/2)} \\ e^{-i(\phi_1/2)} & 0 & 0 \\ -e^{i(\phi_1/2)} & 0 & 0 \end{pmatrix}. \tag{15}$$

With the obvious expression of H_1, we can work out the operator $U_1(t)$. By choosing the evolution time τ_1 such that

$$J_1\tau_1 = \frac{\pi}{\sqrt{2}}, \tag{16}$$

the resulting unitary operator reads

$$U_1(\tau_1) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & e^{-i\phi_1} \\ 0 & e^{i\phi_1} & 0 \end{pmatrix}. \tag{17}$$

Thus, the action of the evolution operator $U_1(\tau_1)$ on the states in the logic subspace $\mathcal{S}^L$ is equivalent to that of the transformation $U_{xz}(\phi_1)$.

In order to ensure that the action of $U_1(\tau_1)$ on $\mathcal{S}^L$ is purely holonomic, we need to check conditions (i) and (ii). Condition (i) is satisfied since the subspace spanned by $\{U_1(\tau_1)|0\rangle_L, U_1(\tau_1)|1\rangle_L\}$ coincides with $\mathcal{S}^L$. Furthermore, as H_1 and $U_1(t)$ commute with each other, condition (ii) reduces to $\langle k|_L H_1 |k'\rangle_L = 0$, where k, $k' = 0$, 1. Thus, both conditions (i) and (ii) are satisfied, and $U_1(\tau_1)$ is therefore a one-qubit holonomic gate in the subspace $\mathcal{S}^L$, $\mathcal{S}^L \subset \mathcal{S}^D$.

Secondly, we demonstrate how to realize the one-qubit holonomic gate

$$U_{zx}(\phi_2) = Z_L e^{i\phi_2 X_L}, \tag{18}$$

where ϕ_2 is an arbitrary phase. In the computational basis $\{|0\rangle_L, |1\rangle_L\}$, we have

$$U_{zx}(\phi_2) = \begin{pmatrix} \cos\phi_2 & i\sin\phi_2 \\ -i\sin\phi_2 & -\cos\phi_2 \end{pmatrix}. \tag{19}$$

To realize U_{zx}, we set $J_{12}^y = J_2 \sin\frac{\phi_2}{2}$, $J_{13}^x = -J_2 \cos\frac{\phi_2}{2}$, and all other $J_{kl}^{x(y)}$ to zero, where J_2 is a time-independent parameter [31]. The Hamiltonian then reads

$$H_2 = J_2\left(R_{12}^y \sin\frac{\phi_2}{2} - R_{13}^x \cos\frac{\phi_2}{2}\right). \tag{20}$$

Again $\mathcal{S}_D$ is an invariant subspace of $U_2(t) = e^{-iH_2 t}$. Expressed in the $\{|a\rangle, |0\rangle_L, |1\rangle_L\}$, the resulting time evolution operator takes the form

$$U_2(\tau_2) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & \cos\phi_2 & i\sin\phi_2 \\ 0 & -i\sin\phi_2 & -\cos\phi_2 \end{pmatrix} \tag{21}$$

by choosing the evolution time τ_2 such that

$$J_2\tau_2 = \pi. \tag{22}$$

Equation (21) shows that the action of the evolution operator $U_2(\tau_2)$ on $\mathcal{S}^L$ is equivalent to that of $U_{zx}(\phi_2)$. Its holonomic nature is demonstrated as above. Thus, $U_2(\tau_2)$ acts as a one-qubit holonomic gate in the subspace $\mathcal{S}^L$.

We note that any single-qubit operation can be written as a combination of the following two types of rotations

$$R_z(\theta) = e^{-i(\theta/2)\sigma^z}, \qquad R_x(\varphi) = e^{-i(\varphi/2)\sigma^x}, \tag{23}$$

where θ, φ are rotation angles and σ^z, σ^x are Pauli operators. Equations (13) and (19) imply

$$\begin{aligned} U_{xz}(0)U_{xz}(-\theta/2) &= e^{-i(\theta/2)Z_L}, \\ U_{zx}(0)U_{zx}(-\varphi/2) &= e^{-i(\varphi/2)X_L}, \end{aligned} \tag{24}$$

where X_L and Z_L are just the Pauli Z and Pauli X operators of the logical qubit. This proves that $U_{xz}(\phi_1)$ and $U_{zx}(\phi_2)$ can realize any single-qubit rotation.

Thirdly, we demonstrate how to realize a nontrivial two-qubit gate. It is worth noting that the Hamiltonian in Eq. (7) serves single-qubit gates but cannot directly be applied to implement two-qubit gates. To implement a holonomic two-qubit gate, four-qubit interactions are needed. Here, we generate the CNOT gate by means of the Hamiltonian,

$$H_3 = J_3(R_{13}^x R_{45}^x - R_{13}^x R_{46}^x), \tag{25}$$

where J_3 is a time-independent parameter [31]. The Hamiltonian H_3 is obtained by setting $J_{13,45}^{xx} = -J_{13,46}^{xx} = J_3$ and all other controllable four-qubit coupling constants to zero. The choice of J_3 is related to the evolution time τ_3. The requirement for J_3 or τ_3 is

$$J_3\tau_3 = \frac{\pi}{\sqrt{2}}. \qquad (26)$$

In this case, $\mathcal{S}^D \otimes \mathcal{S}^D$ is a decoherence-free subspace, in which the small subspace spanned by $\{|a\rangle \otimes |a\rangle, |0\rangle_L \otimes |0\rangle_L, |0\rangle_L \otimes |1\rangle_L, |1\rangle_L \otimes |0\rangle_L, |1\rangle_L \otimes |1\rangle_L\}$ is an invariant subspace of the Hamiltonian H_3. In the invariant subspace, the evolution operator at time $t = \tau_3$ reads

$$U_3(\tau_3) = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}. \qquad (27)$$

Then, the CNOT gate is realized in the subspace $\mathcal{S}^L \otimes \mathcal{S}^L$, i.e., span$\{|0\rangle_L \otimes |0\rangle_L, |0\rangle_L \otimes |1\rangle_L, |1\rangle_L \otimes |0\rangle_L, |1\rangle_L \otimes |1\rangle_L\}$. One may verify that conditions (i) and (ii) are fulfilled too. $U_3(\tau_3)$ plays a two-qubit holonomic CNOT gate in the subspace $\mathcal{S}^L \otimes \mathcal{S}^L$.

We have succeeded in constructing two noncommuting holonomic single-qubit gates U_{xz} and U_{zx} and a holonomic CNOT two-qubit gate in DFSs of a system undergoing collective dephasing. The three gates compose a universal set of nonadiabatic holonomic quantum gates in DFSs. It is worth noting that the scheme proposed here is suitable for scaling up the logic qubits. The Hamiltonian to realize the gates of the nth logic qubit has the same structure as H_1 or H_2 but with the exchanging $R_{12}^{x(y)} \rightarrow R_{3n-2,3n-1}^{x(y)}$ and $R_{13}^{x(y)} \rightarrow R_{3n-2,3n}^{x(y)}$, while the Hamiltonian to realize the CNOT gate between the mth and the nth logic qubits has the same structure as H_3 but with the exchanging $R_{13}^x R_{45}^x \rightarrow R_{3m-2,3m}^x R_{3n-2,3n-1}^x$ and $R_{13}^x R_{46}^x \rightarrow R_{3m-2,3m}^x R_{3n-2,3n}^x$.

In summary, we have put forward a scheme for nonadiabatic holonomic quantum computation in decoherence-free subspaces. By using only three neighboring physical qubits undergoing collective dephasing to encode one logical qubit, we realize a universal set of quantum gates. Our scheme combines the coherence stabilization virtues of decoherence-free subspaces and the fault tolerance of geometric holonomic control. Comparing with the previous schemes, our scheme has removed the long run-time requirement in the adiabatic evolution and can avoid the extra errors and decoherence involved due to long time evolution. Since the Hamiltonian in the scheme may be independent of time, our scheme seems promising in experimental implementation, which may shed light on the applications of holonomic quantum computation in decoherence-free subspaces.

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*tdm@sdu.edu.cn

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