Symmetry Protected Topological Order and Spin Susceptibility in Superfluid ^{3}He-B

Takeshi Mizushima,[1,*] Masatoshi Sato,[2,†] and Kazushige Machida[1]

[1]*Department of Physics, Okayama University, Okayama 700-8530, Japan*

[2]*The Institute for Solid State Physics, The University of Tokyo, Chiba 277-8581, Japan*

(Received 19 April 2012; revised manuscript received 1 August 2012; published 16 October 2012)

We here demonstrate that the superfluid ^{3}He-B under a magnetic field in a particular direction stays topological due to a discrete symmetry, that is, in a symmetry protected topological order. Because of the symmetry protected topological order, helical surface Majorana fermions in the B phase remain gapless and their Ising spin character persists. We unveil that the competition between the Zeeman magnetic field and dipole interaction involves an anomalous quantum phase transition in which a topological phase transition takes place together with spontaneous symmetry breaking. Based on the quasiclassical theory, we illustrate that the phase transition is accompanied by anisotropic quantum criticality of spin susceptibilities on the surface, which is detectable in NMR experiments.

DOI: 10.1103/PhysRevLett.109.165301 PACS numbers: 67.30.H−, 03.65.Vf, 67.30.er, 74.20.Rp

Introduction.—Superfluid ^{3}He-B is one of the most concrete examples of time-reversal invariant topological superfluids [1,2], where the ground state wave function supports a nontrivial bulk topological invariant in three spatial dimensions [3–8]. As a consequence of the bulk-edge correspondence, helical Majorana fermions live on its specular surface, and their self-conjugate property gives rise to an Ising-like anisotropy of spin susceptibility [9–16]. Recently, the surface of a Majorana cone has been detected in experiments [17].

Since the topological superfluidity in ^{3}He-B, which is categorized as class DIII [3], is ensured by time-reversal invariance, it is sometimes stated that any time-reversal breaking such as a finite magnetic field immediately wipes out the topological nature. Indeed, in the presence of a strong magnetic field, the Majorana Ising spin ceases to exist [18]. However, as argued in the Letter, a more careful consideration on the basis of microscopic calculations and symmetry of the system points to a different conclusion. It is worth mentioning that the robustness of a topological phase transition for a topological insulator against the time-reversal breaking is proposed in an extended Kane-Mele model [19].

In this Letter, we show that ^{3}He-B under a magnetic field in a particular direction stays topological as a symmetry protected topological order. In spite of the time-reversal breaking due to the magnetic field, the topological property is retained by a hidden $\boldsymbol{Z}_2$ symmetry that is obtained by a combination of time reversal and an $\mathrm{SO}(3)_{\boldsymbol{L}+\boldsymbol{S}}$ rotation. Because this $\boldsymbol{Z}_2$ symmetry restores a chiral symmetry of the microscopic Hamiltonian, helical surface Majorana fermions in the B phase remain to be gapless, and the Ising spin character of the Majorana fermions persists unless the discrete symmetry is spontaneously broken. Finally, we come to the conclusion that at a critical Zeeman field H^* the system undergoes anomalous quantum phase transition in the sense that topological phase transition takes place together with spontaneous breaking of the $\boldsymbol{Z}_2$ symmetry [Fig. 1(a)].

The phase transition is described by a pair of an order parameter $\hat{\ell}_z$ and a topological number w, which are defined in Fig. 1(b) and in the text below Eq. (4). Conventionally, topological phase transitions are accompanied by creation or destruction of gapless surface states and gap closing of bulk quasiparticle spectra. While the anomalous quantum phase transition in the above involves the destruction of gapless surface states, it does not show the bulk gap closing. Instead, there appears a long range order due to symmetry breaking. Moreover, it takes place between two conceptional different quantum orders: At a critical magnetic field, the system undergoes a transition from a topologically ordered state ($w \neq 0$) to a conventionally ordered one ($\hat{\ell}_z \neq 0$).

The symmetry protected topological order is closely associated with the order parameter manifold of ^{3}He-B. Ignoring the dipole interaction and a Zeeman field, the bulk ^{3}He-B spontaneously reduces the symmetry $\mathrm{SO}(3)_{\boldsymbol{L}} \times \mathrm{SO}(3)_{\boldsymbol{S}} \times \mathrm{U}(1)$ to $\mathrm{SO}(3)_{\boldsymbol{L}+\boldsymbol{S}}$ [20]. The gap function is $\Delta(\hat{\boldsymbol{k}}, \boldsymbol{r}) = i\boldsymbol{\sigma} \cdot \boldsymbol{d}(\hat{\boldsymbol{k}}, \boldsymbol{r})\sigma_y$, where $d_\mu = d_{\mu\nu}\hat{k}_\nu$ and,

$$d_{\mu\nu}(\boldsymbol{r}) = e^{i\vartheta} R_{\mu\nu}(\hat{\boldsymbol{n}}, \varphi)\Delta_\nu(\boldsymbol{r}). \quad (1)$$

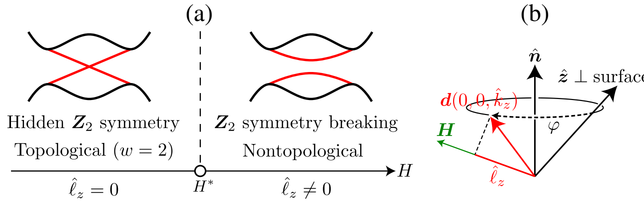

FIG. 1 (color online). (a) Schematic phase diagram of ^{3}He-B under a parallel magnetic field, where H^* involves the topological phase transition with spontaneous symmetry breaking. (b) Relation between $\hat{\ell}_z$ and the orientation of $\boldsymbol{d}(0, 0, \hat{k}_z)$ for an arbitrary $(\hat{\boldsymbol{n}}, \varphi)$ at the surface.

The broken symmetry $SO(3)_{L-S}$, the relative rotation between spin and orbital spaces, is described by $R_{\mu\eta}(\hat{\boldsymbol{n}}, \varphi)$ with the rotation axis $\hat{\boldsymbol{n}}$ and the angle φ. Here the repeated Greek indices imply the sum ($\mu, \nu, \eta = x, y, z$) and σ_μ denotes the Pauli matrices in the spin space. In general, the dipole interaction acting as a small perturbation chooses a particular state of $(\hat{\boldsymbol{n}}, \varphi)$.

To quantitatively determine $\hat{\ell}_z(\hat{\boldsymbol{n}}, \varphi)$ and H^*, we here utilize the quasiclassical theory which takes account of dipole interaction and Zeeman energy on equal footing. In a slab geometry, the finite H^* results from the competition between dipole and magnetic energies, where the former (latter) favors the symmetry protected topological (a nontopological) order. It is found that since the topological order protects the Majorana Ising spins, the phase transition is accompanied by anomalous critical behaviors of spin susceptibilities on the surface.

Surface bound states.—Let us start with the mean-field Hamiltonian density in the Nambu representation,

$$\underline{\mathcal{H}}(\boldsymbol{r}_1, \boldsymbol{r}_2) = \begin{bmatrix} \epsilon(\boldsymbol{r}_1, \boldsymbol{r}_2) & \Delta(\boldsymbol{r}_1, \boldsymbol{r}_2) \\ -\Delta^*(\boldsymbol{r}_1, \boldsymbol{r}_2) & -\epsilon^*(\boldsymbol{r}_1, \boldsymbol{r}_2) \end{bmatrix} + \underline{V}_Z \delta(r_{12}). \tag{2}$$

In this Letter, we set $\hbar = k_B = 1$. Equation (2) consists of $\epsilon(\boldsymbol{r}_1, \boldsymbol{r}_2) = \delta(\boldsymbol{r}_{12})(-\boldsymbol{\nabla}^2/2M - E_F)$ and the Zeeman energy $\underline{V}_Z \equiv -\mu_n H_\mu \mathrm{diag}(\sigma_\mu, -\sigma^*_\mu)$, where M, $E_F = k_F^2/2M$, and μ_n are the mass, Fermi energy, and magnetic moment of ^{3}He atoms. The pair potential for ^{3}He-B, $\Delta(\boldsymbol{k}, \boldsymbol{r}) \equiv \int d\boldsymbol{r}_{12} e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_{12}} \Delta(\boldsymbol{r}_1, \boldsymbol{r}_2)$ with Eq. (1), is simplified to $\Delta(\boldsymbol{k}, \boldsymbol{r}) = U(\hat{\boldsymbol{n}}, \varphi)\Delta_0(\boldsymbol{k}, \boldsymbol{r})U^{\mathrm{T}}(\hat{\boldsymbol{n}}, \varphi)$ with $U(\hat{\boldsymbol{n}}, \varphi) \in SU(2)$ and $\Delta_0(\boldsymbol{k}, \boldsymbol{r}) = i\sigma_\mu \sigma_y \Delta_\mu(\boldsymbol{r})\hat{k}_\mu$.

We first diagonalize Eq. (2) as $\int d\boldsymbol{r}_2 \underline{\mathcal{H}}(\boldsymbol{r}_1, \boldsymbol{r}_2)\boldsymbol{\varphi}_E(\boldsymbol{r}_2) = E\boldsymbol{\varphi}_E(\boldsymbol{r}_1)$, that is, the Bogoliubov–de Gennes (BdG) equation, where E and $\boldsymbol{\varphi}_E$ describe the energy and wave function of quasiparticles. This is solved within the Andreev approximation $\boldsymbol{\nabla}^2 \to i\boldsymbol{v}_F \cdot \boldsymbol{\nabla}$ and the uniform pair potential $\Delta_\mu(\boldsymbol{r}) = \Delta_0$, where $\boldsymbol{v}_F = \hat{\boldsymbol{k}} v_F$ is the Fermi velocity. In this work, we consider the B-phase sandwiched by two specular walls which are normal to the $\hat{z}$-axis. For $\boldsymbol{H} = \boldsymbol{0}$, the dispersion of the surface Andreev bound state (SABS) is given by $E_0(\boldsymbol{k}_\parallel) = \pm \frac{\Delta_0}{k_F}|\boldsymbol{k}_\parallel|$ with $\boldsymbol{k}_\parallel$ being the momentum in the xy plane [8,10,11]. The corresponding wave functions are expressed as $\boldsymbol{\varphi}^{(\pm)}_{0,\boldsymbol{k}_\parallel}(\boldsymbol{r}) \propto e^{i\boldsymbol{k}_\parallel \cdot \boldsymbol{r}_\parallel} e^{-z/\xi} \sin(\sqrt{k_F^2 - \boldsymbol{k}_\parallel^2} z)\underline{\mathcal{U}}(\hat{\boldsymbol{n}}, \varphi)\boldsymbol{\Phi}^{(\pm)}_{\boldsymbol{k}}$, where $(\pm)$ correspond to the positive and negative energy states and we set $\underline{\mathcal{U}} \equiv \mathrm{diag}(U, U^*)$. We also introduce $\boldsymbol{\Phi}^{(+)}_{\boldsymbol{k}} = \underline{\tau}_x \boldsymbol{\Phi}^{(-)^*}_{\boldsymbol{k}_\parallel} \equiv (1, -ie^{i\phi_k}, -e^{i\phi_k}, -i)^T$ with $\phi_{\boldsymbol{k}} = \tan^{-1}(\hat{k}_y/\hat{k}_x)$ and $\underline{\tau}_\mu$ being the Pauli matrices in the Nambu space.

For a finite H, the dispersion of the SABS is obtained from the linear combination, $\boldsymbol{\varphi}_{\boldsymbol{k}_\parallel} = a_+ \boldsymbol{\varphi}^{(+)}_{0,\boldsymbol{k}_\parallel} + a_- \boldsymbol{\varphi}^{(-)}_{0,\boldsymbol{k}_\parallel}$, as

$$E(\boldsymbol{k}_\parallel) = \pm\sqrt{[E_0(\boldsymbol{k}_\parallel)]^2 + [\mu_n H \hat{\ell}_z(\hat{\boldsymbol{n}}, \varphi)]^2}, \tag{3}$$

where $\hat{\ell}_\nu(\hat{\boldsymbol{n}}, \varphi) \equiv \hat{h}_\mu R_{\mu\nu}(\hat{\boldsymbol{n}}, \varphi)$ with $\hat{h}_\mu = H_\mu/H$.

Symmetry protected topological phase.—As we showed in Eq. (3), if $\hat{\ell}_z = 0$, the SABS remains gapless even in the presence of a magnetic field. From a topological point of view, however, this seems to be a puzzle: Because the magnetic field breaks the time-reversal invariance, topological protection as a time-reversal invariant topological superfluids does not work any more. Nevertheless, no gap opens in the SABS if the magnetic field satisfies $\hat{\ell}_z = 0$.

First, one should notice that $\hat{\ell}$ itself could be affected by a Zeeman magnetic field. Therefore, an immediate solution for this puzzle might be that if one applies a magnetic field, $\hat{\ell}$ changes so as $\hat{\ell}_z \neq 0$. However, as is shown below, if the Zeeman field is parallel to the xy plane (say, along $\hat{\boldsymbol{x}}$), this is not the case: There exists a symmetry that ensures $\hat{\ell}_z = 0$. Interestingly, we find that this symmetry resolves the puzzle above at the same time, by providing another topological protection of the SABS.

Let us first consider symmetry of the system. Among $SO(3)_{L+S}$ rotations under which microscopic interactions of ^{3}He atoms are invariant, the slab geometry considered here preserves its subgroup $SO(2)_{L+S}$ rotation $U(\theta)$ in the xy plane. The point is that while the Zeeman term along $\hat{\boldsymbol{x}}$ explicitly breaks both the time-reversal symmetry and the $SO(2)_{L+S}$ rotation symmetry above, it does not break a combination of them. In fact, in this case, the flipped magnetic field by time-reversal $\mathcal{T}$ is recovered by the π rotation in the xy plane. Therefore, the microscopic Hamiltonian of ^{3}He atoms is invariant under the discrete symmetry given by $\mathcal{T}U(\pi)$. We notice here that $\hat{\ell}_z$ mentioned above is transformed nontrivially as $\hat{\ell}_z \to -\hat{\ell}_z$ under this symmetry. Therefore, $\hat{\ell}_z$ is an order parameter of the discrete symmetry, and it should be zero unless the discrete symmetry is spontaneously broken.

Remarkably, one can introduce a topological invariant if the discrete symmetry is not spontaneously broken. In that case, the BdG Hamiltonian (2) in the momentum space is manifestly invariant under the discrete symmetry, $\underline{\mathcal{H}}(k_x, k_y, -k_z) = \mathcal{T}U(\pi)\underline{\mathcal{H}}(k_x, k_y, k_z)U^{-1}(\pi)\mathcal{T}^{-1}$, where $\mathcal{T} = i\sigma_y K$ is time reversal with complex conjugate operator K and $U(\pi) = i\sigma_z \underline{\tau}_z$ is the π rotation. Therefore, combining it with the particle-hole symmetry of the BdG Hamiltonian, $\underline{\mathcal{C}}\,\underline{\mathcal{H}}(\boldsymbol{k})\underline{\mathcal{C}}^\dagger = -\underline{\mathcal{H}}^*(-\boldsymbol{k})$ with $\underline{\mathcal{C}} = \underline{\tau}_x K$, one obtains the relation $\underline{\Gamma}\,\underline{\mathcal{H}}(k_x, k_y, k_z)\underline{\Gamma}^{-1} = -\underline{\mathcal{H}}(-k_x, -k_y, k_z)$ with $\underline{\Gamma} = \sigma_x \underline{\tau}_y$. On the k_z axis, this reduces to the so-called chiral symmetry

$$\{\underline{\Gamma}, \underline{\mathcal{H}}(0, 0, k_z)\} = 0. \tag{4}$$

Thus, following Refs. [21,22], one can introduce the following one-dimensional (1D) winding number $w = -\frac{1}{4\pi i}\int_{-\infty}^{\infty} dk_z \mathrm{tr}[\underline{\Gamma}\,\underline{\mathcal{H}}^{-1}\partial_{k_z}\underline{\mathcal{H}}]|_{\boldsymbol{k}_\parallel = \boldsymbol{0}}$, which are evaluated as $w = 2$ for $\mu_n H < E_F$ ($\Delta_\perp > 0$). Therefore, the system is topologically nontrivial, and the bulk-edge correspondence implies that the SABS satisfies $E(\boldsymbol{k}_\parallel) = 0$ at $\boldsymbol{k}_\parallel = \boldsymbol{0}$ even in the presence of the magnetic field. It should be noted here that one needs the discrete symmetry specific to this system in order to define w. Therefore, a topological phase realized here is a symmetry protected topological order [23,24].

Majorana Ising spin.—In the absence of the magnetic field, it has been known that helical Majorana fermions in ^{3}He-B have Ising-like spin density [9–16]. Now we show that the symmetry protected topological order discussed above retains the Ising spin character of Majorana fermions.

To show this, we use a general symmetry property of the SABS. Let us consider the low energy limit where only the zero energy SABSs at $\boldsymbol{k}_\parallel = \boldsymbol{0}$ contribute. According to the index theorem of Ref. [22], Eq. (4) infers that the zero energy SABSs are eigenstates of Γ, and the relation $w = n_- - n_+$ holds on the surface of superfluid ^{3}He-B, where $n_\pm$ is the number of the zero energy SABSs with the eigenvalue $\Gamma = \pm 1$. In the present case, $w = n_- = 2$ and $n_+ = 0$, and thus the SABS $\boldsymbol{\varphi}^{(a)}_{\boldsymbol{k}_\parallel=\boldsymbol{0}}$ satisfies $\Gamma\boldsymbol{\varphi}^{(a)}_{\boldsymbol{k}_\parallel=\boldsymbol{0}} = -\boldsymbol{\varphi}^{(a)}_{\boldsymbol{k}_\parallel=\boldsymbol{0}}$ ($a = 1, 2$). Using the particle-hole symmetry, one can also place the relation $\tau_x\boldsymbol{\varphi}^{(a)^*}_{\boldsymbol{k}_\parallel=\boldsymbol{0}} = \boldsymbol{\varphi}^{(a)}_{\boldsymbol{k}_\parallel=\boldsymbol{0}}$ at the same time. From these two relations, $\boldsymbol{\varphi}^{(a)}_{\boldsymbol{k}_\parallel=\boldsymbol{0}}$ has a generic form as $\boldsymbol{\varphi}^{(a)}_{\boldsymbol{k}_\parallel=\boldsymbol{0}} = [\xi^{(a)}, i\xi^{(a)*}, \xi^{(a)*}, -i\xi^{(a)}]^T$ with a function $\xi^{(a)}$. Ignoring nonzero energy modes, the quantized field $\boldsymbol{\Psi} = [\hat{\psi}_\uparrow, \hat{\psi}_\downarrow, \hat{\psi}^\dagger_\uparrow, \hat{\psi}^\dagger_\downarrow]^T$ is expanded as $\boldsymbol{\Psi}(z) = \sum_{a=1,2}\boldsymbol{\varphi}^{(a)}_{\boldsymbol{k}_\parallel=\boldsymbol{0}}\gamma^{(a)}$ with real $\gamma^{(a)}$, and from the general form of $\boldsymbol{\varphi}^{(a)}_{\boldsymbol{k}_\parallel=\boldsymbol{0}}$, one obtains $i\hat{\psi}_\uparrow = -\hat{\psi}^\dagger_\downarrow$, which is a general consequence of our symmetry protected topological order.

Now, following Refs. [9,10], one can show that the last relation, $i\hat{\psi}_\uparrow = -\hat{\psi}^\dagger_\downarrow$, yields the Ising character of the SABSs: It is shown that among the local density operator and the spin density operators, which are given by $\rho \equiv \frac{1}{2}[\hat{\psi}^\dagger_a\hat{\psi}_a - \hat{\psi}_a\hat{\psi}^\dagger_a]$ and $S_\mu \equiv \frac{1}{4}[\hat{\psi}^\dagger_a(\sigma_\mu)_{ab}\hat{\psi}_b - \hat{\psi}_a(\sigma^T_\mu)_{ab}\hat{\psi}^\dagger_b]$, respectively, only S_z is nonzero while the other components are identically zero. So, in the low energy limit, the SABSs do not contribute to the local density fluctuation, and its local spin density is Ising-like. Here, note that we only use a general property of the chiral symmetry; thus, the Ising character is a direct consequence of our symmetry protected topological order.

Quasiclassical Eilenberger theory.—Let us now microscopically determine $\hat{\ell}_z$ and H^*. For this purpose, we here utilize the quasiclassical Eilenberger theory, which provides a quantitative theory for superfluid ^{3}He at low pressures [25]. This is based on the quasiclassical Green's functions $\underline{g} \equiv \underline{g}(\hat{\boldsymbol{k}}, \boldsymbol{r}; i\omega_n)$ with the Matsubara frequency $\omega_n = (2n+1)\pi T$ ($n \in \mathbb{Z}$) and the 2×2 unit matrix σ_0

$$\underline{g} = \begin{bmatrix} \sigma_0 g_0 + \sigma_\mu g_\mu & i\sigma_y f_0 + i\sigma_\mu\sigma_y f_\mu \\ i\sigma_y f^\dagger_0 + i\sigma_y\sigma_\mu f^\dagger_\mu & \sigma_0 g^\dagger_0 + \sigma_\mu g^\dagger_\mu \end{bmatrix}. \quad (5)$$

The evolution is governed by the Eilenberger equation $[i\omega_n\underline{\tau}_z - \underline{\mathcal{S}}(\hat{\boldsymbol{k}}, \boldsymbol{r}), \underline{g}] = -i\boldsymbol{v}_F\cdot\boldsymbol{\nabla}\underline{g}$. The 4×4 matrix $\underline{\mathcal{S}}$ consists of the Zeeman energy $\underline{V}_Z$ and the self-energies,

$$\underline{\mathcal{S}}(\hat{\boldsymbol{k}}, \boldsymbol{r}) = \frac{1}{1+F^a_0}\underline{\tau}_z\underline{V}_Z + \begin{bmatrix} \nu_\mu(\hat{\boldsymbol{k}}, \boldsymbol{r})\sigma_\mu & \Delta(\hat{\boldsymbol{k}}, \boldsymbol{r}) \\ \Delta^\dagger(-\hat{\boldsymbol{k}}, \boldsymbol{r}) & \nu^*_\mu(\hat{\boldsymbol{k}}, \boldsymbol{r})\sigma^*_\mu \end{bmatrix}, \quad (6)$$

where ν_μ denotes the Fermi liquid corrections obtained as $\nu_\mu(\hat{\boldsymbol{k}}, \boldsymbol{r}) = \sum_\ell A^a_\ell\langle P_\ell(\hat{\boldsymbol{k}}, \hat{\boldsymbol{k}}')g_\mu(\hat{\boldsymbol{k}}, \boldsymbol{r}; i\omega_n)\rangle_{\hat{\boldsymbol{k}}', \omega_n}$. P_ℓ is the Legendre polynomial and $\langle\cdots\rangle_{\hat{\boldsymbol{k}}, \omega_n} = T\sum_{|\omega_n|<E_c}\int\frac{d\hat{\boldsymbol{k}}}{4\pi}$, with a cutoff E_c. The coefficient $A^a_\ell \equiv F^a_\ell/[1 + F^a_\ell/(2\ell+1)]$ is parameterized with the antisymmetric Fermi liquid parameters, $F^a_0 = -0.695$ and $F^a_1 = -0.5$ [20].

The gap equation is obtained as $\Delta_{ab}(\hat{\boldsymbol{k}}, \boldsymbol{r}) = \langle V^{cd}_{ab}(\hat{\boldsymbol{k}}, \hat{\boldsymbol{k}}')[i\sigma_\mu\sigma_y f_\mu(\hat{\boldsymbol{k}}', \boldsymbol{r}; i\omega_n)]_{cd}\rangle_{\hat{\boldsymbol{k}}', \omega_n}$, with a, b, c, $d = \uparrow, \downarrow$. At the low pressure limit, the pair interaction of ^{3}He atoms is described as $V^{cd}_{ab}(\hat{\boldsymbol{k}}, \hat{\boldsymbol{k}}') = 3|\Lambda|\hat{k}_\mu\hat{k}'_\mu\delta_{ac}\delta_{bd} - Q_{\mu\nu}(\hat{\boldsymbol{k}}, \hat{\boldsymbol{k}}')(\sigma_\mu)_{ac}(\sigma_\nu)_{bd}$. The first term arises from the p-wave interaction with $\mathrm{SO}(3)_{\boldsymbol{S}} \times \mathrm{SO}(3)_{\boldsymbol{L}} \times \mathrm{U}(1)$ and the second term is the dipole interaction, where $Q_{\mu\nu}(\hat{\boldsymbol{k}}, \hat{\boldsymbol{k}}')$ is obtained from $Q_{\mu\nu}(\boldsymbol{k}, \boldsymbol{k}') = g_D R\int r^{-3}(\delta_{\mu\nu} - 3\hat{r}_\mu\hat{r}_\nu)e^{-i(\boldsymbol{k}-\boldsymbol{k}')\cdot\boldsymbol{r}}d\boldsymbol{r}$ with $\boldsymbol{k} \approx \hat{\boldsymbol{k}}k_F$. The factor R includes the contributions of high energy quasiparticles [26]. The dipole interaction can be expressed in terms of the partial wave series (p-, f-, and higher waves). However, since the pairing interaction between ^{3}He atoms is dominated by the $\mathrm{SO}(3)_{\boldsymbol{S}} \times \mathrm{SO}(3)_{\boldsymbol{L}} \times \mathrm{U}(1)$ channel and the dipole interaction can be regarded as a small perturbation, we take account of only the p-wave contribution of $Q_{\mu\nu}(\hat{\boldsymbol{k}}, \hat{\boldsymbol{k}}')$. Then, the gap equation for $\Delta_\mu(\boldsymbol{r})$ is given by $\Delta_\mu(\boldsymbol{r}) = \delta_{\mu\nu}R^{-1}_{\mu\eta}(\hat{\boldsymbol{n}}, \varphi)\{(3|\Lambda| - \tilde{\Lambda}_D)\langle\hat{k}_\nu f_\mu\rangle_{\hat{\boldsymbol{k}}, \omega_n} - 3\tilde{\Lambda}_D[\delta_{\mu\nu}\langle\hat{k}_\mu f_\mu\rangle_{\hat{\boldsymbol{k}}, \omega_n} + \epsilon_{\mu\nu\eta}\langle(\hat{\boldsymbol{k}}\times\boldsymbol{f})_\eta\rangle_{\hat{\boldsymbol{k}}, \omega_n}]\}$, with the dimensionless factor $\tilde{\Lambda}_D \equiv \frac{3\pi}{10}g_D R$. At the thermodynamic limit with $\Delta_x = \Delta_y = \Delta_\parallel$ and $\Delta_z = \Delta_\perp$, the gap equation reproduces $\cos\varphi = -\frac{1}{4}\frac{\Delta_\perp}{\Delta_\parallel}$ [27–29].

The Eilenberger and gap equations with Eq. (1) provide self-consistent equations for $\underline{g}$ under a fixed ($\hat{\boldsymbol{n}}$, φ). The Eilenberger equation with $\underline{g}^2 = -\pi^2\underline{\tau}_0$ is numerically solved with the Riccati parameterization in the system that two specular walls normal to $\hat{\boldsymbol{z}}$ are situated at $z = 0$ and $z = 20\xi$. The numerical procedure is the same as that in Refs. [14,30]. We solve the gap equation with $|\Lambda|^{-1} = \pi T_{c0}\sum_{|\omega_n|<E_c}|\omega_n|^{-1}$ and $\tilde{\Lambda}_D/\Lambda^2 = 2\times 10^{-4}$ (2×10^{-5}), where we set the cutoff $E_c = 20\pi T_{c0}$. Note that since the ratio $\tilde{\Lambda}_D/\Lambda^2$ is associated with the distortion of Δ_μ in the thermodynamic limit [29], it is independent of the energy cutoff. The coherence length $\xi \equiv v_F/\pi T_{c0}$ is estimated

about 80 nm at the zero pressure of ^{3}He-B and $T_{c0} \approx 1$ mK is the critical temperature of the bulk B phase under $H = 0$. In a slab geometry, $\hat{\boldsymbol{n}}$ may be assumed to be spatially uniform, because the length scale of the spatial variation of $\hat{\boldsymbol{n}}$ is macroscopically large [20], compared with the thickness of the sample $20\xi \approx 1.6$ μm in typical experiments [31,32]. Then, the stable configuration of $(\hat{\boldsymbol{n}}, \varphi)$ is determined by minimizing the thermodynamic potential $\delta\Omega[\underline{g}]$ whose explicit form is the same as that given by Vorontsov and Sauls in Ref. [33]. Note that for the thickness 20ξ, the magnetic field induces the first-order phase transition from the B phase to the A or planar phase at the critical field $H_{AB} = 0.09\pi T_{c0}/\mu_n \approx 3.6$ kG [34].

Numerical results.—Since the $\hat{\boldsymbol{n}}$ vector always points to $\hat{\boldsymbol{z}}$ in the case of the perpendicular field $\boldsymbol{H} \parallel \hat{\boldsymbol{z}}$, we here consider a parallel field $\boldsymbol{H} \parallel \hat{\boldsymbol{x}}$. Figures 2(a) and 2(b) describe the energy landscape on the unit sphere of $\hat{\boldsymbol{n}}$. Figure 2(c) depicts the energy gap of the SABS evaluated from Eq. (3). The SABS becomes gapless along the certain trajectory on the sphere of $\hat{\boldsymbol{n}}$, which coincides with the condition of $\hat{\ell}_z = 0$. The stable configuration of $\hat{\boldsymbol{n}}$ is determined as a consequence of the interplay between dipole interaction and Zeeman energy. The former favors the situation where $\hat{\boldsymbol{n}}$ is normal to the surface, namely, $\hat{\ell}_z = 0$, while the condition for minimizing the Zeeman energy [35], $\hat{\ell}_z = 1$, is same as the condition that opens the maximum energy gap in the SABS. Hence, in the magnetic field lower than the dipolar field $H_{\rm D} \approx 30$ G $\sim 0.001\pi T_{c0}/\mu_{\rm n}$, as displayed in Fig. 2(a), $\hat{\boldsymbol{n}}$ points to $\hat{\boldsymbol{z}}$. It is seen in Fig. 2(b) that for the larger H's it tends to tilt from $\hat{\boldsymbol{z}}$ to the direction with $\hat{\ell}_z \neq 0$.

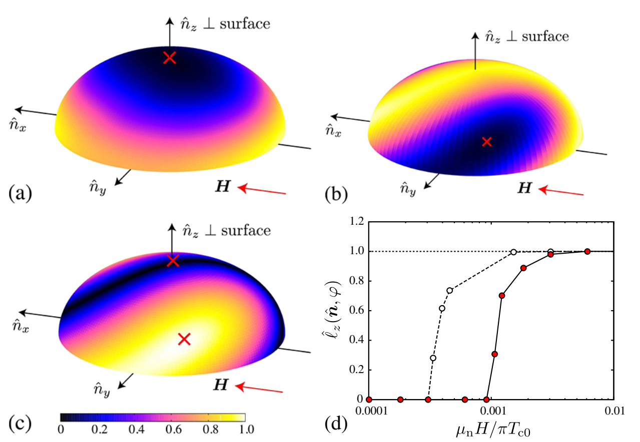

FIG. 2 (color online). Energy landscape on the unit sphere of $\hat{\boldsymbol{n}}$, $\delta\Omega(\hat{\boldsymbol{n}})$, at $\mu_n H/\pi T_{c0} = 9.2 \times 10^{-4}$ (a) and 0.0061 (b) where we fix $\varphi/\pi = -0.5537$ which minimizes the dipole interaction. We also set $\boldsymbol{H} \parallel \hat{\boldsymbol{x}}$ and $T/T_{c0} = 0.2$. The bright (dark) color depicts the higher (lower) energy. The energy gap $\min|E(\boldsymbol{k}_\parallel)|$ of Eq. (3) is displayed in (c). (d) Field dependence of $\hat{\ell}_z$ estimated with the stable $(\hat{\boldsymbol{n}}, \varphi)$ for $\tilde{\Lambda}_D/\Lambda^2 = 2 \times 10^{-4}$ (the solid line) and 2×10^{-5} (the dashed line).

The field dependence of $\hat{\ell}_z$ estimated with the stable configuration of $(\hat{\boldsymbol{n}}, \varphi)$ is displayed in Fig. 2(d). In the limit of the low field, $\hat{\ell}_z$ is locked to be $\hat{\ell}_z = 0$, which ensures the existence of surface Majorana fermions. $\hat{\ell}_z$ stays zero up to the critical value $\mu_n H^*/\pi T_{c0} \approx 0.001$, which is consistent with the argument that the systems with $\hat{\ell}_z = 0$ have the discrete symmetry. At $H \geq H^*$, the symmetry protected topological phase with $\hat{\ell}_z = 0$ undergoes a change to the nontopological phase with $\hat{\ell}_z \neq 0$.

In Fig. 3(a) we plot the field dependence of the local spin susceptibility on the surface, $\tilde{\chi}_{\mu z}(z = 0)$, defined as $\tilde{\chi}_{\mu\nu}(z)/\chi_N \equiv |M_\mu(z)|/M_N$ for a magnetic field $\boldsymbol{H} \parallel \hat{r}_\nu$. The local magnetization $M_\mu(z)$ is estimated as $M_\mu(\boldsymbol{r}) = M_N[\hat{h}_\mu + \frac{1}{\mu_n H}\langle g_\mu(\hat{\boldsymbol{k}}, \boldsymbol{r}; i\omega_m)\rangle_{\hat{k},\omega_m}]$ with that in the normal state $M_N = \frac{2\mu_n^2}{1+F_0^a} N_F H$, where N_F is the density of states of the normal ^{3}He. It is seen from Fig. 3(a) with the solid line that for $\boldsymbol{H} \parallel \hat{\boldsymbol{z}}$, the local spin susceptibility on the surface, $\tilde{\chi}_{zz}(0)$, is considerably enhanced, compared with $\tilde{\chi}_{zz}(z = 10\xi)$ (the dashed line) [34].

In contrast, when the parallel field ($\boldsymbol{H} \parallel \hat{\boldsymbol{x}}$) is applied, the magnetization $M_\mu(z)$ on the surface is sensitive to the orientation of $\hat{\boldsymbol{\ell}}$. It is seen in Fig. 3(b) with the dashed line that $M_x(z)$ at $\mu_n H/\pi T_{c0} = 9.2 \times 10^{-4}$ is strongly suppressed in the surface region, where $\hat{\boldsymbol{n}} \parallel \hat{\boldsymbol{z}}$, that is $\hat{\ell}_z = 0$, is energetically favored. This implies that the SABS does not contribute to $M_x(z)$ on the surface and is consistent with the property of the Majorana Ising spins.

In the relatively high field $\mu_n H/\pi T_{c0} = 0.0018$, however, $M_x(z)$ is enhanced around the surface, while $M_z(z)$ which is perpendicular to $\boldsymbol{H} \parallel \hat{\boldsymbol{x}}$ emerges on the surface. This emergence of $M_z(z)$ on the surface reflects the stable configuration of $(\hat{\boldsymbol{n}}, \varphi)$, where $\hat{\ell}_z = R_{xz}(\hat{\boldsymbol{n}}, \varphi)$ deviates from zero and is less than unity. As displayed in Figs. 3(a) and 3(b), the magnetic field within the range of $0 < \hat{\ell}_z < 1$ significantly induces $M_z(z)$ and $\tilde{\chi}_{zx}(z)$ on the surface, where the SABS opens the finite energy gap and the winding number w is not defined.

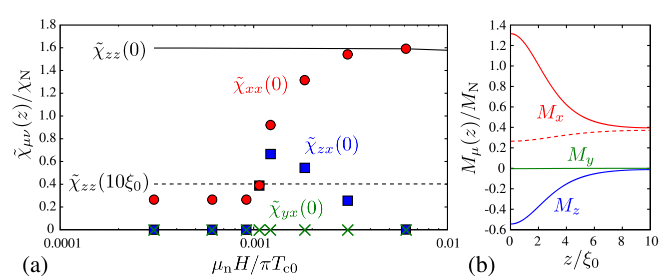

FIG. 3 (color online). (a) Field dependence of $\tilde{\chi}_{\mu\nu}(z)/\chi_N$ at $T = 0.2T_{c0}$. The solid (dashed) lines denote $\tilde{\chi}_{zz}(0)$ ($\tilde{\chi}_{zz}(10\xi)$) for $\boldsymbol{H} \parallel \hat{\boldsymbol{z}}$ and the symbols correspond to $\tilde{\chi}_{\mu x}(0)$ for $\boldsymbol{H} \parallel \hat{\boldsymbol{x}}$. (b) $M_\mu(z)$ for $\boldsymbol{H} \parallel \hat{\boldsymbol{x}}$ at $\mu_n H/\pi T_{c0} = 9.2 \times 10^{-4}$ (dashed line) and 0.0018 (solid lines), where $M_{y,z}$ at $\mu_n H/\pi T_{c0} = 9.2 \times 10^{-4}$ are zero. All data are taken with $\tilde{\Lambda}_D/\Lambda^2 = 2 \times 10^{-4}$.

Conclusions.—Here, we have clarified that the interplay of dipole interaction and a magnetic field in ^{3}He-B involves a new class of quantum phase transition, that is, the topological phase transition with the spontaneous breaking of the hidden $\boldsymbol{Z}_2$ symmetry. Using the quasiclassical theory, we have demonstrated that ^{3}He-B stays topological as the symmetry protected topological order and Majorana Ising spins exist unless the $\boldsymbol{Z}_2$ symmetry is spontaneously broken at the critical field H^*. The quantum phase transition is accompanied by the anomalous behavior of spin susceptibilities, which is observable through NMR experiments [36,37] in a slab geometry.

The authors are grateful to S. Higashitani, M. Ichioka, O. Ishikawa, R. Nomura, J. Saunders, R. Shindou, Y. Okuda, and Y. Tsutsumi for fruitful discussions and comments. This work was supported by JSPS (No. 2074023303, 2134010303 and 22540383) and the MEXT KAKENHI (No. 22103002 and No. 22103005).

*mizushima@mp.okayama-u.ac.jp
†msato@nuap.nagoya-u.ac.jp
Permanent address: Department of Applied Physics, Nagoya University, Nagoya 464-8603, Japan.

[1] Y. Tanaka, M. Sato, and N. Nagaosa, J. Phys. Soc. Jpn. **81**, 011013 (2012).
[2] X. L. Qi and S. C. Zhang, Rev. Mod. Phys. **83**, 1057 (2011).
[3] A. P. Schnyder, S. Ryu, A. Furusaki, and A. W. W. Ludwig, Phys. Rev. B **78**, 195125 (2008).
[4] X. L. Qi, T. L. Hughes, S. Raghu, and S. C. Zhang, Phys. Rev. Lett. **102**, 187001 (2009).
[5] M. Sato, Phys. Rev. B **79**, 214526 (2009).
[6] M. Sato, Phys. Rev. B **81**, 220504(R) (2010).
[7] A. Kitaev, AIP Conf. Proc. **1134**, 22 (2009).
[8] G. E. Volovik, JETP Lett. **90**, 587 (2009).
[9] M. Stone and R. Roy, Phys. Rev. B **69**, 184511 (2004).
[10] S.-B. Chung and S.-C. Zhang, Phys. Rev. Lett. **103**, 235301 (2009).
[11] Y. Nagato, S. Higashitani, and K. Nagai, J. Phys. Soc. Jpn. **78**, 123603 (2009).
[12] G. E. Volovik, JETP Lett. **90**, 398 (2009).
[13] R. Shindou, A. Furusaki, and N. Nagaosa, Phys. Rev. B **82**, 180505(R) (2010).
[14] Y. Tsutsumi, T. Mizushima, M. Ichioka, and K. Machida, J. Phys. Soc. Jpn. **79**, 113601 (2010).
[15] T. Mizushima and K. Machida, J. Low Temp. Phys. **162**, 204 (2011).
[16] M. A. Silaev, Phys. Rev. B **84**, 144508 (2011).
[17] S. Murakawa, Y. Wada, Y. Tamura, M. Wasai, M. Saitoh, Y. Aoki, R. Nomura, Y. Okuda, Y. Nagato, M. Yamamoto, S. Higashitani, and K. Nagai, J. Phys. Soc. Jpn. **80**, 013602 (2011), and references therein.
[18] G. E. Volovik, JETP Lett. **91**, 201 (2010).
[19] Z. Ringel and E. Altman, arXiv:1207.0581.
[20] D. Vollhardt and P. Wölfle, *The Superfluid Phases of Helium-3* (Taylor and Francis, London, 1990).
[21] M. Sato and S. Fujimoto, Phys. Rev. B **79**, 094504 (2009).
[22] M. Sato, Y. Tanaka, K. Yada, and T. Yokoyama, Phys. Rev. B **83**, 224511 (2011).
[23] Z.-C. Gu and X.-G. Wen, Phys. Rev. B **80**, 155131 (2009).
[24] F. Pollmann, E. Berg, A. M. Turner, and M. Oshikawa, Phys. Rev. B **85**, 075125 (2012).
[25] J. W. Serene and D. Rainer, Phys. Rep. **101**, 221 (1983).
[26] A. J. Leggett, J. Phys. C **6**, 3187 (1973); , Ann. Phys. (N.Y.) **85**, 11 (1974).
[27] L. Tewordt and D. Einzel, Phys. Lett. A **56**, 97 (1976).
[28] N. Schopohl, J. Low Temp. Phys. **49**, 347 (1982).
[29] R. S. Fishman, Phys. Rev. B **36**, 79 (1987).
[30] Y. Tsutsumi, M. Ichioka, and K. Machida, Phys. Rev. B **83**, 094510 (2011).
[31] S. Miyawaki, K. Kawasaki, H. Inaba, A. Matsubara, O. Ishikawa, T. Hata, and T. Kodama, Phys. Rev. B **62**, 5855 (2000).
[32] R. G. Bennett, L. V. Levitin, A. Casey, B. Cowan, J. Parpia, and J. Saunders, J. Low Temp. Phys. **158**, 163 (2010).
[33] A. B. Vorontsov and J. A. Sauls, Phys. Rev. B **68**, 064508 (2003).
[34] T. Mizushima, Phys. Rev. B **86**, 094518 (2012).
[35] W. F. Brinkman, H. Smith, D. D. Osheroff, and E. I. Blount, Phys. Rev. Lett. **33**, 624 (1974).
[36] A. I. Ahonen, M. Krusius, and M. A. Paalanen, J. Low Temp. Phys. **25**, 421 (1976).
[37] O. Ishikawa, Y. Sasaki, T. Mizusaki, A. Hirai, and M. Tsubota, J. Low Temp. Phys. **75**, 35 (1989).