

Polyakov Loop and the Hadron Resonance Gas Model

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The Polyakov loop has been used repeatedly as an order parameter in the deconfinement phase transition in QCD. We argue that, in the confined phase, its expectation value can be represented in terms of hadronic states, similarly to the hadron resonance gas model for the pressure. Specifically, $L(T) \approx \frac{1}{2} \sum_{\alpha} g_{\alpha} e^{-\Delta_{\alpha}/T}$, where g_{α} are the degeneracies and Δ_{α} are the masses of hadrons with exactly one heavy quark (the mass of the heavy quark itself being subtracted). We show that this approximate sum rule gives a fair description of available lattice data with $N_f = 2 + 1$ for temperatures in the range $150 \text{ MeV} < T < 190 \text{ MeV}$ with conventional meson and baryon states from two different models. For temperatures below 150 MeV different lattice results disagree. One set of data can be described if exotic hadrons are present in the QCD spectrum while other sets do not require such states.

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Introduction.—The transition from the hadronic phase to the quark-gluon plasma phase has been a recurrent topic in hadronic physics [1]. In pure gluodynamics or, equivalently, for infinitely heavy quarks, this is a true phase transition. The order parameter is identified as the thermal Wilson line or Polyakov loop [2–4],

$$\Omega = \mathbf{P} e^{i \int_0^{1/T} A_0 dx_0}, \quad L(T) = \langle \text{tr} \Omega \rangle, \quad (1)$$

where A_0 is the gluon field, T is the temperature and \mathbf{P} denotes path ordering. $L(T)$ changes abruptly from zero to near N_c (the number of colors) due to the breaking of the center symmetry $\mathbb{Z}(N_c)$ for $T > T_c$. In QCD, i.e., for dynamical quarks, the center symmetry is explicitly broken by the quarks and one has instead a smooth crossover [5] and the critical temperature T_c is usually defined by the condition $L''(T_c) = 0$. Lattice simulations show that chiral symmetry is restored when quarks and gluons are deconfined. These theoretical insights strongly suggested the experimental quest for the quark-gluon plasma in current facilities. The Polyakov loop also serves as a gluonic effective degree of freedom in the successful Polyakov–Nambu–Jona–Lasinio and Polyakov–quark–meson models to describe hot and/or dense QCD [6–10].

Since hadrons (and possibly glueballs) are the physical states in the confined phase, it should be expected, by quark-hadron duality, that physical quantities admit a representation in terms of hadronic states. The QCD pressure presents a prime example of this, through the hadronic resonance gas model (HRGM) [11–17],

$$\frac{1}{V} \log Z = - \int \frac{d^3 p}{(2\pi)^3} \sum_{\alpha} \zeta_{\alpha} g_{\alpha} \log(1 - \zeta_{\alpha} e^{-\sqrt{p^2 + M_{\alpha}^2}/T}), \quad (2)$$

with g_{α} the degeneracy factor, $\zeta_{\alpha} = \pm 1$ for bosons and fermions, respectively, and M_{α} the hadron mass. These resonances are the low-lying states listed in the review by the Particle Data Group (PDG) [18]. Actually, in the large N_c limit this expectation becomes a true theorem in QCD since the flavor resonances become narrow $\Gamma/M = O(1/N_c)$ (see, e.g., Refs. [19,20]). After some controversies, lattice calculations seem to suggest that this is also a good approximation in the physical case $N_c = 3$ [21]. This problem has been addressed within a strong coupling expansion for heavy quarks in Ref. [22].

Despite its prominent theoretical role, $\Omega(\mathbf{x})$ does not appear to be directly accessible in the laboratory, being most naturally defined in the imaginary-time formalism of field theory at finite temperature [23,24] (see Ref. [25] for its definition within the real-time formalism). The realization of the Polyakov loop through a static (or heavy) quark and its relation with the heavy-quark self-energy (or even with the binding energy between a static and a dynamical quark [26]), is not new [4,27], but the phenomenological consequences of this fact have not yet been extracted at a quantitative level. Here we argue that the Polyakov loop in the confined phase can also be represented in terms of hadronic properties in a direct and quantitative way, similarly to the HRGM for the pressure.

Polyakov loop and hadronic spectrum.—In the Hamiltonian formulation of QCD [28,29] the gauge is partially fixed by the condition $A_0 = 0$ and the dynamical degrees of freedom are contained in the spatial gluons \mathbf{A}

and the quarks. The time-independent gauge transformations $g(\mathbf{x})$ are still a residual symmetry of the Hamiltonian acting in the Hilbert space \mathcal{H} of functionals $\Psi(A, q, \bar{q})$. The gauge group $SU(N_c)$ decomposes \mathcal{H} into invariant subspaces labeled by an irreducible representation r at each point \mathbf{x} : $\mathcal{H} = \bigoplus_{\{r(\mathbf{x})\}} \mathcal{H}_{\{r(\mathbf{x})\}}$. In the Euclidean lattice formulation, the role of the integration over A_0 , or equivalently integration over $\Omega(\mathbf{x})$ with the Haar measure, is to project onto the physical subspace, which requires a color singlet at every point \mathbf{x} .

An infinitely heavy quark (of a new flavor) sitting at x_0 is a spectator with spin and color degrees of freedom only. For the gluons and dynamical (as opposed to heavy) quarks, this is equivalent to living in the subspace $\mathcal{H}_{r(x_0)=3}$: a color singlet at every point except x_0 , which is in the fundamental representation ($\mathbf{3}$ for three colors). The projector onto this subspace is obtained by adding the factor $N_c \text{tr}[\Omega(x_0)]$ to the Haar measure [30]. For the expectation value of the Polyakov loop this immediately implies the relation [31,32]

$$L_{\text{bare}}(T) = \frac{1}{2} \frac{\text{Tr}_{h,x_0}(e^{-H/T})}{\text{Tr}_{\text{phys}}(e^{-H/T})}. \quad (3)$$

The factor N_c in the projector represented the trivial degeneracy of the system formed by gluons plus dynamical quarks in the fundamental representation at x_0 , and is canceled when this is combined with the spectator quark to form a color singlet. The factor $1/2$ removes the double counting from the two spin states of the spectator quark. The left-hand side is independent of the heavy quark spin (as the Polyakov loop carries no spin) and this is fully consistent with the well-known heavy-quark spin symmetry present in QCD [33,34]. The (infinite) mass of the spectator quark is not included in H .

Equation (3) is exact for the bare Polyakov loop and the partition functions in $\mathcal{H}_{\text{phys}}$ and $\mathcal{H}_{r(x_0)=3}$ on the lattice. In the renormalized continuum limit the relation still holds, after removing the additional specific UV divergence introduced by the heavy quark self-energy in $L(T)$ and $\text{Tr}_{h,x_0}(e^{-H/T})$. Such removal leaves a nonperturbative ambiguity by an additive constant in the Polyakov loop free energy $F(T) = -T \log L(T)$ [21,27,35–38].

The (renormalized) partition functions in Eq. (3) are saturated by states of the spectrum. Since the spectator quark can be reached smoothly by taking the infinite mass limit of a heavy quark at rest, this implies

$$L(T) = \lim_{m_h \rightarrow \infty} \frac{1}{2} \frac{\sum_{\alpha} g_{h\alpha} e^{-(E_{h\alpha} - m_h)/T}}{\sum_{\alpha} g_{\alpha} e^{-E_{\alpha}/T}}, \quad (4)$$

where m_h denotes the heavy quark mass. Here, the sum in the denominator is just the QCD partition function and so it includes all possible states made of gluons and dynamical quarks (labeled by α). On the other hand, the sum in the numerator includes all possible QCD states with exactly

one heavy quark h at rest (such a statement would be meaningless for dynamical quarks, but not for infinitely heavy quarks), plus gluons and dynamical quarks (jointly labeled by $h\alpha$). The difference $E_{h\alpha} - m_h$ explicitly removes the heavy quark mass from the total energy of the state.

For temperatures well below the crossover, we expect the previous states to be of hadronic type (and possibly glueballs). In particular, the heavy quark h will form a hadron with the dynamical quarks, typically, a meson of hybrid type, i.e., formed by the heavy quark and a dynamical antiquark, $h\bar{q}$, or a hybrid baryon with the heavy quark and two dynamical quarks, hqq .

The HRGM for the QCD partition function follows naturally from assuming that the QCD interaction primarily confines quarks into hadrons and that purely hadronic interactions can be neglected. Under this assumption, the numerator of Eq. (4) contains one hybrid heavy-light hadron at rest plus exactly the same multi-hadron states occurring in the denominator. This yields a cancellation between numerator and denominator. Therefore, within the same approximations leading to the HRGM, we expect the following relation to hold between the Polyakov loop expectation value in the confined phase and the hadronic spectrum

$$L(T) \approx \frac{1}{2} \sum_{\alpha} g_{h\alpha} e^{-\Delta_{h\alpha}/T}, \quad \Delta_{h,\alpha} = M_{h\alpha} - m_h. \quad (5)$$

Here the sum is over all states made just of a single hybrid hadron at rest (with exactly one heavy flavor quark, mass $M_{h\alpha}$ and degeneracy $g_{h\alpha}$), and no additional hadrons.

Of course, neither the sum rule in Eq. (5) nor the HRGM can be accurate when unconfined states of the spectrum start to play a role, that is, for temperatures in the crossover region or above. For instance, $L(T)$ is a decreasing function at high enough temperatures, in the perturbative regime [39], while the Boltzmann distribution in the right-hand side of Eq. (5) is always increasing as a function of T . On the other hand, the mass of the heavy-light hadron is an observable (a renormalization invariant) but depends on the heavy flavor, while $\Delta_{h\alpha}$ is universal in the heavy quark limit but has some running from m_h , which is itself not an observable. The Polyakov loop is renormalization invariant and well defined, modulo the above mentioned shift ambiguity in the free energy. This can be compensated by a corresponding shift in the heavy quark mass.

Estimates from the physical spectrum.—In order to compare different Polyakov loop determinations with the hadronic sum rule, they have to be brought to a common renormalization condition. Two such determinations are related by $L^I(T) = e^{C/T} L(T)$, for some constant energy shift C . In Fig. 1 we compile five Polyakov loop data sets, obtained with physical quark masses and three flavors on the lattice [17,21]. The plot shows that four of them agree after applying suitable finite renormalizations (no

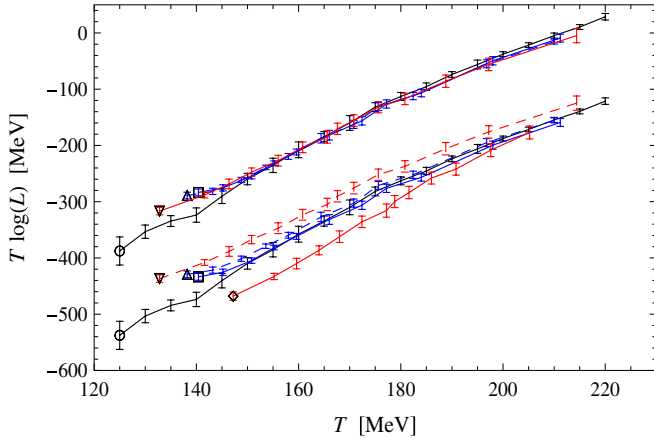


FIG. 1 (color online). $T \log[L(T)]$ as a function of T (units in MeV) from simulations on the lattice with $2+1$ physical dynamical quark masses. Lower set of lines: data after a common shift $C = -150$ MeV (just for displaying purposes) for continuum extrapolated stout (black solid line, “circle”) [21], HISQ/tree action $N_t = 12$ scale set r_1 (blue solid line, “square”) and f_k (blue dashed line, “up triangle”), and asqtad action $N_t = 12$ scale set r_1 (red solid line, “down triangle”) and f_k (red dashed line, “rhombus”) [17]. Upper set of lines: same data (except asqtad scale set r_1) with shifts $C = 0, 0, -10$ MeV and -30 MeV, for stout, HISQ/tree scale r_1 , HISQ/tree scale f_k , and asqtad scale f_k , respectively.

attempt has been made to optimize the agreement), in a wide range of temperatures. Unfortunately, the agreement deteriorates at lower temperatures, below $T \approx 150$ MeV. This region is relevant for comparison with the hadronic sum rule. Since there is no “true” value of $L(T)$, a finite renormalization, or choice of heavy quark mass, should be admitted too in the hadronic sum rule. Nevertheless, large renormalizations (compared with the hadronic scale) would be unnatural, and would probably signal an inadequacy of the sum rule, or of the renormalization prescription used for the Polyakov loop. A neat way to remove any ambiguities is to work with the derivative of $T \log[L(T)]$ with respect to T . This slope is sensitive to the effective number of states at a given temperature. Although with some noise, Fig. 2 shows that a signal can be extracted in this way.

A natural step is to check to what extent the hadronic sum rule is fulfilled by experimental states compiled in the PDG. Several sources of error should be kept in mind when doing this, among others, that not all needed states may have been compiled, that the heavy quarks in nature have a finite mass, that their current mass is scale dependent, and that the quark masses on the lattice may not be identical to the physical ones. Hadrons with a bottom quark would be optimal, due to the large quark mass compared to Λ_{QCD} , but the available data are scarce, so we turn to charmed hadrons. Specifically, we consider the lowest lying single-charmed mesons and baryons with u , d , and s as the dynamical flavors, with quarks in relative s wave inside

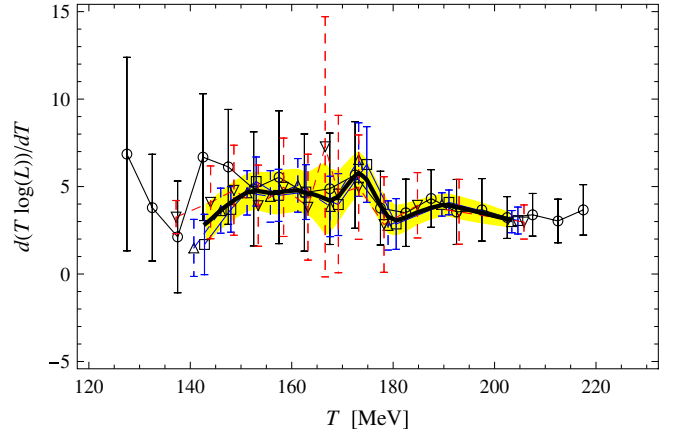


FIG. 2 (color online). Extraction of the slope $\frac{d}{dT}\{T \log[L(T)]\}$ (dimensionless) as a function of T (in MeV) from the same four lattice data sets of Fig. 1. The error bars in the slope were obtained from assuming a linear interpolation between measured points. The black thick line indicates the average in the range of temperatures common to the four sets. The yellow strip indicates the uncertainty.

the hadron. For mesons, these are usually identified with the states (spin-isospin multiplets) \bar{D} , \bar{D}_s , \bar{D}^* (2010) and \bar{D}_s^* , and for baryons, with Λ_c , $\Sigma_c(2455)$, Ξ_c , Ξ'_c , Ω_c , $\Sigma_c(2520)$, $\Xi_c(2645)$, and $\Omega(2770)$. A total of 12 meson states and 42 baryon states.

The plot in Fig. 3 shows that the lowest-lying states fall short to saturate the sum rule, regardless of the choice of mass of the charmed quark, m_c . This is not surprising as any model predicts many excited states on top of the lowest-lying ones, as is also the case for light-quark hadrons. Adding more states from the PDG does not

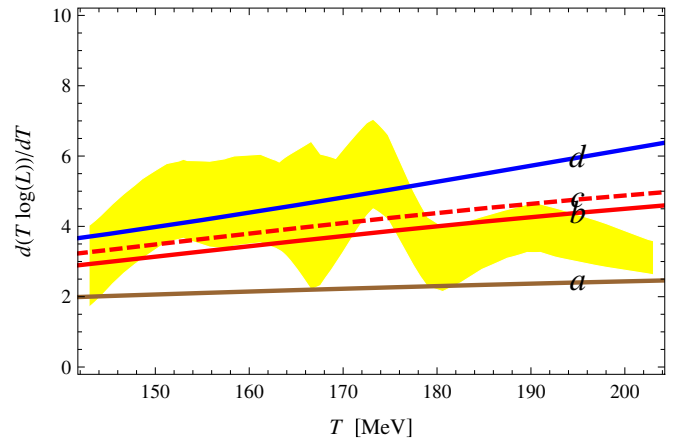


FIG. 3 (color online). Comparison of $\frac{d}{dT}\{T \log[L(T)]\}$ (yellow strip) with $\frac{d}{dT}(T \log(\frac{1}{2} \sum_{\alpha} g_{h\alpha} e^{-\Delta_{h\alpha}/T}))$ from hadronic states (mesons plus baryons): lowest-lying hadrons from PDG (solid brown line, label a), RQM states from Refs. [40,41] with quark c (solid red line, label b), and with quark b (dashed red line, label c), and bag model estimate including states up to $\Delta = 5500$ MeV (solid blue line, label d).

seem practical due to the fragmentary information available. Instead we turn to hadronic models. The aim is not so much to have a detailed description of the various states but to give a sufficiently good overall description of the whole spectrum. To this end we consider the relativized quark model (RQM) [40,41], and the bag model [42,43]. We have verified that the RQM provides a good account of the trace anomaly in Ref. [21]. The total number of hadron states computed in Ref. [40,41] with one c quark is 117 for mesons and 1470 for baryons, corresponding to a maximum value of $\Delta = M - m_c$ about 1500 MeV. For hadrons with one b quark, 87 mesonic and 1740 baryonic states, with a similar upper bound for Δ . In both cases we have supplemented missing states with strange quarks by means of the equal spacing rule [44] and a s quark mass of 109 MeV (extracted from the lowest-lying hadrons masses). The prediction based on these hadronic states is displayed in Fig. 3. The two sum rules are closer to the Polyakov loop result but still tend to stay below it in the range $T < 175$ MeV, a consequence of the truncation of states to $\Delta < 1500$ MeV. It is noteworthy that the bottom sum rule gives a better value, as it would be expected due to the larger mass of the b quark.

The other model we consider is more schematic but allows us to easily include a larger number of states. This is a simplified MIT bag model [42,43], in order to correctly count the number of states without fine details such as multiplet splittings. As bag energy we take

$$\Delta = \frac{\sum_i n_i \omega_i - Z}{R} + \frac{4\pi}{3} R^3 B + \sum_i m_i, \quad (6)$$

where n_i , ω_i , and m_i are the occupation number, bag frequency, and current quark mass ($m_u = m_d = 0$, $m_s = 109$ MeV). The model gives directly Δ after adjustment of R , without center of mass corrections, since the heavy quark has actually infinite mass (not included), sits at the center and plays no active role. The sum over i runs over just one antiquark for $h\bar{q}$ mesons and two quarks for hqq baryons. We set $B = (166 \text{ MeV})^4$ [43], and find $m_b = 4660$ MeV and $Z = 0.3$ from a fit to the single-bottom lowest-lying mesons and baryons masses. The result for the bag model with $\Delta < 5500$ MeV is displayed in Fig. 3. The more complete description of the hadronic spectrum gives a better account of the Polyakov loop data in the range $145 \text{ MeV} < T < 175 \text{ MeV}$. We have checked the following. (i) For this range of temperatures heavier states become irrelevant for the sum rule. We see this by projecting the cumulative number of states assuming a power law dependence for mesons and for baryons [45]. (ii) Truncation to $\Delta < 1500$ MeV gives a result quite consistent with that of RQM. (iii) A similar power-law projection of the spectrum for the RQM, to estimate the effect of adding the states above $\Delta = 1500$ MeV, also reproduces quantitatively the bag model result. And (iv) using single-charm lowest-lying hadrons to fix the bag model

parameters yields tiny differences in Fig. 3. The hadronic sum rule eventually overshoots the Polyakov loop result (see Fig. 3) as the crossover to the deconfined phase sets in. This mimics the same behavior in the HRGM [21,46].

In Fig. 4 we display lattice data for $T \log L(T)$ vs the hadronic sum rule. The hadronic estimate describes well the various lattice data sets in the range $150 \text{ MeV} < T < 190 \text{ MeV}$, and even lower temperature data from Ref. [17]. However, the steeper slope displayed by the stout action data [21] for $T < 150$ MeV cannot possibly be saturated with conventional mesons and hadrons, since all these states have already been accounted for. Inclusion of exotic hadrons, $hq\bar{q}\bar{q}$ (tetraquarks) and $hqqq\bar{q}$ (pentaquarks), do actually produce the required slope (see Fig. 4), and it would imply a much shorter temperature range of applicability of the hadronic sum rule. At present, the various sets of lattice data disagree at the lowest temperatures, and also it is unsettled whether exotic hadrons are present in the QCD spectrum or not [47]. Our analysis implies that resolution of one of these issues would shed light on the other.

Final comments.—From QCD considerations, we derive a Boltzmann distribution formula in terms of hadrons for the expectation value of the Polyakov loop in the confined phase, as required by quark-hadron duality, where its real and positive character becomes manifest. Our derivation exposes the obvious fact that the Polyakov loop gets its expectation value from the dressing with dynamical quarks or antiquarks. Since $U(N_f)$ is an exact global symmetry, the numerator in Eq. (3), and hence $L(T)$ itself, can be decomposed into separate contributions from different flavors and different baryon numbers. Such decompositions are in principle accessible to lattice calculations (although with a difficulty similar to that of introducing a chemical potential) and they would provide further information

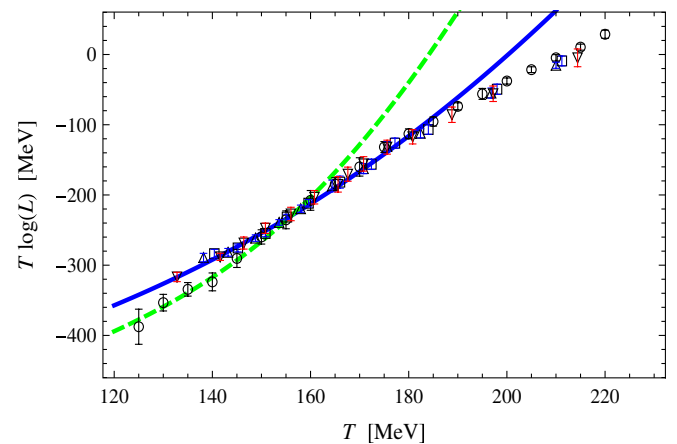


FIG. 4 (color online). $T \log[L(T)]$ (in MeV) for the four lattice data sets of Fig. 1 compared to the hadronic sum rule from the bag model. Solid blue line: estimate from conventional mesons and baryons applying a shift $C = 55$ MeV. Dashed green line: estimate when exotic hadrons are included, with $C = 15$ MeV.

about the interplay between the QCD thermal state and the heavy quark spectrum.

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