Quantum Magnetomechanics: Ultrahigh-Q-Levitated Mechanical Oscillators

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Engineering nanomechanical quantum systems possessing ultralong motional coherence times allows for applications in precision quantum sensing and quantum interfaces, but to achieve ultrahigh motional Q one must work hard to remove all forms of motional noise and heating. We examine a magneto-meso-mechanical quantum system that consists of a 3D arrangement of miniature superconducting loops which is stably levitated in a static inhomogeneous magnetic field. The motional decoherence is predominantly due to loss from induced eddy currents in the magnetized sphere which provides the trapping field ultimately yielding $Q \sim 10^9$ with motional oscillation frequencies of several hundreds of kilohertz. By inductively coupling this levitating object to a nearby driven flux qubit one can cool its motion very close to the ground state and this may permit the generation of macroscopic entangled motional states of multiple clusters.

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Recently there has been considerable effort towards mapping the boundary between the classical and the quantum world by exploring the physics of mesoscopic and macroscopic mechanical systems. From an applications point of view, as precision measurement of position and acceleration generally involve some kind of motion, the necessity of building smaller and more sensitive devices has required a more careful exploration of the classical-quantum limit. The possibility to couple, control, and measure mesomechanical motion in a wide range of different physical systems leads to new experimental applications in diverse fields such as measuring forces between individual biomolecules [1–3], magnetic forces from single spins [4], perturbations due to the mass fluctuations involving single atoms and molecules [5], pressure [6] and acceleration [6], fundamental constants [7], small changes in electrical charge [8], gravitational wave detection [9], and applications in quantum computation [10], quantum optics [11], and condensed matter physics [12,13].

Observing any quantum properties of a mechanical system is a challenge. Under typical conditions, energy losses, thermal noise, and decoherence processes make it impossible to observe any motional quantum effects. To observe quantum mechanical motional effects the system has to be close enough to its ground state and it has to preserve this quantum coherence for a reasonable amount of time. This leads to the necessity of engineering ultralow dissipative systems which, in oscillating systems, is measured by the quality factor Q representing the energy lost per cycle. To achieve this one must engineer a system which is mechanically isolated from its surroundings to an extreme level. On the other hand, one must also find a way to cool down the motion close to its motional ground state which necessities coupling that system to another in order to dump entropy. Numerous mesomechanical oscillating systems have been studied recently, such as cavity optomechanical experiments employing cantilevers [14], micromirrors [15,16], microcavities [17,18], nanomembranes [19], macroscopic mirror modes [20], and optically levitated microspheres [21] and nanospheres [22] (see [23]). As shown in Ref. [24], it has been possible to create and control quantum states, but, except in a few cases, reaching large Q for nano- to microscopic sized motional devices is still an open problem. In fact, while a mechanical oscillator usually involves many coupled degrees of freedom, we are interested in the quantum behavior of one of them: the center of mass motion.

We present a theoretical model for a mesoscopic mechanical oscillator. In our setup, a strongly inhomogeneous static magnetic field generated by a magnetized sphere is placed above a cluster, hereafter the resonator, of three orthogonal superconducting loops (see Fig. 1). Coupling one of the loops inductively to a superconducting flux qubit, we describe a protocol for cooling the center of mass translational degree of freedom for vertical motion close to the ground state. More specifically, the low mechanical frequency is on resonance with the dressed frequency of the (driven) qubit. Together with the high mechanical quality factor, this resonance condition allows for the energy to be dissipated in the qubit environment. We also assume the resonator and the qubit to be connected to separate thermal baths with temperatures, respectively, T_r and T_q, and these can be different, for instance, due to additional motional noise associated with, e.g., actively controlling the position of the magnetic sphere.

The total Hamiltonian H is the sum of terms involving the resonator (H_r), the qubit (H_q), and the interaction (H_I): $H = H_r + H_q + H_I$. The Hamiltonian of the resonator can be written in the following form: $H_r = K_{\mathrm{c.m.}} + K_{\mathrm{rot}} + V$, where $K_{\mathrm{c.m.}}$ is the kinetic energy due to the translational motion of the center of mass, K_{rot} is the rotational kinetic

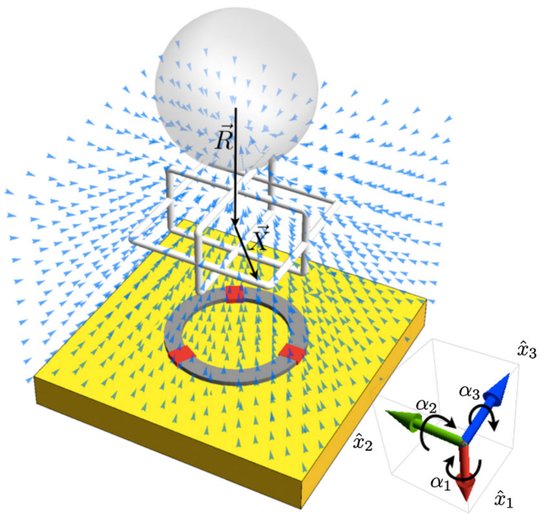

FIG. 1 (color online). Cluster of three insulated superconducting loops levitating in a magnetic field generated by a magnetized sphere with magnetization vector along $\hat{x}_1$. In the laboratory frame the axes are labeled $\{\hat{x}_j\}_{j=1}^3$ with angular coordinates α_j around each. The reference system has the center of the sphere as origin. We depict the magnetic vector field generated by the spherical magnet and the nearby flux qubit (which we take to be a 3-junction phase qubit), sitting under the cluster on a yellow substrate.

energy, and V is the effective potential energy as a function of the translational and rotational degrees of freedom. In the following m is the mass of the resonator, $\vec{X}$ the vector position of the various part of the resonator in a corotating reference frame with origin at the center of mass, $\vec{R}$ the coordinate of the center of mass in the laboratory reference frame, $\tilde{\vec{r}}$ the vector position of the various part of the resonator in the laboratory frame, and $\vec{r} = \tilde{\vec{r}} - \vec{R}$. The origin of the laboratory frame is taken to be at the center of the magnetic sphere (see Fig. 1).

Because of the rigid body properties, the only allowed relation between $\vec{r}(\vec{X}, t)$ and $\vec{X}$ is given by $\vec{r}(\vec{X}, t) = O(t)\vec{X}$, where $O = e^{\sum_i \alpha_i(t) T_i}$ [with $(T_i)_{jk} = \epsilon_{ijk}$] is a rotation matrix and $\alpha_i \in [-\pi, \pi)$. We can then define the angular velocity vectors with components $\Omega_i = \dot{\alpha}_i$. The motion of the rigid body is thus completely determined by the 6 degrees of freedom $(\vec{R}, \vec{\alpha})$. The inertia tensor of the resonator is $\mathbb{I}_{ij} = \int dV_r \rho(\vec{X})(\vec{X}^2 \delta_{ij} - \vec{X}_i \cdot \vec{X}_j)$. The kinetic energies are defined with respect to the reference system in Fig. 1 as $K_{\rm c.m.} = \frac{1}{2} m \sum_i \dot{R}_i^2$ and $K_{\rm rot} = \frac{1}{2}\sum_{i,j} \mathbb{I}_{ij}\Omega_i\Omega_j$.

The potential V is just the sum of the flux energy due to the current flowing in the loops and the gravitational potential energy: $V = \frac{1}{2}\sum_{a=1}^3 L_a I_a^2 - mgR_1$, where the index $a = 1, 2, 3$ labels the normal to the plane of the loop, L_a is the inductance of loop a, and I_a is the current flowing in loop a. By the symmetry of the resonator the mutual inductances between the loops is zero. The currents are obtained by stationary flux condition enforced by the Meissner effect:

$$\Delta\phi_a(\vec{R}, \vec{\alpha}) + L_a I_a(\vec{R}, \vec{\alpha}) = 0, \tag{1}$$

where $\Delta\phi_a(\vec{R}, \vec{\alpha}) = \phi_a(\vec{R}, \vec{\alpha}) - \phi_a(\vec{R}(0), \vec{\alpha}(0))$ is the difference in magnetic flux threading loop a when the system is in the configuration labeled by $(\vec{R}, \vec{\alpha})$ and when the system is in its initial configuration $(\vec{R}(0), \vec{\alpha}(0))$. Any infinitesimal change in flux due to an infinitesimal displacement or rotation induces a supercurrent whose action is to restore the loop's position or orientation. The stronger this restoring force is, the higher the oscillation frequency will be. Denoting $\vec{\Sigma}_a$ the area vector of loop a, then the flux through this loop is $\phi_a(\vec{R}, \vec{\alpha}) = \int_\Sigma \vec{B} \cdot d\vec{\Sigma}_a = \int_{\partial\Sigma_a} \vec{A} \cdot d\vec{r}$, where $\vec{A}(\vec{r}; \vec{\mathcal{M}}, \vec{R}) = \frac{\mu_0}{4\pi} \frac{1}{|\vec{r}+\vec{R}|^3} \vec{\mathcal{M}} \times (\vec{r} + \vec{R})$ is the vector potential generated by a sphere with homogeneous magnetization vector $\vec{\mathcal{M}}$ calculated at the point $\vec{r} + \vec{R}$. From this one can obtain an expression for the potential energy V as a function of the coordinates $\vec{\zeta} = (x, y, z, \alpha_x, \alpha_y, \alpha_z)$. For example, at first order in $\vec{\zeta}$,

$$\Delta\phi_a = \frac{\mu_0}{4\pi}\{\vec{K}_a \cdot [\vec{r} - \vec{R}(0) \times \vec{\alpha}] + \vec{Q}_a \cdot [\vec{\mathcal{M}} \times \vec{\alpha}]\}, \tag{2}$$

where the vectors $\vec{K}_a$ and $\vec{Q}_a$ are calculated from the magnetic field and the sphere magnetization. This leads to a second order expansion of the potential energy $V = \frac{1}{2} V_{ij} \zeta^i \zeta^j$.

The effect of the gravity in the potential energy causes a small shift of the potential minimum. By considering the magnetization of the sphere to be aligned along the vertical x direction, zero modes appear from the second order expansion of the potential energy around its minimum which ostensibly would allow the system to drift away, yet stability is restored thanks to higher order contributions. The "typical" dimensions of the system are radius of sphere $R_s = 10\ \mu$m; resonator dimensions $(1, 10, 10)\ \mu$m with a wire thickness of $0.1\ \mu$m, distance between the center of the sphere and the top of the cluster of loops $1.2R_s$. For NbTi nanowires [25] and a sphere made of soft ferrite NiZn with residual magnetization $\mu_0|\mathcal{M}| = 0.29$ T [26], the translational frequencies are $(600, 75, 75)$ kHz. The high frequency motion is a consequence of the high intensity magnetic field allowed by the choice of the NbTi superconductor, which, in normal conditions, has very high critical magnetic field strength $B_C \sim 5$ T [25]. In the small oscillations regime, the trapping does not disappear (yet it become less tight) as the system grows in size, so that, for example, increasing the combined resonator and sphere system by a factor of 10 decreases these oscillation frequencies by ~ 10 [27].

To second order perturbation theory (assuming the system's initial configuration is not too far from equilibrium [27]), the vertical direction (with associated variable R_1) is decoupled from the other degrees of freedom. As we show below, its frequency is also well separated relative to the linewidth of the other mode frequencies; hence, it is well resolved by the qubit coupling. We will consider the quantized variable corresponding to small deviations from the initial position along the vertical direction: $\hat{x} = \sqrt{\frac{\hbar}{2m\omega_r}}(\hat{a} + \hat{a}^\dagger)$. The dynamics of this variable is governed by an harmonic Hamiltonian whose frequency ω_r can be obtained by diagonalizing the potential V.

There are several potential sources of loss which limit the motional Q for the resonator. The energy dissipated from the moving resonator due to its action as a dipole emitter of radiation is negligible. Similarly, coupling to other phonon motional modes of the resonator is far off resonant and insignificant. The dominate modes that would be coupled to are the flexural, radial, and torsional modes for the horizontally aligned loop. Approximating the loop as a circle of radius r_ℓ, the lowest frequency mode is the torsional mode, which is $\nu_{\rm tor} = \frac{1}{\sqrt{2}\pi}\frac{a}{r_\ell}$, where a is the velocity of sound along the wire [28]. Using $r_\ell = 5\ \mu$m we find $\nu_{\rm tor} \sim 200$ MHz, more than 2 orders of magnitude larger than the resonator frequency. Other sources of dissipation are the viscous drag of flux lines oscillating inside the pinning wells inside the superconducting wires and background gas friction. Schilling [7] has calculated the losses to flux line dragging and damping in a rarefied gaseous atmosphere for the two-dimensional version of our resonator with loss rate γ, and for our oscillator frequency the motional quality factor is $Q = \omega_r/\gamma \approx 10^{11}$. Ultimately, the dominant source of loss is due to inductive coupling to damped eddy currents in the magnetized sphere. In order to estimate this effect let us consider infinitesimal horizontal loops of radius R' inside the sphere and placed at a distance h from the bottom of the sphere. The motional electromotive force induced in each of such loops is given by $|\epsilon| = M_{\ell,s}(R', h)\frac{dI}{dt}$, where $M_{\ell,s}(R', h)$ is the mutual inductance between the horizontal loop of the resonator and the horizontal infinitesimal loop of the sphere. We can compute the power loss by assuming the currents inside the resonator have instantaneous effects on the sphere, and integrating over the solid sphere s:

$$P = \int_{\rm s} \frac{[M_{\ell,s}(R', h)\frac{dI}{dt}]^2}{\rho 2\pi R'} dR' dh \le \int_{\rm s} \frac{[M_{\ell,s}(R', h) I \omega_r]^2}{\rho 2\pi R'} dR' dh, \tag{3}$$

where ρ is the resistivity. A more careful calculation including retardation effects has a negligible correction [27]. For a sphere made out of the soft ferrite NiZn with $\rho = 10^7\ \Omega$m, relative magnetic permeability $\mu_r = 250$, and $\mu_0|\mathcal{M}| = 0.29$ T [26], the effects due to eddy current loss give a value of $Q = E\omega_r/P = 10^9$, where the ground state energy $E = \hbar\omega_r$. Theory has predicted that one can expect motion Q factors to be a large as 10^{12} in levitated systems [22], and experiments with a magnetized sphere levitating at frequencies of 100–300 Hz in the presence of superconducting electrodes of a parallel plate capacitor have observed Q values of up to 10^6, limited primarily due to loss from flux pin dragging [29]. The prospect for ultralarge motional Q at moderately high motional frequencies is one of the primary benefits of our scheme.

We now describe how to cool the resonator by coupling it to a superconducting flux qubit (see the setup in Fig. 1). We envisage a three stage process. First, begin with the resonator placed on the surface near the qubit and at a temperature above its T_c so it is a normal metal. Next magnetize the sphere off-line and bring it into the vicinity of the resonator and proceed to the cool the resonator to below T_c by bringing it into contact with a cold reservoir. Finally, raise the sphere which by virtue of the Meissner effect will lift the resonator above the surface. When the sphere-resonator complex is high enough off the surface, the magnetic field due to the sphere will be small enough (below the critical field strength) at the location of the superconducting flux qubit on the surface. In practice, a height of $d \sim 30\ \mu$m from the qubit to the center of the resonator provides for $B < 0.005\ {\rm T} \ll B_c$, with $B_c \sim 0.01$ T for an aluminum superconducting flux qubit [30].

By denoting the qubit bare frequency with ω_q and by introducing a driving by a classical field of frequency ω_d detuned from ω_q by $\delta = \omega_d - \omega_q$ and with Rabi frequency Ω, the qubit Hamiltonian in the rotating frame with frequency ω_d is $\hat{H}_q = -\frac{\hbar\delta}{2}\hat{\sigma}_z + \frac{\hbar\Omega}{2}\hat{\sigma}_x$. The classical interaction Hamiltonian for the inductive coupling between the horizontal resonator loop and the loop of the flux qubit is given by $\hat{H}_I = M_{\ell,q} I_\ell I_q$, where I_ℓ, I_q are the currents flowing in that loop and in the qubit and the mutual inductance is $M_{\ell,q} = \frac{\mu_0}{4\pi}\oint\oint\frac{d\vec{s}\cdot d\vec{s}'}{r}$, where $d\vec{s}$ and $d\vec{s}'$ are vectors tangent to two points on the path of the two loops and r is the distance between the points. Because we are considering small deviations from the equilibrium initial position, the mutual inductance between the other loops or due to the angular motion of the resonator is negligible. We expand I_ℓ to first order in small deviations from the initial position using Eqs. (1) and (2). In the quantized version we also replace I_q with $I_q\hat{\sigma}_z$ (see, for example, [31]). Denoting $D_I(0) = \frac{\partial I_\ell}{\partial x}|_{\vec{R}(0)}$ as the derivative of the current evaluated at the initial cluster position, the quantized interaction Hamiltonian is $\hat{H}_I = \hbar\lambda(\hat{a} + \hat{a}^\dagger)\hat{\sigma}_z/2$, with $\lambda = \sqrt{\frac{2}{m\hbar\omega_r}} M_{\ell,q} D_I(0) I_q$. By considering a vertical magnetized sphere with magnetization $|\vec{\mathcal{M}}|$, and approximating the resonator loops to be point-like, one finds that the coupling roughly behaves as $\lambda \propto \frac{A_q\sqrt{|\vec{\mathcal{M}}|}}{d^3}$, where A_q is the area of the qubit loop and d is the distance between the qubit and the center of the resonator, which, for this calculation, we supposed to be much bigger than the other spatial dimensions involved. The final quantized expression for the Hamiltonian describing the motion of the cluster in the vertical direction and its interaction with the qubit is ($\hbar = 1$):

$$\hat{H} = -\frac{\delta}{2}\hat{\sigma}_z + \frac{\Omega}{2}\hat{\sigma}_x + \omega_r \hat{a}^\dagger \hat{a} + \frac{\lambda}{2}(\hat{a} + \hat{a}^\dagger)\hat{\sigma}_z. \quad (4)$$

The coupling strength can be adjusted over quite a large range ($\lambda/2\pi \in [10^2, 10^5]$ Hz) by fixing the distance ($[60, 2]$ μm) between the center of the resonator and the qubit, taken to be a circle of radius 5 μm, and the resonator. We choose qubit parameters satisfying $\sqrt{\Omega^2 + \delta^2} := \omega_r$, which sets the scale for resonant coupling to the resonator. The resonator and the qubit are generically coupled to separate thermal baths and interact with each other through the coupling H_I. The joint state of the qubit and resonator is denoted $\hat{\rho}$ and evolves under the following master equation:

$$\dot{\hat{\rho}} = -i[\hat{H}, \hat{\rho}] + \hat{\mathcal{L}}_\Gamma(\hat{\rho}) + \hat{\mathcal{L}}_\gamma(\hat{\rho}). \quad (5)$$

By introducing an energy exchanging, Markovian coupling between the qubit and a bosonic thermal bath and by defining the map $D[\hat{O}](\hat{\rho}) \equiv (2\hat{O}\hat{\rho}O^\dagger - \{\hat{O}^\dagger\hat{O}, \hat{\rho}\})$, we can describe the dynamics of the two level system in the limit $\omega_d \gg \omega_r$ [27] through the following Liouville operator: $\hat{\mathcal{L}}_\Gamma(\hat{\rho}) = \frac{\Gamma_\perp}{2}(N_q + 1)D[\sigma_-] + \frac{\Gamma_\perp}{2}N_q D[\sigma_+] + \frac{\Gamma_{||}}{2}D[\sigma_z]$, where the dissipation factor $\Gamma_\perp$ together with an additional dephasing term depending on an excess dephasing factor $\Gamma_{||}$ have been introduced, and where the equilibrium phonon occupation number is $N_q = (e^{\hbar\omega_q/k_B T_q} - 1)^{-1}$, where T_q is the qubit phonon bath temperature. By tracing out the qubit we obtain an effective equation of motion for the resonator. From this we will see that under certain cases we can obtain a cooling process bringing the cluster towards its motional ground state.

Under the temperature independent assumptions ($\lambda \ll \Gamma_\perp, \omega_r$), and assuming the resonators initial state is a pure coherent state $|\alpha\rangle$, the final phonon occupation number for the resonator is solved for in Ref. [32]:

$$n_f = N_{\text{th}}[\zeta + (1 - \zeta)/(1 + \zeta e^{I_1/(N_{\text{th}}\zeta\eta^2)})]. \quad (6)$$

Here the "renormalized" cooling rate is $\Gamma(\alpha) = i\lambda(\vec{S}_1^z/\alpha - \vec{S}_{-1}^z/\alpha^*)$, with $\zeta = \Gamma_c(0)/\gamma$, $I_1 = 2\int_0^\infty d\alpha\alpha\tilde{\Gamma}_c(\alpha/\eta)$, $\tilde{\Gamma}_c = \Gamma_c(\alpha)/\Gamma_c(0)$, $\eta = \lambda/\omega_r$, and $N_{\text{th}} = (e^{\hbar\omega_r/k_B T_r} - 1)^{-1}$, where T_r is the resonator phonon bath temperature. The qubit polarization Fourier components, $\vec{S}_1^z$ and $\vec{S}_{-1}^z$, are given by the solutions to Bloch equations (for the solution, see [27]). In the low temperature limit, ($\lambda\sqrt{N_{\text{th}} + 1/2} \ll \Gamma_\perp, \omega_r$), which is equivalent to the Lamb-Dicke regime, one can obtain an effective master equation for the resonator after tracing out the qubit (see Ref. [32] and references therein). This gives a new effective resonator damping rate $\Gamma = \Gamma_C + \gamma$ with $\Gamma_C = S(\omega_r) - S(-\omega_r)$, where $S(\omega)$ denotes the qubit fluctuation spectrum $S(\omega) = \frac{\lambda^2}{2}\,\text{Re}\int_0^\infty e^{i\omega\tau}d\tau\{\langle\hat{\sigma}_z(\tau)\hat{\sigma}_z(0)\rangle_0 - \langle\hat{\sigma}_z\rangle_0^2\}$ and $\langle\cdot\rangle_0$ denotes the steady state expectation. The resulting steady state occupation of the resonator is

$$n_{\text{LD}} = \gamma N_{\text{th}}/\Gamma_C + N_0, \quad (7)$$

where $N_0 = S(-\omega_r)/\Gamma_C$. Figure 2 shows the final expected cooled motional Fock number n_f in the on resonant case and for two different qubit bath temperatures. For comparison, we also plot the dashed line labeled as n_{LD} obtained from the low temperature theory extrapolated to high temperature [as given by Eq. (7)]. One can see that for a large temperature range the expression n_{LD} is valid; however, above a certain bath temperature cooling is no longer possible. By coupling the resonator to the qubit, the system shows a significant enhancement in the cooling rate relative to letting it cool by dissipative loss alone. The cooling rate goes from $\gamma/2\pi = 6 \times 10^{-4}$ Hz to $(\gamma + \Gamma_C)/2\pi \sim 1$ Hz when in contact with a reservoir at temperature $T_q = 10^{-2}$ K and to $(\gamma + \Gamma_C)/2\pi \sim 1$ Hz when in contact with a reservoir at $T_q = 10^{-1}$ K (with the other parameters fixed as described in the caption of Fig. 2). The net result is that, even if the resonator is connected to a very hot thermal bath ($N_{\text{th}} \sim 10^5$), one can cool the system to a final average phonon number $n_f \sim 1$. As a final note, performances would improve with increased resonator frequency. This can be obtained by replacing the magnetized sphere with a magnetic tip [33] with much larger magnetic field gradients.

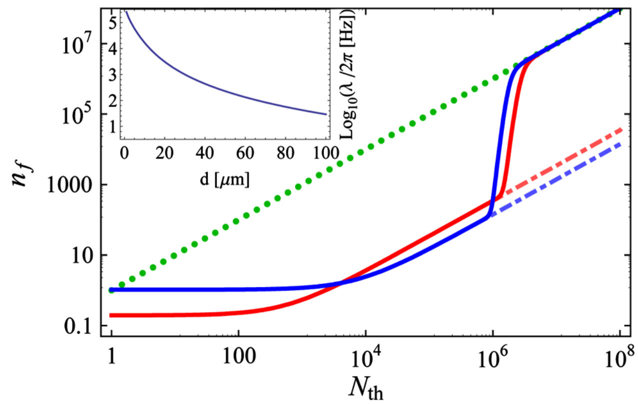

FIG. 2 (color online). Cooling performance. Final resonator phonon occupation number n_f as a function of the initial occupation number N_{th} for cooling via dissipation. The phonon number N_{th} corresponds to the fixed temperature T_r of the resonator bath. The dot-dashed lines represent the low temperature limit n_{LD}. When n_f is below the identity dotted line, some cooling is achieved. The system parameters for the light gray (red) curves are $(\omega_r, \lambda, \Gamma_\perp)/2\pi = (6 \times 10^5, 1.6 \times 10^3, 10^5)$ Hz and $(\Omega, \delta, T_q) = (0.5\omega_r, -\sqrt{\omega_r^2 - \Omega^2}, 10\text{ mK})$. The dark gray (blue) curves represents a case where the bath is at higher temperature but the decay rate is smaller: $(\omega_r, \lambda, \Gamma_\perp)/2\pi = (6 \times 10^5, 1.6 \times 10^3, 10^4)$ Hz and $(\Omega, \delta, T_q) = (0.5\omega_r, -\sqrt{\omega_r^2 - \Omega^2}, 100\text{ mK})$. The excess dephasing has been set to satisfy $\Gamma_{||} = \Gamma_\perp$. Inset: Resonator-qubit coupling constant λ as a function of the distance d between the qubit and the center of the cluster of loops. For large distances the coupling scales as d^{-3} following its linear dependency on the mutual inductance between the horizontal resonator loop and the qubit loop.

We have presented a mechanical oscillating system having very low dissipation utilizing superconducting material levitated in a vacuum via the Meissner effect. By inductive coupling to a flux qubit we showed how to cool down the motion of the system both in high and low temperature environments using engineered dissipation of the qubit. The overall performance improves with stronger or higher gradient magnetic fields, which could be achieved by replacing the magnetized sphere with other geometric magnetized objects. It is also possible to use the qubit to provide an indirect coupling between spatially separated levitated resonators. This would allow for the generation of spatially extended (several microns), macroscopic motional multimode superposition states that could be used for high precision measurements of force gradients, e.g., gravity. For two oscillators coupled by one qubit, one can use the mechanism in Ref. [34] to generate two-mode motional Yurke-Stoler cat states, while for a chain of oscillators and interspaced coupling qubits it is possible [35] to generate very large spatially extended entangled motional states.

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Note added.—Recently, we became aware of similar recent work [36] which focuses on the quantum motional cooling of a microsphere levitated by the Meissner effect.

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