# Is Black-Hole Ringdown a Memory of Its Progenitor?

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We perform an extensive numerical study of coalescing black-hole binaries to understand the gravitational-wave spectrum of quasinormal modes excited in the merged black hole. Remarkably, we find that the masses and spins of the progenitor are clearly encoded in the mode spectrum of the ringdown signal. Some of the mode amplitudes carry the signature of the binary's mass ratio, while others depend critically on the spins. Simulations of precessing binaries suggest that our results carry over to *generic* systems. Using Bayesian inference, we demonstrate that it is possible to accurately measure the mass ratio and a proper combination of spins even when the binary is itself invisible to a detector. Using a mapping of the binary masses and spins to the final black-hole spin allows us to further extract the spin components of the progenitor. Our results could have tremendous implications for gravitational astronomy by facilitating novel tests of general relativity using merging black holes.

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*Introduction.*—A black-hole-binary merger produces a single black hole that quickly "rings down" to the Kerr solution, fully characterized by its mass and angular momentum. It is well known that the frequencies and damping times of the ringdown gravitational waves (GWs) are described by the same two parameters (see, e.g., Ref. [1] and references therein). However, the mode distribution of the ringdown amplitudes depends on the progenitor. Recently Kamaretsos *et al.* [2] suggested that we could exploit this fact to measure properties of the progenitor from the ringdown signal. This was demonstrated by using a set of numerical-relativity simulations of nonspinning binaries parametrized by the mass ratio and constructing a signal model reflecting the clear mass ratio dependence of the ringdown mode amplitudes.

It follows that in general the ringdown amplitudes will depend on all eight binary parameters (the two masses, plus the vector components of each black hole's spin). In this Letter we report two remarkable results. First, that at least some of the mode amplitudes depend *only* on the mass ratio of the progenitor binary, largely independent of the spins. Therefore, we should be able to use the ringdown to measure the individual masses of a binary even when we cannot observe the binary itself. Second, one other mode amplitude carries a clear signature of the *spins* of the progenitor black holes and depends on an effective spin parameter related to the difference in the spin magnitudes. In the case of aligned spins (i.e., nonprecessing binaries), this fact, along with a mapping of the progenitor configuration to the final black-hole spin [3], allows us to determine the individual spin components from the ringdown phase alone.

We show that progenitor parameters *can* be measured with good accuracy with the Einstein Telescope (ET) [4]. If mode amplitudes can be extracted from GW observations, they could be used to test strong-field general relativity, study the nature of the merged object, especially if it is a naked singularity, and as the only means to observe the formation of black holes when the inspiral phase of the signal is outside a detector's sensitivity band.

The physical origin of the mode-amplitude relations is unclear; but we note a relation to post-Newtonian inspiral results, raising questions for future research.

*Background.*—For a black hole of mass $M$, located at a distance $D$, the plus and cross polarizations, $h_+$ and $h_\times$, of GWs emitted due to quasinormal mode oscillations can be written to a good approximation as

$$h_+(t) = +\frac{M}{D}\sum_{\ell,m} A_{\ell m} Y_+^{\ell m} e^{-t/\tau_{\ell m}} \cos(\omega_{\ell m} t - m\phi + \varphi_{\ell m}),$$

$$h_\times(t) = -\frac{M}{D}\sum_{\ell,m} A_{\ell m} Y_\times^{\ell m} e^{-t/\tau_{\ell m}} \sin(\omega_{\ell m} t - m\phi + \varphi_{\ell m}),$$

for $t \geq 0$, where only the first (least damped) overtone is kept and the rest are omitted. Here $A_{\ell m}$, $\omega_{\ell m}$, $\tau_{\ell m}$ are the mode amplitudes, frequencies and damping times, respectively, $Y_{+,\times}^{\ell m}(\iota)$ are related to $-2$ spin-weighted spherical harmonics that depend only on the inclination $\iota$ of the black hole's spin axis to the observer's line-of-sight [5], $\phi$ is the azimuth angle at which the black hole is observed with respect to a suitably chosen frame, and $\varphi_{\ell m}$ the initial phase angles of the modes.

Black hole perturbation theory can be used to compute the mode frequencies and damping times [6], but *not* the mode amplitudes $A_{\ell m}$, which depend on the nature of the perturbation—in our case, a highly distorted black hole that results from the merger. Instead, we must use numerical simulations to calculate the mode spectrum and its dependence on the progenitor parameters [2,7,8].

*Numerical results.*—We explored the effect of spins with a large number of numerical binary simulations that

 

consisted of 2–4 inspiral orbits before merger. There were three sets of simulations: (1) binaries with nonprecessing equal spins $\chi_i = S_i/m_i^2 = \{0, \pm 0.3, \pm 0.5, \pm 0.7\}$ and mass ratios $q = m_1/m_2 = \{2, 4\}$, (2) systems with antialigned nonprecessing spins such that the final black-hole spin was the same as that for the corresponding nonspinning binary for $(q, \chi_{\rm fin}) = (2, 0.62)$, (3, 0.54), and (4, 0.47), using the final-spin fits in Refs. [3,9] and (3) four $q = 2$ precessing binaries having equal initial spins with $(x, y, z)$ components equal to (0.2, 0, 0), (0, 0.4, 0), (0.6, 0, 0) and (0.2, 0.2, 0.1), where the orbital plane lies on $xy$. There were a total of 40 configurations, not including additional tests to verify that the results were robust against changes in the number of inspiral orbits.

All simulations were performed with the BAM code [10]. As is standard, the error bars in the amplitudes were estimated by varying the numerical resolution and GW extraction radius. The highest resolution near the black holes was $\sim m/35$, where $m$ is the mass of the smallest black hole, and the GW signal was typically calculated at $140M_{\rm in}$ from the source. The ringdown amplitudes $A_{\ell m}$ were computed by fitting an exponential decay function to the data from $t = 10M$ after the peak of the (2, 2) luminosity, until the point where the signal was dominated by numerical noise. $A_{22}$ and $A_{21}$ are typically accurate to within 2%, and $A_{33}$ and $A_{32}$ to within 10%. The weaker modes are too noisy to be measured accurately, and are shown only for qualitative comparison.

Figure 1 shows the results for the first set of simulations, of equal spin binaries. The amplitudes of the seven strongest modes ($A_{\ell m} = A_{\ell -m}$ for nonprecessing binaries) are plotted as a function of a total spin parameter $\chi_+ = (m_1\chi_1 + m_2\chi_2)/M_{\rm in}$, where $M_{\rm in} = m_1 + m_2$ and $\chi_+ = \chi_i$ for these cases. This is the same spin parameter that has been used in recent phenomenological models of binary waveforms [11,12]. The amplitudes are all relative to the 22 mode, for which we show the absolute amplitude.

We see immediately that $A_{22}$ and $A_{33}$ change with mass ratio, but vary only weakly with respect to spin. In contrast, $A_{21}$ varies strongly with spin. Figure 1, therefore, suggests that the 22 and 33 modes carry information about the progenitor mass ratio, and the 21 mode carries information about the effective total spin.

The second series of simulations tests this hypothesis. For each mass ratio, this set generates approximately the same final black hole with different progenitor spin configurations. The goal was to show that the mode amplitudes carried a signature of the progenitor spins independently of the final black-hole spin. The mode amplitudes for the $q = 2$ case are shown in the left panel of Fig. 2, as a function of $\chi_+$. As before, 22 and 33 show little variation, but the 21 mode changes by nearly a factor of five. This is strong evidence that the final black holes in this set are *not* really degenerate: although their mode frequencies and damping times will be identical, they will differ from one another in the 21 mode amplitude. This is consistent with studies of black-hole recoil: the recoil is mostly due to the interplay of the (2, ±2) and (2, ±1) modes [13], and both the recoil and (2, ±1) mode amplitudes depend strongly on the progenitor spins.

Unfortunately, the trend of 21 is now the opposite of that in Fig. 1 with respect to $\chi_+$, implying that the 21 mode amplitude is not determined by $\chi_+$. Consider instead the effective spin parameter

$$\chi_{\rm eff} = \frac{1}{2}(\sqrt{1-4\nu}\chi_1 + \chi_-), \qquad \chi_- = \frac{m_1\chi_1 - m_2\chi_2}{M_{\rm in}}.$$

The right panel of Fig. 2 shows the amplitude of 21 as a function of $\chi_{\rm eff}$ for all the simulations discussed so far. In *all* cases they are well approximated by

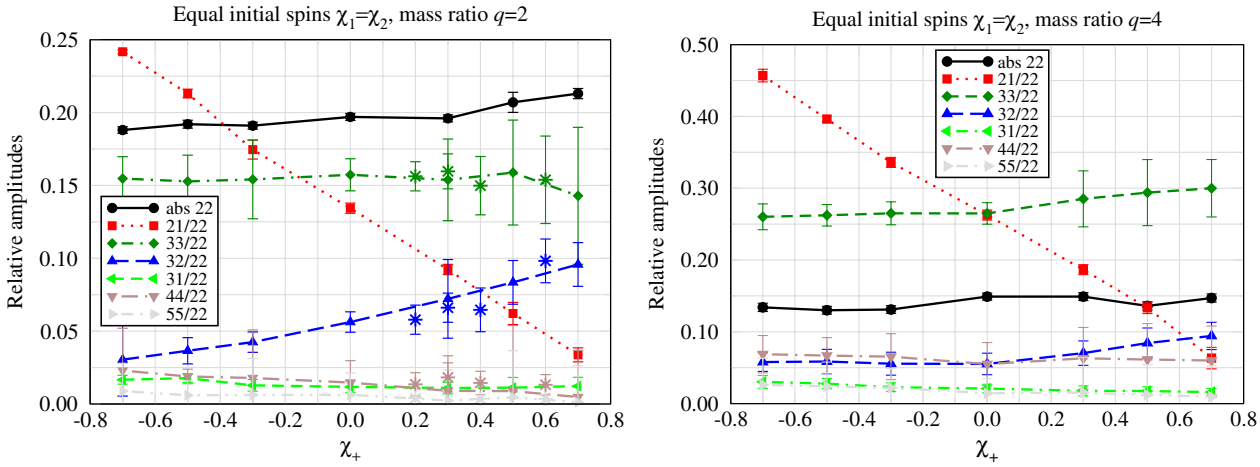


FIG. 1 (color online). Quasinormal mode amplitudes of binaries with aligned spins and mass ratio $q = 2$ (or $\nu = 2/9$, left panel) and $q = 4$ (or $\nu = 4/25$, right panel). The values from the nonspinning binary simulations are at $\chi_+ = 0$. Also shown in the left panel, with asterisks, are the results from the $q = 2$ equal initial $\chi_i$ precessing simulations. Note that for the 22 mode, the absolute amplitudes are always shown, scaled according to the final black-hole mass, that is $(r/M)h_{22}$.

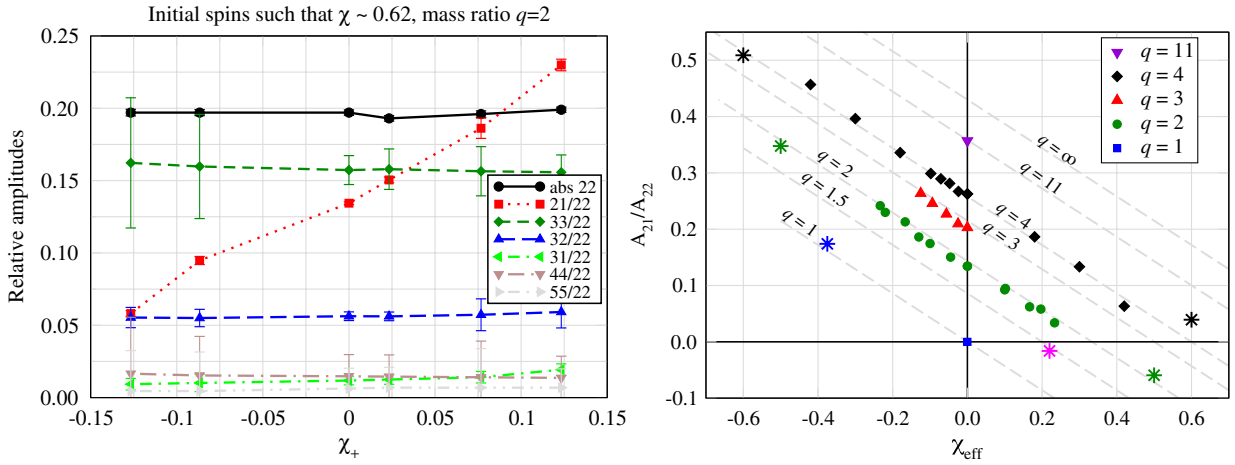


FIG. 2 (color online). Left panel plots the amplitude of the various modes as a function of the total spin parameter $\chi_+$ for the $q = 2$ simulations that end in a black hole of $\chi \simeq 0.62$. Modes 22, 33 are again rather insensitive to progenitor spins, while 21 varies by nearly a factor of 5. Right panel plots the 21 amplitudes from all simulation sets as a function of an effective spin term $\chi_{\rm eff}$ allowing us to estimate this parameter from a measurement. We verified our predictions with additional simulations marked with asterisks.

$$\hat{A}_{21} \equiv A_{21}/A_{22} = 0.43[\sqrt{1-4\nu} - \chi_{\rm eff}], \qquad (1)$$

which is shown by dashed lines in Fig. 2 for different values of $q$. The above equation is consistent with the expectation that $A_{21}$ will be excited in the case of equal mass binaries when $\chi_1 \neq \chi_2$, and also predicts that in general it will be zero when $\chi_{\rm eff} = \sqrt{1-4\nu} = |m_1 - m_2|/M_{\rm in}$. We tested these predictions with six additional simulations, shown in Table I. The predicted amplitudes $\hat{A}^P_{21}$ agree with the computed amplitudes $\hat{A}^M_{21}$ within error bars. Negative values indicate that the 21 phase is offset by 180° with respect to the 22 phase; in the equal-mass cases this is equivalent to swapping $\chi_1$ and $\chi_2$, or rotating the initial data by half an orbit.

All of these results apply to nonprecessing binaries: the progenitor spins and final spin were all parallel or antiparallel to the binary's orbital angular momentum. This will not be true in general; the spins and orbital plane will precess during the inspiral, and the final black hole's spin will be misaligned with respect to the premerger orbital plane. Even if the ringdown modes were rotated into an optimal frame by a procedure like that introduced in Ref. [14], there would be an asymmetry between the $+m$ and $-m$ modes, since this is a signature of the out-of-plane recoil (see Sec. III.A in Ref. [15]). However, it is possible that if the ringdown modes were described in the optimal frame, then their *average* would satisfy the relations we have observed. To test this, we simulated four precessing binaries. In each case the final spin was misaligned with the initial orbital plane, but only slightly, so that to a first approximation we could still consider the average of the (2, ±2) and (3, ±3) modes. The results for these cases are shown in Fig. 1, and, remarkably, satisfy the same relations we have observed for nonprecessing binaries. This provides strong evidence that our results carry over to *generic* binaries.

TABLE I. Additional simulations to test Eq. (1).

| $q$ | 1 | 1.5 | 2 | 2 | 4 | 4 |
|---|---|---|---|---|---|---|
| $\chi_{\rm eff}$ | −0.375 | 0.220 | −0.500 | 0.500 | −0.600 | 0.600 |
| $\hat{A}^P_{21}$ | 0.161 | −0.005 | 0.358 | −0.070 | 0.516 | 0.000 |
| $\hat{A}^M_{21}$ | 0.174 | −0.016 | 0.348 | −0.059 | 0.509 | 0.039 |

*Interpretation.*—Post-Newtonian (PN) theory provides a clue to the behavior of the amplitudes of the various modes. It is quite possible that the various modes excited during the inspiral phase retain the memory of their structure through to the ringdown phase. (There are signs that this will be true from, e.g., Fig. 11 in Ref. [16] for nonspinning binaries.) It is, therefore, instructive to look at the inspiral mode amplitudes. In particular, the 21 mode reads [17]

$$h_{21} \propto \frac{\nu M_{\rm in} v^3}{D}\left(\sqrt{1-4\nu} - \frac{3}{2} v \chi_-\right). \qquad (2)$$

Here $v$ is the PN expansion parameter, namely the orbital speed. There are three points to note. First, for nonspinning systems, the 21 amplitude has *identical* dependence on the mass ratio during the inspiral and ringdown phases. Second, the spin terms in the 22 and 33 modes (indeed, all modes for which $l + m$ is even) appear at 1.5 PN order beyond the leading order and so spins have a negligible effect. For $v = 1/\sqrt{3}$, 22 and 33 vary by about ∼20% when $\chi_1$ and $\chi_2$ change from −0.8 to +0.8. However, for 21 (and all odd $l + m$ modes) the spin effect occurs at 0.5 PN order beyond the leading order; spins affect odd $l + m$ modes far more strongly than they do even $l + m$ modes. For $v = 1/\sqrt{3}$, the 21 mode varies by a factor of

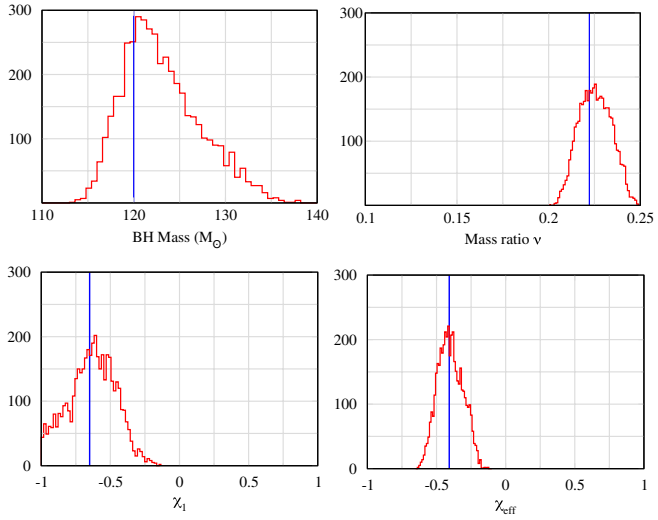


FIG. 3 (color online). Posterior distributions are plotted for a ringdown signal detected with the ET. The vertical lines correspond to the parameters of the signal injected into the ET mock data stream. The source is 1 Gpc from the detector; the signal to noise ratio is ~27.

4.5, and the 32 mode by 72%, when spins vary from $-0.8$ to $+0.8$. Finally, the dominant spin effect in the 21 mode amplitude is determined by the quantity $\chi_-$. It is really *not* the total spin that determines the amplitude, but the difference of spins, as in the ringdown phase.

*Measurement.*—To estimate how well the progenitor spins and mass ratio can be measured we injected a ringdown signal in background noise with power spectral density as expected in the Einstein Telescope, ET-B [18], and used Bayesian inference with nested sampling [19,20] to detect and measure its parameters. Our signal consists of the first three dominant modes, 22, 21, and 33, with the 21 mode amplitude given by Eq. (1) and for the 22 and 33 modes we took $A_{22} = 0.864\nu$, $\hat{A}_{33} = 0.44(1 - 4\nu)^{0.45}$. The signal and the template are both characterized by six parameters, $(M, \nu, \chi_1, \chi_2, D, t_0)$, where $t_0$ is the time of arrival of the signal at the detector. The angles describing the location of the source on the sky $(\theta, \varphi)$, the inclination $\iota$ of the binary and polarization angle $\psi$, are all assumed to be known. The azimuth angle $\phi$ and the initial phases of the various modes $\varphi_{\ell m}$ are all also assumed to be zero. These angles have strong correlations with the distance to the binary but not the intrinsic parameters. Thus, relaxing the above assumptions is not likely to have a big impact in the measurement of the intrinsic parameters of the source.

The posterior distributions for four of the parameters from one of our runs are plotted in Fig. 3, which show that the parameters of the progenitor can be quite accurately measured by using just the ringdown signal. A more detailed study is needed to fully characterize the measurement accuracies over the full parameter space, by incorporating other parameters such as the sky position of the source and its inclination, assumed to be known in this work.

*Discussion.*—In this Letter we have addressed a question implied in Ref. [2]: Can we measure the mass ratio of a generic binary from the ringdown signal alone? We have found two remarkable results. First, we *can* measure the mass ratio from the ringdown signal, and second, we may also be able to measure the individual black-hole spins. In other words, both the masses and spins of the two component black holes could be measured purely from the rapidly decaying perturbation that they leave on the final merged black hole.

The first result is demonstrated with a large numerical study of nonprecessing binaries to show that the ratio of the amplitudes of the $(\ell = 3, |m| = 3)$ and $(\ell = 2, |m| = 2)$ ringdown modes carry a clear signature of the mass ratio. Furthermore, we have evidence from a small set of precessing-binary configurations that this signature is retained in generic binaries. And finally, we have shown that this signature could be accurately measured in observations with the ET.

The second result is restricted to nonprecessing binaries. We found that the ratio of the $(\ell = 2, |m| = 1)$ and $(\ell = 2, |m| = 2)$ mode amplitudes depends on a certain difference between the individual black-hole spins. We produced a model of this spin dependence in terms of an effective spin parameter $\chi_{\rm eff}$, which is accurate across a wide sampling of the nonprecessing-binary parameter space. In a parameter-estimation exercise, where this model is injected into simulated ET noise, measurements of the final mass and spin, and of $\chi_{\rm eff}$, can be used in conjunction with a final-spin fit [3,9] to determine the individual black-hole spins.

Many questions remain open for future research. What is the physical origin of the observed ringdown spectrum? How do we fully model the ringdown signal from generic binaries? And, of most significance, what additional astrophysics will these results allow us to learn in third generation GW detectors, and how precisely will we be able to test general relativity?

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