Apparent Low-Energy Scale Invariance in Two-Dimensional Fermi Gases

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Recent experiments on a 2D Fermi gas find an undamped breathing mode at twice the trap frequency over a wide range of parameters. To understand this seemingly scale-invariant behavior in a system with a scale, we derive two exact results valid across the entire Bardeen-Cooper-Schrieffer–Bose-Einstein condensation (BCS–BEC) crossover at all temperatures. First, we relate the shift of the mode frequency from its scale-invariant value to $\gamma_d \equiv (1 + 2/d)P - \rho(\partial P/\partial \rho)_s$ in d dimensions. Next, we relate γ_d to dissipation via a new low-energy bulk viscosity sum rule. We argue that 2D is special, with its logarithmic dependence of the interaction on density, and thus γ_2 is small in both the BCS–BEC regimes, even though $P - 2\varepsilon/d$, sensitive to the dimer binding energy that breaks scale invariance, is not.

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Systems exhibiting scaling symmetry or conformal invariance are very special. In all laboratory realizations, one needs to tune one or more physical parameters (temperature, chemical potential, coupling) to observe scale-invariant behavior, for instance, in the vicinity of a quantum critical point [1]. Another example is provided by strongly interacting Fermi gases in three spatial dimensions (3D), which display remarkable scale invariance properties at unitarity, where the s-wave scattering length diverges by tuning to the Feshbach resonance. This is manifested in universal thermodynamics [2], the vanishing of the dc bulk viscosity [3], and the entire bulk viscosity spectral function $\zeta(\omega, T)$ [4] at unitarity. There may also be tantalizing connections between the ratio of the shear viscosity η to the entropy density s of the unitary Fermi gas [5,6] and the bound for η/s conjectured on the basis of gauge-gravity duality [7].

For the unitary gas in a 3D isotropic harmonic trap, scale invariance manifests itself most dramatically as an undamped monopole breathing mode oscillating at twice the trap frequency ω_0 independent of temperature [8,9]. This mode corresponds to an isotropic *dilation* of the gas wherein the coordinates in the many-body wave function are scaled $\propto \cos(\omega t)$. Scale invariance implies that this wave function is an exact eigenstate of the Hamiltonian and oscillates at a frequency $2\omega_0$ without damping [8].

In a recent experiment [10], collective modes in a two-dimensional (2D) Fermi gas were measured over a broad range of temperatures and interaction strengths. Remarkably, the breathing mode was found to oscillate without any observable damping at $\simeq 2\omega_0$ for $0.37 \lesssim T/T_F \lesssim 0.9$ and $0 \lesssim \ln(k_F a_2) \lesssim 500$, where a_2 is the 2D scattering length. This observation is extremely surprising, given that there is no *a priori* reason to expect scale-invariant behavior in a system which has a scale, namely, the dimer binding energy in 2D.

Our goal is to understand why the 2D Fermi gas appears to show nearly scale invariant behavior over a very broad range of parameters without the need for fine-tuning. Understanding this may give insight into related problems such as why, in some quantum field theories with conformal invariance broken by a mass term, the sound speed and bulk viscosity remain close to their conformal-limit values for a wide range of energies [11].

We emphasize that this question is distinct from that of small deviations from scale invariance in weakly interacting 2D Bose gases. Quantum gases with an unregularized delta-function interaction have an SO(2,1) symmetry [12] and exhibit scale invariance. However, the cutoff essential to describe an actual short-range interaction leads to a violation of scale invariance (analogous to an anomaly in quantum field theory) and an interaction-dependent shift in the breathing mode frequency from $2\omega_0$ [13] in a 2D Bose gas. The 2D Bose gas experiments that see nearly scale-invariant behavior are in the weakly interacting regime [14–16], where deviations are expected to be small. In contrast, the 2D Fermi case that we focus on is not weakly interacting and we must take into account strong interactions.

Results.—We begin by summarizing our approach and main results. We consider a dilute Fermi gas in $d = 2, 3$ dimensions with a short-range s-wave interaction, arising from a broad Feshbach resonance between two spin species, each with density $n/2$. The dimensionless interaction g_d is expressed as $g_3 = -1/k_F a_3$ in 3D and $g_2 = \log(k_F a_2)$ in 2D; a_d is the s-wave scattering length that sets the dimer binding energy $\varepsilon_b = -1/ma_d^2$, when $a_d > 0$. The fermions have mass m, density $n \sim k_F^d$, and we set $\hbar = 1$.

The quantity of central interest in our analysis is

$$\gamma_d \equiv (1 + 2/d)P - \rho(\partial P/\partial \rho)_s, \tag{1}$$

which is the deviation of the adiabatic compressibility $\rho(\partial P/\partial \rho)_s$ from its value $(1 + 2/d)P$ in a scale-invariant system, where the pressure $P \propto \rho^{(1+2/d)}$. Here $s = S/N$ is the entropy per particle and $\rho = mn$ the mass density.

First, we show that γ_d governs the difference between the frequency ω_m of the hydrodynamic monopole breathing mode and $2\omega_0$ for a Fermi gas in an isotropic harmonic trap $V_{\rm ext}(r) = m\omega_0^2 r^2/2$. We find

$$\omega_m^2/4\omega_0^2 = 1 - \frac{d^2}{8}\int d^d\mathbf{r}\gamma_d(r)\Big/\int d^d\mathbf{r}n(r)V_{\rm ext}(r). \quad (2)$$

Second, we show that γ_d is related to the exact sum rule for the bulk viscosity spectral function $\zeta(\omega)$:

$$(2/\pi)\int_0^\infty d\omega[\zeta(\omega) - C\zeta_0(\omega)/C_0] = \gamma_d. \quad (3)$$

Here, C is the "contact" [17,18] with $\zeta_0(\omega)$ and C_0 the bulk viscosity and contact in the zero-density $n \to 0$ limit at $T = 0$. The subtraction on the left-hand side removes the large-ω tail of $\zeta(\omega)$ (see Fig. 1) and the sum rule thus measures the availability of *low-energy* ($\lesssim |\varepsilon_b|$) spectral weight for excitations that break scale invariance.

The experimental observations of Ref. [10] in 2D Fermi gases imply that $\gamma_2 \ll n\epsilon_F$. Our goal is to understand *why* γ_2 is so small for a wide range of interaction strengths, even though other measures of the departure from scale invariance, such as $P - 2\varepsilon/d$ (where ε is the energy density), are *not* small. γ_d strictly vanishes only at the unitary point $g_3 = 0$ in 3D, and in the weak-coupling BCS limit $g_2 \to \infty$ in 2D. However, we will argue that there is considerable evidence for an anomalously small γ_2 across the entire BCS-BEC crossover. Remarkably, within mean field theory (MFT) [19] $\gamma_2^{\rm MF} = 0$ for all values of g_2 (and only in $d = 2$). In addition, the available $T = 0$ quantum Monte Carlo (QMC) data in 2D [20] leads to an estimate of γ_2 that is consistent with zero over the entire crossover, except possibly near $g_2 = 0$. We reach the same conclusion at finite T using a scaling argument, and argue that this is due to the logarithmic dependence of g_2 on density. Using the sum rule (3), we will argue that a small

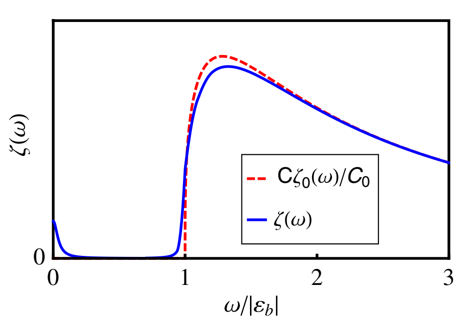

FIG. 1 (color online). Schematic plot of the bulk viscosity spectral function $\zeta(\omega)$ (solid line) and the scaled "vacuum" contribution $C\zeta_0(\omega)/C_0$ (dashed line) given by (6). The smallness of the subtracted sum rule (3) for $[\zeta(\omega) - C\zeta_0(\omega)/C_0]$ implies very little spectral weight below the dimer binding energy $|\varepsilon_b|$ for excitations that break scale invariance.

γ_2 also gives insight into the negligible viscous damping of the monopole mode.

Monopole breathing mode.—The normal mode solutions of the hydrodynamic equations with frequency ω are obtained from the Lagrangian [9]

$$\mathcal{L}[\boldsymbol{u}] = \omega^2\int d\mathbf{r}\rho_0\boldsymbol{u}^2(\mathbf{r}) - \int d\mathbf{r}[\rho_0^{-1}(\partial P/\partial\rho)_s(\delta\rho)^2 + 2\rho_0(\partial T/\partial\rho)_s\delta\rho\delta s + \rho_0(\partial T/\partial s)_\rho(\delta s)^2], \quad (4)$$

describing quadratic fluctuations in entropy δs and density $\delta\rho$ about their equilibrium values, s_0 and ρ_0. The displacement field $\boldsymbol{u}(\mathbf{r}, t)$ is related to the velocity $\mathbf{v}$ by $\partial\boldsymbol{u}/\partial t = \mathbf{v}$. Conservation of density and entropy gives $\delta\rho = -\boldsymbol{\nabla}\cdot(\rho_0\boldsymbol{u})$ and $\delta s = -\boldsymbol{u}\cdot\boldsymbol{\nabla}s_0$. Equation (4) is valid in both the normal as well as the superfluid phase, where it describes first sound with $\mathbf{v}_n = \mathbf{v}_s$ [9].

We obtain the result (2) for the breathing mode frequency using the scaling ansatz $\boldsymbol{u}(\mathbf{r}, t) = u\mathbf{r}\cos(\omega t)$ in (4), together with the Maxwell relation $(\partial P/\partial s)_\rho = \rho_0^2(\partial T/\partial\rho)_s$ and the equilibrium identities $\nabla P_0 = (\partial P/\partial\rho)_s\nabla\rho_0 + (\partial P/\partial s)_\rho\nabla s_0 = -n_0\nabla V_{\rm ext}$ for $V_{\rm ext} = m\omega_0^2r^2/2$, and $\nabla T_0 = (\partial T/\partial\rho)_s\nabla\rho_0 + (\partial T/\partial s)_\rho\nabla s_0 = 0$. The above scaling ansatz provides a rigorous upper bound on the mode frequency [9]. Generalizing the variational ansatz to $\boldsymbol{u} = \mathbf{r}\sum_{n=0}u_n r^{2n}\cos(\omega t)$, it is easy to show that the corrections to (2) are governed by higher powers of γ_d. Thus, γ_d rigorously determines the deviation of the monopole frequency ω_m from $2\omega_0$.

We next relate γ_d to the contact C, given by $C = 2\pi ma_2(\partial\varepsilon/a_2)_s$ in 2D [21,22] and $C = 4\pi ma_3^2(\partial\varepsilon/\partial a_3)_s$ in 3D [18]. We find $\gamma_2 = -[C + \frac{a_2}{2}(\partial C/\partial a_2)_s]/4\pi m$ and $\gamma_3 = -[C + a_3(\partial C/\partial a_3)_s]/36\pi ma_3$. This makes it clear that $\omega_m = 2\omega_0$ is strictly valid only for $a_2 \to \infty$, the BCS limit in 2D, where $C \to 0$, and at unitarity in 3D, where $|a_3| \to \infty$. On the other hand, the breathing mode frequency (2) is very sensitive to $\gamma_d \neq 0$ in both 2D and 3D. Using $\int d^d\mathbf{r}nV_{\rm ext} \sim \mathcal{O}(N\epsilon_F)$, we estimate that a value of γ_d as small as $0.1n\epsilon_F$ would give rise to a 5% shift in ω_m. The fact that no such shift is observed [10] in 2D indicates that we must understand why $\gamma_2 \ll n\epsilon_F$ for a wide range of g_2 and T.

Viscosity sum rules.—The bulk viscosity ζ is the only transport coefficient that damps the scaling flow $\boldsymbol{u} \propto \mathbf{r}$ [23]. To gain insight into why it is small in 2D, we derive a new bulk viscosity sum rule that relates γ_2 to the low-energy spectral weight for excitations that break scale-invariance symmetry.

The bulk viscosity spectral function $\zeta(\omega)$ is related by a Kubo formula to the transverse $\chi_T(\mathbf{q}, \omega)$ and longitudinal $\chi_L(\mathbf{q}, \omega)$ current correlators: $\zeta(\omega) = \lim_{q\to 0}m^2\omega[{\rm Im}\chi_L - (2 - 2/d){\rm Im}\chi_T]/q^2$. Generalizing Ref. [4] to arbitrary d, we obtain the exact sum rule

$$\frac{2}{\pi}\int_0^\infty d\omega\zeta(\omega) = -(2 - 2/d)X_T + X_L - \rho c_s^2. \quad (5)$$

Here, $X_{T(L)} = \lim_{q\to 0}\langle[\hat{j}^x_{-\mathbf{q}}, [\hat{H}, \hat{j}^x_{\mathbf{q}}]]\rangle_{T(L)}/q^2$ with the current $\hat{j}^x_{\mathbf{q}} = \sum_{\mathbf{k}\sigma}[(2\mathbf{k}+\mathbf{q})_x/2m]\hat{c}^\dagger_{\mathbf{k}\sigma}\hat{c}_{\mathbf{k}+\mathbf{q}\sigma}$. The subscript $T(L)$ denotes the transverse (longitudinal) $q \to 0$ limit [24], and $c_s \equiv (\partial P/\partial\rho)_s^{1/2}$ is the adiabatic sound speed. Evaluating the commutators in (5) for an isotropic pair potential with range r_0, we find the 2D result $(2/\pi)\int d\omega\zeta(\omega) = 2\varepsilon - \rho c_s^2 + \alpha C/m + \beta C\ln\Lambda/m$. Here, $\Lambda = 1/r_0$ is an ultraviolet (UV) cutoff, and α, β are constants. In 3D [4], the terms proportional to C are of the form $\alpha C/ma_3 + \beta C\Lambda/m$.

The key insight that allows us to obtain physical results independent of Λ is that an UV divergence of precisely the same form must arise in the two-body problem. The sum rule for ζ_0 has the same form as above, but with energy density and contact replaced by their zero-density, $T = 0$ values, ε_0 and C_0, while $c_s = 0$ for $n \to 0$. The exact solution ζ_0 of the two-body problem can then be used to regularize the divergence in the many-body problem. The same idea underlies the standard replacement of the bare interaction with the two-particle s-wave scattering length in the study of dilute gases [25].

The $T = 0$, zero-density limit $\zeta_0(\omega)$ of the viscosity spectral function has an *exact* representation in terms of the sum of all particle-particle ladder diagrams with two external current vertices. These are the well-known [26] Aslamazov-Larkin, Maki-Thompson, and self-energy diagrams. We thus obtain the 2D result [27]

$$\zeta_0(\omega) = \frac{C_0}{4m\omega}\frac{\Theta(\omega - |\varepsilon_b|)}{\ln^2(\omega/|\varepsilon_b| - 1) + \pi^2}, \tag{6}$$

for $\omega > 0$, and $\zeta_0(-\omega) = \zeta_0(\omega)$. In 2D, there is a bound state for all values of the scattering length [19]. Thus, in the zero-density (single dimer) and temperature limit, $\varepsilon_0 = \varepsilon_b = -1/ma_2^2$ and $C_0 = 4\pi/a_2^2$. The absence of spectral weight in ζ_0 below $|\varepsilon_b|$ is due to the fact that the only excitations in this limit involve pair disassociation with a gap $|\varepsilon_b|$ at $T = 0$.

The UV divergences can now be removed by looking at the difference between the sum rule for the interacting many-body system and that for the $T = 0$, $n \to 0$ limit, scaled by (C/C_0). In 2D, we find $(2/\pi)\int d\omega[\zeta(\omega) - C\zeta_0(\omega)/C_0] = 2\varepsilon - \rho c_s^2 - 2C\varepsilon_0/C_0$. Using $C = 4\pi m(P - \varepsilon)$ and $\varepsilon_0/C_0 = -1/(4\pi m)$, we obtain (3) in 2D. The same methodology can also be used to obtain corresponding results for the bulk viscosity in 3D as well as the shear viscosity in any d [27].

Our main focus will be on (3), which quantifies the *low-energy* spectral weight in $\zeta(\omega)$ with the high-energy tail $C\zeta_0(\omega)/C_0$ subtracted out; see Fig. 1. However, we can also obtain the total spectral weight in $\zeta(\omega)$:

$$S_{2\mathrm{D}} \equiv \frac{2}{\pi}\int_0^\infty d\omega\zeta(\omega) = 3P - \varepsilon - \rho c_s^2 = -\frac{1}{8\pi m}\left(\frac{\partial C}{\partial g_2}\right)_s. \tag{7}$$

$S_{2\mathrm{D}} \geq 0$ for all g_2 [21], as required by $\zeta(\omega) \geq 0\,\forall\,\omega$ [4]. In Fig. 2, we plot $S_{2\mathrm{D}}$ as a function of $g_2 = \log(k_F a_2)$ using $T = 0$ QMC data [20] to evaluate the right-hand side of (7). Both $S_{2\mathrm{D}}$ and its 3D counterpart $S_{3\mathrm{D}}$ [4] (inset of Fig. 2 using the QMC data of Ref. [28]) are $\ll n\epsilon_F$ in the BCS region $g_d \gtrsim 1$, but become significantly larger on the BEC side.

Apparent scale invariance.—We now have all the results in hand to discuss deviations from scale invariance. So far, we have shown that γ_d controls the deviation of the monopole ω_m from $2\omega_0$ and also governs the availability of low-energy bulk viscosity spectral weight $\zeta(\omega)$. We can intuitively understand the exact relation (3) between a ζ-sum rule and the shift in the mode frequency as a Kramers–Kronig transform of the dc bulk viscosity $\zeta(0)$ that damps the monopole mode.

What, if anything, is special about 2D that leads to the strong experimental signatures [10] of scale invariance? We begin by addressing this question at $T = 0$ and then generalize to finite temperatures. The first clue comes from MFT, which in 2D has a transparent solution [19] across the entire $T = 0$ BCS-BEC crossover: $\varepsilon = n\epsilon_F/2 - n|\varepsilon_b|/2$. This leads to $P = n\epsilon_F/2$ and thus $\gamma_2^{\mathrm{MFT}} \equiv 0$ for *all* couplings g_2. Contrast this with the 2D MFT result $P - \varepsilon = n|\varepsilon_b|/2$, which is very small in the BCS regime but very large on the BEC side. This is our first hint of something we will see again: γ_d is small in part because it does not involve physics on the scale of the dimer binding energy, whereas $P - \varepsilon$ does.

To understand how quantum fluctuations beyond MFT affect the result for γ_2, we use $T = 0$ QMC data [20]. We find that the QMC-derived γ_2 is vanishingly small in both BCS and BEC regimes, and even for $g_2 \sim 0$, $\gamma_2 \sim 0$ (within large error bars) as shown in Fig. 3. We also see from this figure that the 2D result is quite different from the 3D case. The QMC estimate for γ_3 (using data from Ref. [28]), though quite small on the BCS side of the crossover, is large in the BEC region in 3D.

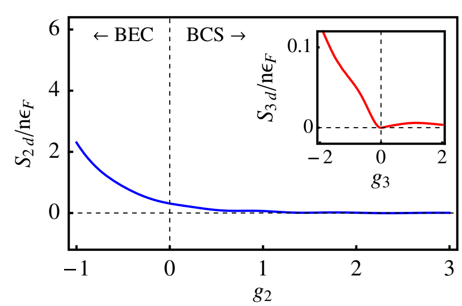

FIG. 2 (color online). *Bulk viscosity sum rule:*—The 2D sum rule $S_{2\mathrm{D}}$ at $T = 0$ in units of $n\epsilon_F$ plotted as a function of the coupling $g_2 = \ln(k_F a_2)$. Inset: The corresponding 3D result [4] as a function of $g_3 = -1/(k_F a_3)$. In both 2D and 3D, $g_d \to +\infty$, the BCS limit, while $g_d \to -\infty$ is the BEC limit.

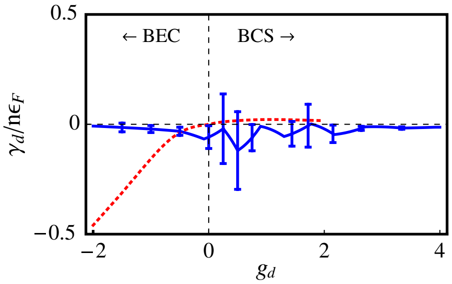

FIG. 3 (color online). γ_2 (solid blue line) and γ_3 (dashed red line) shown in units of $n\epsilon_F$, where n and ϵ_F are the two and three dimensional density and Fermi energy, respectively. The error bars on γ_2 are associated with numerical derivatives of QMC data (with errors) [20]. The coupling g_d is $\ln(k_F a_2)$ for $d = 2$ and $-1/(k_F a_3)$ for $d = 3$.

We now show that the difference between 2D and 3D is tied to the form of the dimensionless couplings $g_2 = \log(k_F a_2)$ and $g_3 = -1/k_F a_3$. In the BCS limit ($g_d \gg 1$), the equation of state has the form $\varepsilon = (n\epsilon_F/2)[1 + A/g_d + B/g_d^2 + \cdots]$ in both 2D and 3D. The perturbative Hartree plus "Fermi liquid" corrections are larger than the pairing contribution not shown. (The only qualitative difference is that A is negative in 3D but positive in 2D [29].) Both γ_d, calculated using (1), and $P - \varepsilon = (a_d/d)(\partial\varepsilon/\partial a_d)$, are small in the BCS limit. In the BEC limit ($g_d < 0$ and $|g_d| \gg 1$), we get $\varepsilon = -n|\varepsilon_b|/2 + \cdots$, which is the energy density of $n/2$ dimers with perturbative corrections in powers of $1/|g_d|$. The key difference between 2D and 3D is in the g_d-dependence of the binding energy $|\varepsilon_b|$, which $\sim \exp(|g_2|)$ in 2D and $\sim 1/|g_3|^2$ in 3D.

To understand the effects of finite temperature, we write the pressure and energy density, related by $P = n(\partial\varepsilon/\partial n)_s - \varepsilon$, in the scaling forms $P = n\epsilon_F \mathcal{F}(g_d, s)$ and $\varepsilon = n\epsilon_F \mathcal{E}(g_d, s)$. There is a qualitative difference between the g_2-dependence of the scaling functions $\mathcal{F}$ and $\mathcal{E}$ in 2D. The pressure does not have a contribution on the scale of the dimer binding energy $|\varepsilon_b| = 1/ma_2^2$; i.e., it does not have a potentially exponentially large contribution in $g_2 = \log(k_F a_2)$, while the energy density does. We have already seen this in the $T = 0$ MFT results, and the same is also observed in the 2D virial expansion [30]. We conjecture that the scaling function $\mathcal{F}$ is a slowly varying function of g_2 at all temperatures in 2D (except in the immediate vicinity of a weak singularity at T_c). The equation of state is then $P \sim n^2$ up to logarithmic corrections, leading to a small γ_2.

The absence of high energy contributions on the scale of the dimer binding energy to P and the compressibility is also consistent with γ_2 being related to the low energy spectral weight as shown by our sum rule. Once high energy excitations on the scale of $|\varepsilon_b|$ are excluded, low energy phonons (with a near scale-invariant dispersion $\omega_{\mathbf{q}}(n) \sim \sqrt{n}q$), for instance, dominate the equation of state leading to $P \sim n^2$ and a small γ_2. Unlike γ_2, however, $P - \varepsilon$ is not small, as it involves high energy contributions on the scale of $|\varepsilon_b|$ in the BEC regime.

Another way to characterize the deviation from scale invariance, analogous to the "trace anomaly" in quantum field theory, is to rewrite (1) as $\gamma_d = -(\partial P/\partial g_d)\beta(g_d)/d$, where $\beta(g_d) \equiv k_F(\partial g_d/\partial k_F)$ describes the scaling of the coupling g_d with respect to the momentum scale k_F. We see that $\beta(g_2) = 1$ while $\beta(g_3) = g_3$, reflecting the difference between the logarithmic and power-law dependence on the density in 2D and 3D, respectively. In both the BCS ($g_d \gg 1$) and BEC ($g_d \ll -1$) regions, the 2D beta function is much smaller than its 3D counterpart.

Finally, using the sum rules (7) and (3), we discuss the damping of the monopole mode, controlled by $\zeta(0)$. Although a small value for the sum rule by itself does not rigorously provide an upper-bound on ζ, any physically reasonable functional form for the spectral function (e.g., a Drude form for $\omega \lesssim |\varepsilon_b|$; see Fig. 1) would lead to a very small value for $\zeta(0)$. We see from Fig. 2 that in the BCS regime $g_d \gtrsim 1$, the sum rule $\int d\omega\zeta(\omega) \ll n\epsilon_F$ in both 2D and in 3D. We would thus expect a very small $\zeta \ll n$ here in both 2D and 3D. This sum rule is quite large on the BEC side and does not lead to any restriction on ζ. From the low-energy sum rule (3), however, we see that the large value of $S_{2\mathrm{D}}$ in the BEC limit is entirely dominated by the high-energy tail on scales larger than the dimer binding energy. Once this is subtracted out, the low-energy integrated spectral weight, equal to γ_2, is very small even in the BEC regime (see Fig. 1). Thus in 2D, we expect the bulk viscosity ζ to be very small both in the BCS and in the BEC regimes.

Conclusions.—We have shown that the parameter γ_d controls the deviation of the breathing mode frequency ω_m from its scale-invariant value $2\omega_0$ and also quantifies the low-energy spectral weight for excitations that break scale invariance, using an exact sum rule. We argue that 2D is special, with a coupling that depends logarithmically on density, leading to a very small γ_2, even in the BEC regime where scale invariance is strongly broken by the large dimer binding energy (and hence $P - \varepsilon$ is large). The small γ_2 also implies, via the 2D sum rule, weak damping of the monopole mode in both the BCS and BEC regimes. The regime very near $g_2 = 0$ deserves further theoretical and experimental investigation, but the available evidence suggests that γ_2 might be small there as well.

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Note added in proof.—While completing this manuscript we became aware of Ref. [31], which analyzes the experiment of Ref. [10] from the different perspective of quantum anomalies at $T = 0$. In the only area of substantial overlap, our general result (2) for the breathing mode reduces to that of Ref. [31] if we assume a polytropic equation of state.

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