Interface Phase Transition Induced by a Driven Line in Two Dimensions

Tridib Sadhu,* Zvi Shapira,† and David Mukamel‡

Department of physics of complex systems, Weizmann institute of science, Rehovot 76100, Israel
(Received 12 July 2012; revised manuscript received 19 August 2012; published 27 September 2012)

The effect of a localized drive on the steady state of an interface separating two phases in coexistence is studied. This is done using a spin-conserving kinetic Ising model on a two-dimensional lattice with cylindrical boundary conditions, where a drive is applied along a single ring on which the interface separating the two phases is centered. The drive is found to induce an interface spontaneous symmetry breaking whereby the magnetization of the driven ring becomes nonzero. The width of the interface becomes finite and its fluctuations around the driven ring are nonsymmetric. The dynamical origin of these properties is analyzed in an adiabatic limit, which allows the evaluation of the large deviation function of the magnetization of the driven ring.

DOI: 10.1103/PhysRevLett.109.130601 PACS numbers: 05.70.Np, 05.50.+q, 05.70.Ln

The effect of local drive on the properties of an interface separating two coexisting phases has recently been explored as a simple example of systems driven away from equilibrium. Much of the attention is due to the surprising experimental results on a colloidal gas-liquid interface subjected to a shear flow parallel to the interface [1]. It was found that the shear drive applied away from the interface strongly suppresses the fluctuations of the interface, making it smoother. This long-distance effect of the drive is due to long-range correlations that characterize driven systems [2–8]. An interesting theoretical approach for studying this phenomenon has been introduced by Smith *et al.* [9], who considered a two-dimensional version of the system, and modeled it by an Ising lattice gas below its transition temperature. Using spin-conserving Kawasaki dynamics and applying shear flow at the boundaries parallel to the interface, it was observed that the interface indeed becomes narrower, although its width still increases with the length of the interface. In closely related studies, the effect induced by a current-carrying line on a neighboring nondriven one has also been analyzed [10–13].

In this Letter, we consider a drive localized along an interface that separates two coexisting phases, and study the resulting interface properties. This is done using a 2D Ising model on a square lattice with cylindrical boundary conditions (Fig. 1) that evolves under spin-conserving dynamics. The drive acts along the ring around which the interface is centered. We find that the drive induces an interface phase transition, which involves spontaneous symmetry breaking, resulting in a nonzero magnetization of the driven ring. In this transition, the macroscopic 2D steady state remains unchanged; however, spontaneous symmetry breaking takes place, involving the steady state of a 1D stripe centered on the driven ring. This is in sharp contrast with the equilibrium setup of an interface subjected to a localizing potential along a ring, where the ring magnetization vanishes at all temperatures and no interface spontaneous symmetry breaking takes place. Moreover, we find that the drive suppresses the fluctuations of the interface, leading to an interface with a finite width that does not scale with the system size. Also, due to the broken symmetry on the driven ring, the interface fluctuations are highly asymmetric. The interface fluctuates more strongly into the bulk phase whose magnetization is opposite to that of the driven ring. These results are first demonstrated using numerical simulations. The model is then analyzed in a special limit, which allows an analytical computation of the large deviation function (LDF) [14] of magnetization of the driven ring, demonstrating the existence of spontaneous symmetry breaking.

To proceed, we consider Ising spins $\boldsymbol{\sigma} \equiv \{\sigma_{\mathbf{r}}\}$ on sites $\mathbf{r} \equiv (x, y)$ of an $L \times (2M + 1)$ square lattice, with periodic boundary condition in the x-direction, while the two open boundaries $y = \pm M$ are coupled to rows from above $(y = M + 1)$ and below $(y = -M - 1)$, respectively, with fixed spins $\sigma_{x,\pm M\pm 1} = \mp 1$ (Fig. 1). The model has nearest-neighbor ferromagnetic interactions, and a drive is introduced by a force field $\mathbf{E} \equiv (E, 0)$, applied on the $y = 0$ ring. The field favors the positive spins to move counterclockwise along the ring, and as a result drives the system out of equilibrium.

We consider a modified Metropolis algorithm [3] where, in every step, a pair of nearest-neighbor sites $\mathbf{r}$ and $\mathbf{r}'$ are

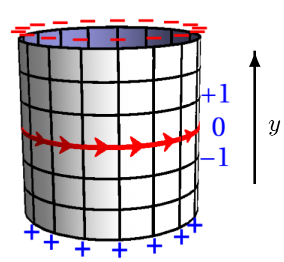

FIG. 1 (color online). Square lattice with cylindrical boundary condition, with the drive on the central ring and the boundary conditions indicated.

chosen at random, and their spins are exchanged with probability $\min\{1, \exp(-\beta\Delta\mathcal{H})\}$, where β is an inverse temperature and $\Delta\mathcal{H}$ is the energy difference between the final and initial configurations. Thus, for exchanging $\sigma_{\mathbf{r}}$ and $\sigma_{\mathbf{r}'}$,

$$\Delta\mathcal{H} = \begin{cases} \Delta H - (\sigma_{\mathbf{r}} - \sigma_{\mathbf{r}'})(\mathbf{r}' - \mathbf{r})\cdot\mathbf{E} & \text{if } \mathbf{r}, \mathbf{r}' \in 0\text{th ring}, \\ \Delta H & \text{elsewhere}, \end{cases}$$

where ΔH is calculated using the Ising Hamiltonian $H = -J\sum_{\langle\mathbf{r},\mathbf{r}'\rangle}\sigma_{\mathbf{r}}\sigma_{\mathbf{r}'}$, with $J > 0$. One Monte Carlo time step is constituted of $L(4M+1)$ such updates. In all the numerical results presented in this Letter, we use a large driving field $E \geq 10J$.

In the absence of a driving field, the model is in equilibrium. At subcritical temperatures ($T < T_c \simeq 2.2692J/k_B$), the equilibrium state is composed of two oppositely magnetized phases, separated by an interface. For an initial configuration with zero overall magnetization, the magnetization profile in the y direction, $m_y \equiv 1/L\sum_x \sigma_{x,y}$, is antisymmetric with respect to $y = 0$. The interface fluctuates symmetrically around the driven ring, leading to zero magnetization on the ring, $m_0 = 0$. In the large L, M limit with fixed aspect ratio L/M, the width of the interface scales as $\sqrt{L}$ [15].

Introducing a drive does not modify the overall macroscopic structure of the steady state. As is naively expected, the steady state is still composed of two oppositely magnetized phases separated by a fluctuating interface around $y = 0$. However, numerical studies of the model reveal some profound changes in the structure of the interface itself. In particular we find the following: (a) In the thermodynamic limit, the magnetization of the driven line, m_0, is nonzero, taking one of two oppositely directed values. It thus breaks the $\sigma_{\mathbf{r}} \to -\sigma_{-\mathbf{r}}$ symmetry of the model. (b) The interface is localized around the driven line, and its width stays finite in the thermodynamic limit, and (c) the fluctuations of the interface into the two bulk phases are highly asymmetric.

In a typical microscopic configuration of the model, the driven line is predominantly occupied by either positive or negative spins representing its two possible ordered states. As the system evolves, the magnetization m_0 fluctuates around one of the nonzero values for a long time. It then switches to the oppositely magnetized state over a much shorter time scale, as shown in the inset of Fig. 2. The time the system spends in each of the two states is the same when averaged over a large number of switches.

The numerical result for the average time between two such successive switches, t_s, is shown in Fig. 2. The data suggest that t_s grows exponentially with L, with $t_s \sim \exp(0.06L)$. The data for each L are averaged over n number of switches that are observed in available computation time (n varies from around 12, 000 to 10 as L changes from 30 to 130); n decreases with L, yielding increasing error bars of order $1/\sqrt{n}$ with L. Although the range of the system size studied is insufficient for conclusive evidence of exponential growth, this form is justified by the theoretical results presented below. The exponential growth implies that in the thermodynamic limit, the two nonzero values of m_0 correspond to two thermodynamically stable phases.

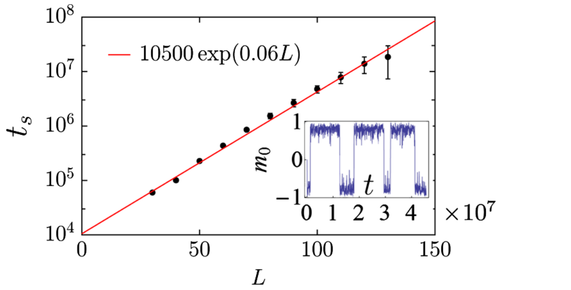

FIG. 2 (color online). The average time between consecutive switches $t_s(L)$ in m_0 for $2M = L$ and $T = 0.6T_c$. A typical time evolution of m_0 for $L = 2M = 100$ is shown in the inset, where time is measured in Monte Carlo steps.

The width of the interface is evaluated by averaging $|y|$ weighted by the derivative $-dm_y/dy$ that peaks at the interface position. The result is shown in the inset of Fig. 3 for both the driven and nondriven cases. A comparison of the two cases clearly indicates that the interface fluctuations are drastically reduced in the presence of drive. As will be shown by the theoretical analysis presented below, the width of the interface remains finite at large L. Such a smoothening of the interface has also been observed in the presence of global drive parallel to the interface [16]. An interesting difference here is that the interfacial fluctuations are asymmetric, resulting in an asymmetric magnetization profile around the driven line (see Fig. 3).

In order to make an analytical analysis of the model feasible, we generalize the model by introducing a

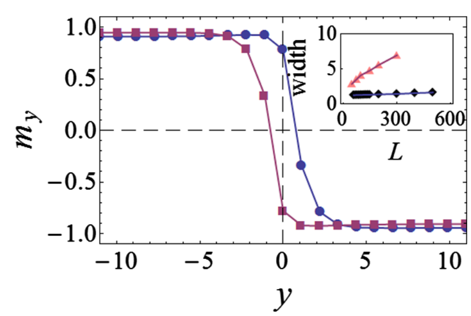

FIG. 3 (color online). The average magnetization profile m_y corresponding to the two phases, close to $y = 0$. The asymmetry around $y = 0$ is clearly seen. The profiles are generated on a 100×101 lattice at $T = 0.85T_c$, averaging over 10^5 configurations at regular intervals of 1000 Monte Carlo steps. The figure in the inset shows the growth of the width of the interface with increasing length L for zero drive (top curve) and for driving strength $E = 10J$ (lower curve).

parameter γ that controls the dynamical rate of the processes involving spin exchange between the $y = 0$ ring and the neighboring rings $y = \pm 1$. For these processes, the rate becomes $\min\{\gamma, \gamma \exp(-\beta\Delta\mathcal{H})\}$, with $\gamma > 0$. The other rates remain unchanged. This does not modify the steady state of the equilibrium case ($E = 0$), but it helps analyzing the nonequilibrium steady state. We now consider the steady state in the following special limit: (a) slow exchange rates ($\gamma \ll L^{-3}$) between the driven and the neighboring rings, (b) an infinite driving field ($E \to \infty$), and (c) low temperature ($\exp(-\beta J) \ll 1$). We show below that in this limit the stationary probability distribution $P(m_0)$ of the magnetization m_0 of the driven line has the form $P(m_0) \propto \exp(-L\phi(m_0))$. The LDF $\phi(m_0)$ is then computed and shown to possess two degenerate minima at nonvanishing values of the magnetization $m_0 = \pm m_0^\star$ (see Fig. 4), implying a spontaneous symmetry breaking on the ring.

We proceed by noting that due to the slow exchange rate γ, there are no significant exchanges between the driven line and its neighboring rings on a time scale $t \ll \Delta t = (\gamma L)^{-1}$. On such a time scale, the lattice may be considered as composed of three subsystems: the driven line, and the upper u ($y > 0$) and lower ℓ ($y < 0$) sublattices. They evolve while keeping their own specific magnetization m_0, m_u, and m_ℓ unchanged, reaching the steady state corresponding to fixed subsystem magnetization. On a longer time scale, $t \gtrsim \Delta t$, the magnetizations m_0, m_u, and m_ℓ evolve as spins are exchanged between the subsystems.

We now define a coarse-grained time variable $\tau = t/\Delta t$ such that the subsystem magnetization evolves with increasing τ; however, at any given τ each subsystem is effectively in the steady state corresponding to its magnetization. This separation of slow and fast processes is analogous to the adiabatic approximation in quantum mechanics [17], and has also been applied in related models [18,19].

Let us characterize the steady states corresponding to fixed subsystem magnetization m_0, m_u and m_ℓ. First, consider the driven ring. In the limit $E \to \infty$, the dynamics within this ring is independent of the two other subsystems

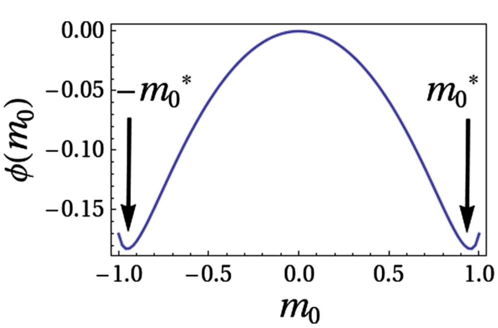

FIG. 4 (color online). The LDF $\phi(m_0)$ calculated using Eq. (3) and (6).

and reduces to that of the totally asymmetric simple exclusion process (TASEP). In its steady state, all spin configurations with fixed magnetization m_0 are equally probable, leading to uniform magnetization and zero spin-spin correlation along the driven line. This steady state is reached in a time of $\mathcal{O}(L^{3/2})$ which, for $\gamma \ll L^{-3}$, is smaller than the typical time of exchange processes between the driven ring and its neighboring ones [20]. Then, the driven ring provides an effective boundary magnetic field Jm_0 on the u and ℓ subsystems. Thus, the steady state of these two subsystems is the equilibrium state of the Ising model subjected to a boundary field. The boundary field results in a magnetization profile m_y which, for large $|y|$, approaches the bulk magnetization values $-m_B$ and m_B for the u and ℓ subsystems, respectively. The length scale of this approach is of the order of the spin-spin correlation length $\xi(T)$ of the 2D Ising model. Since this length is finite at all temperatures except at T_c, it demonstrates that the width of the interface remains finite for large L.

Let $P_\tau(m_0)d\tau$ be the probability of the driven line magnetization to have the value m_0 between coarse-grained time τ and $\tau + d\tau$, while m_u and m_ℓ have already reached stationary values. The probability function evolves as the spins are exchanged between the subsystems. At each exchange process between the driven ring and the bulk, m_0 changes by $\pm 2/L$. Let $p(m_0)$ and $q(m_0)$ be the increasing and decreasing rates of m_0, respectively. Then, the dynamics of m_0 is that of a random walker with position-dependent forward and backward jump rates $p(m_0)$ and $q(m_0)$, respectively, and with boundary condition $P_\tau(m_0) = 0$ for $|m_0| > 1$.

The stationary distribution of this motion is an equilibrium distribution function,

$$P(m_0) = P(0)\exp[-L\phi(m_0)]. \tag{1}$$

The LDF $\phi(m_0)$ is an even function of m_0, and it can be determined using the detailed balance condition $p(m_0)P(m_0) = q(m_0 + 2/L)P(m_0 + 2/L)$, which for $0 < m_0 \leq 1$ yields

$$P\left(m_0 = \frac{2n}{L}\right) = P(0)\prod_{k=1}^{n}\frac{p[\frac{2}{L}(k-1)]}{q[\frac{2}{L}k]}, \tag{2}$$

with $n = 1, \ldots, L/2$. In the large L limit, this yields the LDF for $m \geq 0$,

$$\phi(m_0) = \frac{1}{2}\int_0^{m_0} dm \ln\left[\frac{p(m)}{q(m)}\right], \tag{3}$$

with $\phi(-m_0) = \phi(m_0)$.

The rates $p(m_0)$ and $q(m_0)$ are determined as follows: Consider a spin exchange process between the driven ring and its two neighboring ones, in which the microscopic configuration changes from $\boldsymbol{\sigma}$ to $\boldsymbol{\sigma}'$ and m_0 increases by $2/L$. The rate of this process is

$\omega(\boldsymbol{\sigma} \to \boldsymbol{\sigma}')P(\boldsymbol{\sigma}|m_0, -m_B, m_B)$, where $\omega(\boldsymbol{\sigma} \to \boldsymbol{\sigma}') = \min\{1, \exp(-\beta\Delta H)\}$ is the Metropolis success rate in coarse grain time variable τ and $P(\boldsymbol{\sigma}|m_0, -m_B, m_B)$ is the steady-state probability of configuration $\boldsymbol{\sigma}$ corresponding to subsystem magnetization m_0, $m_u = -m_B$ and $m_\ell = m_B$. Summing over all such exchanges, one obtains

$$p(m_0) = \sum_{\boldsymbol{\sigma},\boldsymbol{\sigma}'} \omega(\boldsymbol{\sigma} \to \boldsymbol{\sigma}')P(\boldsymbol{\sigma}|m_0, -m_B, m_B), \qquad (4)$$

where the sum is over configurations $\boldsymbol{\sigma}'$ whose m_0 is higher than that of $\boldsymbol{\sigma}$ by $2/L$. The magnetization decreasing rate $q(m_0)$ can be readily obtained by noting that because of the invariance of the dynamics to space-time inversion $\sigma_{\mathbf{r}} \to -\sigma_{-\mathbf{r}}$, one has $q(m_0) = p(-m_0)$.

In the slow exchange limit $\gamma \ll L^{-3}$, the probability $P(\boldsymbol{\sigma}|m_0, -m_B, m_B)$ can be expressed in terms of the probability of the subsystem configurations as

$$P(\boldsymbol{\sigma}|m_0, -m_B, m_B) \simeq P(\boldsymbol{\sigma}_0|m_0)P(\boldsymbol{\sigma}_u| - m_B, m_0) \times P(\boldsymbol{\sigma}_\ell|m_B, m_0), \qquad (5)$$

where $\boldsymbol{\sigma}_0$, $\boldsymbol{\sigma}_u$, and $\boldsymbol{\sigma}_\ell$ are the microscopic spin configurations of the three subsystems corresponding to the configuration $\boldsymbol{\sigma}$. Here, $P(\boldsymbol{\sigma}_0|m_0)$ is the steady-state distribution of the driven line with fixed magnetization m_0, which is the same as the steady state of a totally asymmetric simple exclusion process, and $P(\boldsymbol{\sigma}_u| - m_B, m_0)$ and $P(\boldsymbol{\sigma}_\ell|m_B, m_0)$ are the equilibrium distributions of the other two subsystems.

In general, calculating all the terms in Eq. (4) is not straightforward. However, the calculation becomes feasible in the low temperature limit where these rates may be expanded in powers of $\exp(-\beta J)$. In order to keep track of the terms in this expansion, it is convenient to generalize the model by considering an interaction strength between the driven ring and its neighboring ones as $J_1 \leq J$. It is easy to see that the leading contribution to $p(m_0)$ in Eq. (4) results from the exchange process shown in Fig. 5, where both the subsystems u and ℓ are in their respective ground state, $m_\ell = -m_u = 1$. For this process, $\omega(\boldsymbol{\sigma} \to \boldsymbol{\sigma}') = \exp[-2\beta(J + J_1)]$ and $P(\boldsymbol{\sigma}|m_0, -m_B, m_B) = [(1 + m_0)^2 \times (1 - m_0)/8][1 - \mathcal{O}(e^{-6\beta J})]$. Higher-order contributions can be determined similarly from other exchange events. Computing $p(m_0)$ up to $\mathcal{O}(\exp(-6\beta J))$ yields, for $-1 \leq m_0 \leq 1$,

$$\begin{aligned} p(m_0) = {} & \frac{1}{8}[(1 + m_0)^2(1 - m_0)e^{-2\beta J_1}]e^{-2\beta J} \\ & + \frac{1}{8}[(1 + m_0)^2(1 - m_0)(2e^{-2\beta J_1 m_0} + e^{2\beta J_1 m_0}) \\ & + 2(1 + m_0)(1 - m_0)^2(e^{-2\beta J_1} + e^{2\beta J_1 m_0}) \\ & + (1 - m_0)^3 e^{2J_1 m_0}]e^{-6\beta J} + \mathcal{O}(e^{-8\beta J}). \end{aligned} \qquad (6)$$

The LDF $\phi(m_0)$ calculated using the rate in Eq. (6) is plotted in Fig. 4 for $\beta J_1 = \beta J = 3/4$ ($T \simeq 0.6T_c$). This function has two minima, which correspond to the two thermodynamic phases with nonzero m_0. We expect the bimodality of the LDF to disappear below a critical nonvanishing value of E, which cannot be determined analytically with the current theoretical approach.

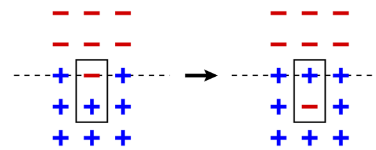

FIG. 5 (color online). Spin exchange process yielding the leading order contribution to $p(m_0)$. The driven ring is denoted by a dotted line, and the pair of spins exchanged are indicated.

In terms of Monte Carlo steps, the average magnetization switching time is given by $t_s \sim \gamma^{-1}\exp(\epsilon L)$, where ϵ is the height barrier of the LDF. For the parameters of Fig. 4, one has $\epsilon = 0.18$, which is of the same order as that obtained numerically in Fig. 2. For a better comparison, higher-order terms in the low-temperature expansion are required.

The analysis presented in this Letter demonstrates that a local drive can induce a phase transition that involves spontaneous symmetry breaking of an interface separating two coexisting phases. In a forthcoming publication, we report studies of the model with periodic boundary conditions in both the x and y directions. In this case, the model exhibits two interfaces, and our studies have shown that either one of them is attracted by the driven ring, resulting in a macroscopic symmetry breaking, in addition to that of the interface [21]. This demonstrates that drive can attract and localize a fluctuating interface.

We thank A. Bar, O. Cohen, M. R. Evans, O. Hirschberg, S. N. Majumdar, A. Maciołek, and S. Prolhac for helpful discussions. The support of Israel Science Foundation (ISF) is gratefully acknowledged.

*tsadhu@gmail.com
†zvi.shapira@weizmann.ac.il
‡david.mukamel@weizmann.ac.il

[1] D. Derks, D. G. A. L. Aarts, D. Bonn, H. N. W. Lekkerkerker, and A. Imhof, Phys. Rev. Lett. **97**, 038301 (2006).
[2] T. Sadhu, S. N. Majumdar, and D. Mukamel, Phys. Rev. E **84**, 051136 (2011).
[3] S. Katz, J. L. Lebowitz, and H. Spohn, Phys. Rev. B **28**, 1655 (1983); J. Stat. Phys. **34**, 497 (1984).
[4] H. Spohn, J. Phys. A **16**, 4275 (1983).
[5] M. Q. Zhang, J. S. Wang, J. L. Lebowitz, and J. L. Vallés, J. Stat. Phys. **52**, 1461 (1988).
[6] P. L. Garrido, J. L. Lebowitz, C. Maes, and H. Spohn, Phys. Rev. A **42**, 1954 (1990).
[7] I. Pagonabarraga and J. M. Rubí, Phys. Rev. E **49**, 267 (1994).

[8] B. Schmittmann and R. Zia, in *Statistical Mechanics of Driven Diffusive System, Phase Transitions and Critical Phenomena*, edited by C. Domb and J. Lebowitz (Academic Press, London, 1995) Vol. 17.
[9] T. H. R. Smith, O. Vasilyev, D. B. Abraham, A. Maciołek, and M. Schmidt, Phys. Rev. Lett. **101**, 067203 (2008).
[10] R. Dickman and R. R. Vidigal, J. Stat. Mech. (2007) P05003.
[11] K. Tsekouras and A. B. Kolomeisky, J. Phys. A **41**, 465001 (2008).
[12] A. Hucht, Phys. Rev. E **80**, 061138 (2009).
[13] H. J. Hilhorst, J. Stat. Mech. (2011) P04009.
[14] H. Touchette, Phys. Rep. **478**, 1 (2009).
[15] D. B. Abraham and P. Reed, Phys. Rev. Lett. **33**, 377 (1974).
[16] K.-t. Leung, K. K. Mon, J. L. Vallés, and R. K. P. Zia, Phys. Rev. Lett. **61**, 1744 (1988); Phys. Rev. B **39**, 9312 (1989).
[17] A. Messiah, *Quantum Mechanics* (North-Holland, Amsterdam, 1962), 1st ed., Chap. XVII, p. 750.
[18] H. van Beijeren and L. S. Schulman, Phys. Rev. Lett. **53**, 806 (1984).
[19] O. Cohen and D. Mukamel, Phys. Rev. Lett. **108**, 060602 (2012).
[20] O. Golinelli and K. Mallick, J. Phys. A **37**, 3321 (2004).
[21] Z. Shapira, M. Sc. thesis, Weizmann Institute of Science, 2012.