Majorana Modes in Driven-Dissipative Atomic Superfluids with a Zero Chern Number

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We investigate dissipation-induced p-wave paired states of fermions in two dimensions and show the existence of spatially separated Majorana zero modes in a phase with vanishing Chern number. We construct an explicit and natural model of a dissipative vortex that traps a single of these modes, and establish its topological origin by mapping the problem to a chiral one-dimensional wire where we observe a nonequilibrium topological phase transition characterized by an abrupt change of a topological invariant (winding number). We show that the existence of a single Majorana zero mode in the vortex core is intimately tied to the dissipative nature of our model. Engineered dissipation opens up possibilities for experimentally realizing such states with no Hamiltonian counterpart.

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The search for topological phases of matter in which elementary excitations exhibit non-Abelian statistics has brought two-dimensional (2D) p-wave paired superfluids and superconductors to the forefront of theoretical and experimental condensed-matter research [1–5]. The bulk of these systems is fundamentally intriguing in that it reveals physics beyond the Landau paradigm: different phases are characterized by distinct values of a nonlocal, *topological* order parameter known as the Chern number, and phase transitions occur whenever the topology changes, signaled by discontinuities in this integer-valued topological invariant. In topologically nontrivial phases corresponding to an *odd* Chern number, vortices with odd vorticity have been predicted to carry *unpaired* Majorana fermions, and to exhibit non-Abelian exchange statistics as a result [6].

In this Letter, we explore the concept of topological order and its connection to the edge physics in a nonequilibrium scenario based on engineered dissipation [7,8]. Prior work on quantum-state engineering in driven-dissipative systems has shown that topologically nontrivial states of many-body Hamiltonians can also be prepared as steady states of a dissipative dynamics [9]. In contrast, we demonstrate that dissipation can lead to a novel manifestation of topological order with no Hamiltonian counterpart. Specifically, we show that spatially separated Majorana zero modes (MZMs) can be obtained in a 2D, dissipation-induced p-wave paired phase of spin-polarized fermions with *vanishing* Chern number. Remarkably, a phase whose topological nature is seemingly trivial—according to the standard diagnostic tool provided by the Chern number—can therefore exhibit phenomenological features characteristic of a nontrivial one, which ultimately leads to the counterintuitive fact that vortices with odd vorticity may obey non-Abelian exchange statistics in a bulk with zero Chern number.

We demonstrate these results in a simple model motivated by an implementation scheme based on cold atoms and optical vortex imprinting [10], where fermion parity is microscopically conserved. We show that they hold over an extended parameter range, in which we identify a topologically nontrivial phase missed by Chern number considerations. Critical points are revealed upon introducing a vortex, in which case we establish the phenomenology of a nonequilibrium topological phase transition characterized by (i) a discontinuity in a topological invariant (winding number), (ii) divergent length and time scales [8,11–13], and (iii) a divergent localization length associated with a MZM bound to the vortex core.

The basic mechanism behind our findings relies on the fact that the introduction of a (dissipative) vortex changes the system in two crucial respects: it modifies its topology, as argued in Refs. [14,15], and imposes specific (dissipative) boundary conditions. Here we show that our model of a vortex with odd vorticity can be mapped to a 1D chiral fermion problem [16] characterized by a nontrivial topological invariant (winding number) $\nu_{1D} = 2$ despite a vanishing bulk Chern number. In such a situation, bulk-edge correspondence arguments [4,17–19] suggest the existence of a *pair* of MZMs in the vortex core. However, owing to the dissipative boundary conditions imposed by the geometry of the vortex core alone—which underpins the universal nature of our findings—a *single* MZM only is found in the core. This phenomenon crucially relies on dissipation, and therefore has no Hamiltonian counterpart. It shows that the potentially harmful effect of dissipation on MZMs [20–22] need not be entirely destructive, but may instead give rise to intriguing novel effects.

Dissipative framework.—We consider a system of N fermionic sites $a_i^\dagger$, a_i evolving under a purely dissipative dynamics governed by a Lindblad master equation

$$\partial_t \rho = \kappa \sum_{i=1}^{N} \Big(L_i \rho L_i^\dagger - \frac{1}{2}\{L_i^\dagger L_i, \rho\}\Big), \qquad (1)$$

where ρ is the system density matrix, κ the damping rate, and L_i are Lindblad operators which are linear in the fermionic operators. The steady state of such dynamics is pure if and only if the Lindblad operators form a set of anticommuting operators, i.e., $\{L_i, L_j\} = 0$ for all i, $j = 1, \ldots, N$. If so, it can be identified with the ground state of the parent Hamiltonian $H_{\text{parent}} \equiv \sum_i L_i^\dagger L_i$. Although purity is not required for the existence of topological order [9], we will only encounter steady states that are pure in the bulk, whose (bulk) topological properties can be inferred from H_{parent} alone. In our dissipative setting, the analog of a gap in a Hamiltonian spectrum is a *dissipative* gap in the Liouvillian spectrum, which *dynamically* isolates the subspaces corresponding to the bulk and edge modes, thereby providing the counterpart of a gap protection through the quantum Zeno effect [23]. Most importantly, the counterpart of topological ground-state degeneracy is the existence of a nonlocal decoherence-free subspace associated with zero-damping Majorana modes $\gamma = \gamma^\dagger$ which satisfy the orthogonality condition $\{L_i, \gamma\} = \{L_i^\dagger, \gamma\} = 0$ for all i (see Supplemental Material [24]). Clearly, this condition is more restrictive than the one for a zero-energy Majorana mode of a Hamiltonian, which reads (for H_{parent}) $[H_{\text{parent}}, \gamma] = \sum_i (L_i^\dagger \{L_i, \gamma\} - \{L_i^\dagger, \gamma\} L_i) = 0$. The crucial difference stems from the first "recycling" term on the right-hand side of Eq. (1), and is the reason why dissipation can crucially modify the Majorana physics found in the Hamiltonian context.

The model.—We consider a square-lattice system driven by so-called "cross" Lindblad operators defined as the following quasilocal linear superposition of fermionic creation and annihilation operators $a_i^\dagger$, a_i:

$$\begin{aligned} L_i &\equiv C_i^\dagger + \alpha e^{i\phi} A_i \\ &= \beta a_i^\dagger + (a_{i_1}^\dagger + i a_{i_2}^\dagger + i a_{i_3}^\dagger + i a_{i_4}^\dagger) \\ &\quad + \alpha e^{i\phi}(a_{i_1} + i a_{i_2} - i a_{i_3} - i a_{i_4}), \end{aligned} \qquad (2)$$

where $\beta \in \mathbb{R}$, $\alpha > 0$, $\phi \in [0, 2\pi)$, and i_1, i_2, i_3 and i_4 are the four clockwise-ordered nearest-neighboring sites of i. The creation and annihilation parts of L_i have s- and p-wave symmetries, respectively. The dissipative dynamics generated by such Lindblad operators can be obtained, for example, as the long-time limit of a microscopically number-conserving (quartic) dissipative dynamics generating phase-locked paired states (see Supplemental Material [24]). In that context, the global relative phase ϕ between the creation and annihilation parts of L_i emerges through spontaneous breaking of the global $U(1)$ symmetry, and the relative strength α is determined by the average particle number. The dimensionless parameter β, on the other hand, can be used to tune the system across phase transitions, as will be shown below.

The steady-state bulk properties of the system are most easily revealed in the infinite-size limit. Defining coefficients u_{ij}, v_{ij} such that $L_i = \sum_j (u_{ij} a_j + v_{ij} a_j^\dagger)$, the momentum-space cross Lindblad operators take the form $L_{\mathbf{k}} = u_{\mathbf{k}} a_{\mathbf{k}} + v_{\mathbf{k}} a_{-\mathbf{k}}^\dagger$, where $u_{\mathbf{k}}$, $v_{\mathbf{k}}$ are the Fourier transforms of u_{ij}, v_{ij}. Properly normalized, they become Bogoliubov quasiparticle operators associated with damping rates $\kappa_{\mathbf{k}} = \kappa \mathcal{N}_{\mathbf{k}}$, where $\mathcal{N}_{\mathbf{k}} = \xi_{\mathbf{k}}^\dagger \xi_{\mathbf{k}}$ with $\xi_{\mathbf{k}}^T = (u_{\mathbf{k}}, v_{\mathbf{k}})$. One can easily verify that the cross Lindblad operators form a complete set of anticommuting operators, so that the system is driven into a unique and pure complex p-wave paired state $|\Omega\rangle$ defined by the condition $L_{\mathbf{k}}|\Omega\rangle = 0$ (for all $\mathbf{k}$) and fully characterized by the real vector $\mathbf{n}_{\mathbf{k}} = \xi_{\mathbf{k}}^\dagger \boldsymbol{\sigma} \xi_{\mathbf{k}}$, where $\boldsymbol{\sigma}$ is a vector of Pauli matrices. Since (i) this steady state is pure (i.e., $|\mathbf{n}_{\mathbf{k}}| = 1$ for all $\mathbf{k}$) and (ii) time-reversal symmetry is broken due to the complex p-wave nature of the state [25], the topological invariant relevant to that case coincides with the Chern number $\nu_{2\text{D}}$ commonly used in 2D Hamiltonian systems (see e.g., Ref. [26]). Here we find that the Chern number vanishes [27], namely,

$$\nu_{2\text{D}} \equiv \frac{1}{4\pi} \int_{\text{BZ}} d^2\mathbf{k}\, \mathbf{n}_{\mathbf{k}} \cdot (\partial_{k_x} \mathbf{n}_{\mathbf{k}} \times \partial_{k_y} \mathbf{n}_{\mathbf{k}}) = 0 \qquad (3)$$

(where BZ stands for "Brillouin zone") for all values of β except at the isolated points $\beta = 0$ and $\beta = \pm 4$ where the dissipative gap closes (see Supplemental Material [24]). Since there is no extended parameter range with nontrivial topological order, one naively expects topological features such as isolated MZMs to be absent when edges or vortices are introduced. We will show below that such conclusions are premature: single, unpaired MZMs are generally found in the parameter range $0 < |\beta| < 4$ when dissipative vortices with odd vorticity are introduced.

Physically, the special values $\beta = 0, \pm 4$ appear as critical points since the dissipative gap closes at these values, leading to divergent length and time scales characteristic of a second-order phase transition [12,13]. However, the symmetry of the steady state (encoded in $\mathbf{n}_{\mathbf{k}}$) and the value of the topological invariant $\nu_{2\text{D}}$ both are identical in the neighborhood of those points. It thus seems as if the apparent critical behavior can neither be traced to a conventional phase transition (with broken symmetry), nor to a topological one (with a discontinuity in the topological invariant). Below we will show that the introduction of a vortex makes it possible to define another topological invariant. This will allow us to identify $\beta = \pm 4$ as genuine critical points associated with topological phase transitions.

Introducing a dissipative vortex.—We now introduce a *dissipative* vortex, by modifying the annihilation part $A_i = \sum_j u_{ij} a_j$ of the above cross Lindblad operators. More specifically, we replace the translation-invariant coefficients

$u_{ij} \equiv \tilde{u}_{ij}$ considered so far by position-dependent coefficients of the form $u_{ij} = f(r_j)e^{-i\ell\varphi_j}\tilde{u}_{ij}$, where (r_j, φ_j) are polar coordinates defined with respect to a particular site i_0 chosen as the center of the vortex core. Two crucial ingredients appear in this definition: (i) a real, rotation-invariant *vortex profile function* $f(r)$ which goes to zero in the vortex core, and (ii) a vortex phase that winds ℓ times around the origin i_0 (ℓ being the *vorticity*). These properties of a dissipative vortex naturally arise in an implementation with ultracold atoms based on optical vortex imprinting (see Supplemental Material [24]). They are fully analogous to those of a vortex in a Hamiltonian scenario. In particular, a π flux is optically imprinted onto the matter system in the case $\ell = 1$.

In what follows, we show that an *odd* dissipative vortex (i.e., with odd vorticity) *generically* traps a single, isolated MZM despite the seemingly trivial topological nature of the bulk. We focus on the simplest case of a single vortex with $\ell = 1$ and proceed in two main steps. First, we demonstrate that the vortex phase crucially modifies the topology of the system and construct a mapping to an effective 1D model. Second, we examine the effect of the dissipative "boundary" conditions imposed in the vortex core by the profile function $f(r)$, and show that the latter are responsible for the existence of a single MZM in the core.

Mapping to a chiral 1D wire.—To exemplify how an odd dissipative vortex changes the topology of the system, we consider the case $\ell = 1$ and assume, without loss of generality, that the vortex profile function $f(r)$ vanishes as $r \to 0$ and satisfies $f(r) = 1$ for $r > r_c$. We identify the region $r < r_c$ as the vortex core. In order to capture the generic properties of the bulk, we first focus on an annular region centered around the vortex core [Fig. 1(a), left] where $f(r)$ is constant and the relevant p-wave operator carries the vortex phase, $e^{-i\varphi}(\partial_x + i\partial_y)$ (in the continuum limit, for simplicity). As made explicit in the Supplemental Material [24], our model is, in this region, formally equivalent to a cylinder model with a p-wave operator of the form $\partial_{x'} + i\partial_{y'}$ [(x', y') being Cartesian coordinates on the cylinder; see Fig. 1(a), right] as in the original translation-invariant model on the plane [see Eq. (2)]. There exists therefore a one-to-one correspondence between the original planar model with an $\ell = 1$ vortex and the same model in cylinder geometry with no vortex. Physically, this stems from the fact that the gauge field $e^{-i\varphi}$ imposed on the plane to describe the vortex naturally arises on the cylinder owing to the extrinsic curvature of the latter.

In order to further extract the essence of the single-vortex problem, we revert to the original lattice description and take advantage of the translation invariance along the y' direction. Defining Fourier-transformed Lindblad operators $L_{x'}(k_{y'}) \propto \sum_{y'} e^{-ik_{y'}y'} L_{x',y'}$, we reduce the system to a stack of 1D wires that can be investigated along the lines of Ref. [9]. In the two momentum sectors corresponding to $k_{y'} = 0$ and π, the relevant 1D Lindblad operators take the form

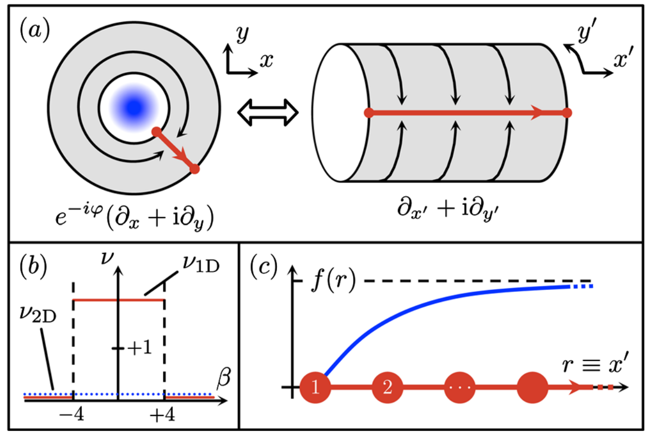

FIG. 1 (color online). Mechanism ensuring the existence of a *single* MZM in the core of an odd dissipative vortex. (a), Left: $\ell = 1$ vortex on the plane, characterized by a core (blue gradient) and a phase factor premultiplying the p-wave operator $\partial_x + i\partial_y$. Right: Mapping from the (grey) annular region around the vortex core to the cylinder—where the vortex phase disappears from the relevant p-wave operator—and reduction to a (chiral) 1D wire problem (thick red). (b) Topological phase diagram for the planar model of Eq. (2) with no vortex (dotted blue, characterized by the Chern number ν_{2D}) and with a single $\ell = 1$ vortex (red, characterized by the winding number ν_{1D}). (c) Dissipative boundary conditions of the 1D wire inherited from the vortex core profile (see text).

$$L_i = \beta' a_i^\dagger + (a_{i-1}^\dagger + a_{i+1}^\dagger) + (-a_{i-1} + a_{i+1}), \tag{4}$$

where i indexes the lattice sites in the x' direction [such that $L_i \equiv L_{x'}(k_{y'} = 0 \text{ or } \pi)$] and $\beta' \equiv \beta + 2$ $(\beta - 2)$ for $k_{y'} = 0$ (π). In these two particular sectors, the system thus reduces to a 1D wire with chiral symmetry [18]. The steady-state bulk properties of this wire can be unveiled in the infinite-size limit, in which case the Lindblad operators form a complete set of anticommuting operators, leading to a pure steady state described by a real unit vector $\mathbf{n}_k$ as above [see Eq. (3)]. Owing to chiral symmetry, this state can be characterized by a "winding number" topological invariant ν_{1D} [29]. As detailed in the Supplemental Material [24], we obtain

$$\nu_{1D} \equiv \frac{1}{2\pi}\int_{\rm BZ} dk\mathbf{a}\cdot(\mathbf{n}_k \times \partial_k \mathbf{n}_k) = 2 \tag{5}$$

or $|\beta'| < 2$ (i.e., for $0 < |\beta| < 4$), and $\nu_{1D} = 0$ otherwise ($\mathbf{a}$ being a unit vector orthogonal to $\mathbf{n}_k$ whose existence is guaranteed by chiral symmetry). We thus find nontrivial topological order in the parameter range delimited by the special points $\beta = 0$ and ± 4 where the Chern number ν_{2D} exhibits discontinuities. Figure 1(b) illustrates the resulting topological phase diagram: nonequilibrium topological quantum phase transitions occur at the critical values $\beta = \pm 4$, while $\beta = 0$ corresponds to an isolated point separating two topologically equivalent phases. In agreement with the full 2D model, the dissipative gap of the 1D wire closes at each of these values (see Supplemental Material [24]).

Dissipative boundary conditions.—The fact that we can identify a nontrivial topological invariant in an effective 1D model describing the region surrounding the vortex core strongly supports the existence of interesting "edge" physics inside the latter. The bulk steady state of the 1D wire, which is pure, can equivalently be described as the ground state of the parent Hamiltonian $H_{\text{parent}} = \sum_i L_i^\dagger L_i$. Bulk-edge correspondence arguments based on H_{parent} would therefore suggest the existence of $\nu_{1D} = 2$ MZMs at the edges of the wire—i.e., in particular, in the vortex core. In our general dissipative setting, however, bulk-edge correspondence arguments can only be formulated in the presence of a dissipative gap *and* in the absence of modes corresponding to subspaces in which the steady state is completely mixed, which we refer to as *purity zero modes* (see Supplemental Material [24]). Keeping in mind that the presence of *dissipative* boundary conditions can potentially give rise to such modes, we now proceed to examine the "edge" physics that emerges in the vortex core as a result of the dissipative boundary conditions imposed by the vortex profile function $f(r)$. To this end, we first extend the above mapping by reducing the inner radius of the annular region shown in Fig. 1(a) to zero. The resulting extended 1D wire is depicted in Fig. 1(c); its first, leftmost site corresponds to the center of the vortex core where $f(r)$ vanishes, as shown by the blue curve. The fact that $f(r)$ varies from site to site in the vortex core crucially leads to a violation of the purity condition $\{L_i, L_j\} = 0$ (for all i, j) which is satisfied in the bulk, as mentioned above. One can easily verify that the anticommutator $\{L_i, L_j\}$ increasingly deviates from zero for $|i - j| \leq 2$ upon approaching the left edge of the wire. As a consequence, the steady state increasingly loses purity and departs from the ground state of H_{parent} featuring a pair of MZMs. Remarkably, *one and only one* MZM survives at the edge of the wire—or, equivalently, in the vortex core—in the full parameter range $0 < |\beta| < 4$ associated with a nontrivial topological phase. This mode is explicitly constructed in the Supplemental Material [24], and is shown to be exponentially localized, on a characteristic length scale $\xi = \xi(|\beta'|) \sim 1/|\log(|\beta'|/2)|$ which diverges at $|\beta'| = 2$, i.e., at $\beta = \pm 4$. At these values which coincide with the critical points found above, the norm of the wave function associated with the MZM diverges with the length of the wire, while the dissipative gap closes in the bulk. This further confirms the onset of a nonequilibrium topological quantum phase transition. The second MZM naively expected in the vortex core, by contrast, acquires a finite damping rate and becomes an environmental degree of freedom with no correlations with the rest of the system. As shown in the Supplemental Material [24], any zero mode of H_{parent} which acquires a finite damping rate is effectively traced out of the system in steady state, *independently* of the initial conditions. We refer to such a mode with no

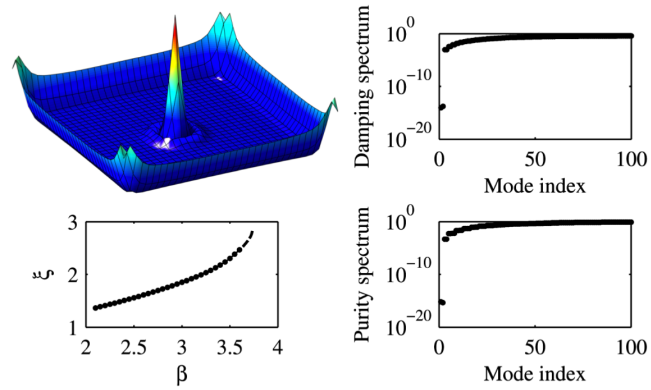

FIG. 2 (color online). Numerical results for a single dissipative vortex with $\ell = 1$ on a square lattice of 35×35 sites with unit spacing and $\kappa = 1$. Left: Typical form of the MZMs localized in the vortex core and on the edge, respectively (here for $\beta = 3$), and localization length scale ξ associated with the MZM trapped in the core. Right: Low-lying part of the damping and purity spectra for $\beta = 2$, both featuring a gap with two zero eigenvalues. All results were obtained for a vortex profile $f(r) = (r/d)e^{-(r/\xi)^2}$ with $d = 10$ and $\xi = 20$.

Hamiltonian counterpart as an *intrinsic* purity zero mode, in accordance with the fact that its existence is intrinsic to the dissipative dynamics itself.

The above findings are supported by extensive numerical simulations, confirming the existence of a single MZM trapped in the vortex core in the full parameter range $0 < |\beta| < 4$ for arbitrary vortex profile functions, as well as the divergence of the corresponding localization length upon approaching the critical points $\beta = \pm 4$ (see Fig. 2). As expected, an intrinsic purity zero mode is found in the vortex core. Similar features are obtained for odd vortices with $\ell > 1$; even vortices, in contrast, do not exhibit any MZMs, as expected from the fact that the vortex phase can be gauged away in that case (see, e.g., Ref. [1]). Our numerical results generally support the main conclusion that odd dissipative vortices generically trap single, isolated MZMs despite the seemingly trivial topological nature of the system. Following the arguments of Ref. [9], we expect such vortices to exhibit non-Abelian exchange statistics when braided around each other through adiabatic parameter changes.

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