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First Constraints on the Running of Non-Gaussianity

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We use data from the Wilkinson Microwave Anisotropy probe temperature maps to constrain a scaledependent generalization of the popular "local" model for primordial non-Gaussianity. In the model where the parameter $f_{\rm NL}$ is allowed to run with scale k, $f_{\rm NL}(k) = f_{\rm NL}^* (k/k_{\rm piv})^{n_{f_{\rm NL}}}$, we constrain the running to be $n_{f_{\rm NL}} = 0.30^{+1.9}_{-1.2}$ at 95% confidence, marginalized over the amplitude $f_{\rm NL}^*$. The constraints depend somewhat on the prior probabilities assigned to the two parameters. In the near future, constraints from a combination of Planck and large-scale structure surveys are expected to improve this limit by about an order of magnitude and usefully constrain classes of inflationary models.

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Introduction.-Non-Gaussianity in the distribution of we seek to constrain the two-parameter power-law subclass of the generalized models [26] $f_{\rm NL}(k) = f_{\rm NL}^* \left(\frac{k}{k}\right)^{n_{f_{\rm NL}}},$

where k_* is an arbitrary fixed parameter, leaving f_{NL}^* and $n_{f_{\rm NI}}$ as the parameters of interest in this model. Such scaling is expected in inflation when more than one field dominates or when there is self-interaction, and its signatures in the CMB and LSS have been discussed in the literature [25,26,30]. The parameter $n_{f_{NL}}$ is often, though certainly not always, expected to be $\leq O(1)$ in inflationary models, but in our phenomenological model it is allowed to take any value.

Bispectrum and f_{NL}^* estimator.—The primordial bispectrum of the $f_{\rm NL}(k)$ model from Eq. (1) is straightforward to calculate:

$$F(\vec{k}_1, \vec{k}_2, \vec{k}_3) = 2[f_{\rm NL}(k_1)P(k_2)P(k_3) + \text{perm.}], \quad (2)$$

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where the full bispectrum is $B(\vec{k}_1, \vec{k}_2, \vec{k}_3) \equiv (2\pi)^3 \delta(\vec{k}_1 +$ $\vec{k}_2 + \vec{k}_3 F(\vec{k}_1, \vec{k}_2, \vec{k}_3)$. Here *P* is the power spectrum of the primordial curvature perturbations, and δ is the Dirac delta function.

Constraining the running parameter $n_{f_{NL}}$ seems difficult because of the apparent requirement to find an estimator for a parameter in an exponent. To avoid this, we resort to an indirect approach where, for a *fixed* value of $n_{f_{NL}}$, we estimate the parameter $f_{\rm NL}^*$ using modifications of the wellknown KSW estimator [31], which is known to be nearly optimal [32,33]. We then iterate over the values of the running $n_{f_{\rm NL}}$ to obtain the full likelihood $\mathcal{L}(f_{\rm NL}^*, n_{f_{\rm NT}})$.

The theoretical expectation for the bispectrum of the temperature anisotropies in the cosmic microwave background can be explicitly evaluated, starting from the definition of the generalized non-Gaussian local model in Eq. (1) to account for the running $n_{f_{NT}}$:

primordial density fluctuations provides a unique window into the physics of inflation. The magnitude of primordial non-Gaussianity and its dependence on scale provide information about the dynamics of scalar field(s), their interactions, and the speed of sound during inflation. Constraints on non-Gaussianity have traditionally come from the measurements of the three-point correlation function of the cosmic microwave background (CMB) temperature anisotropies. Upper limits from COBE [1] have been improved by two orders of magnitude by the Wilkinson Microwave Anisotropy probe (WMAP) experiment [2]. Moreover, clustering of galaxies and galaxy clusters has also been identified as a powerful probe of non-Gaussianity [3], already leading to interesting constraints that are complementary in their information content to the CMB measurements.

So far, most attention has been devoted to the "local" model of primordial non-Gaussianity, where the primordial Newtonian potential $\phi(x)$ is modified with a quadratic term: $\phi = \phi_G + f_{\rm NL}(\phi_G^2 - \langle \phi_G^2 \rangle)$, where ϕ_G is a Gaussian potential [4]. The parameter $f_{\rm NL}$ is currently constrained to be 32 ± 21 by WMAP ([2]; see also Refs. [5,6]) and 28 ± 23 by the large-scale structure [7–9]. Several other non-Gaussian models have been constrained as well (e.g., Refs. [10,11]). However, the "running" with physical scale of these models, which may carry important information about the number of inflationary fields and their interactions [12–23], has not yet been constrained with current data (except for a very rough estimate of the angularmultipole dependence of $f_{\rm NL}$ [11] and implicit constraints on a braneworld-motivated model [24]). Such constraints have only been forecasted for future experiments [25–29]. Constraining the running of non-Gaussianity therefore presents a major new opportunity to probe inflationary physics and is just becoming feasible. In this Letter, we present the first such constraints.

Model.-In this work we consider a physically motivated generalization of the local model, where the parameter $f_{\rm NL}$ is promoted to a function of scale k. In particular,

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$$B_{\ell_{1}\ell_{2}\ell_{3}}^{\text{theory}}(f_{\text{NL}}^{*}, n_{f_{\text{NL}}}) = 2f_{\text{NL}}^{*}I_{\ell_{1}\ell_{2}\ell_{3}} \times \int_{0}^{\infty} r^{2}dr(\alpha_{\ell_{1}}(n_{f_{\text{NL}}}, r) \times \beta_{\ell_{2}}(r)\beta_{\ell_{3}}(r) + \text{perm.}),$$
(3)

where $I_{\ell_1 \ell_2 \ell_3}$ is the Gaunt integral and

$$\alpha_{\ell}(r) \equiv \frac{2}{\pi} \frac{1}{k_{\text{piv}}^{n_{f_{\text{NL}}}}} \int k^{2+n_{f_{\text{NL}}}} t_{\ell}(k) j_{\ell}(kr) dk \qquad (4)$$

$$\beta_{\ell}(r) \equiv \frac{2}{\pi} \int k^2 P_{\Phi}(k) t_{\ell}(k) j_{\ell}(kr) dk.$$
 (5)

Here, t_{ℓ} is the radiation transfer function, which can be calculated using CAMB [34]. Following KSW [31] we can define new, filtered maps $A(\hat{\mathbf{n}}, r)$ and $B(\hat{\mathbf{n}}, r)$,

$$A(\hat{\mathbf{n}}, r) \equiv \sum_{\ell, m} \alpha_{\ell}(n_{f_{\rm NL}}, r) \frac{b_{\ell}}{\tilde{C}_{\ell}} a_{\ell m} Y_{\ell m}(\hat{\mathbf{n}}), \qquad (6)$$

$$B(\hat{\mathbf{n}}, r) \equiv \sum_{\ell, m} \beta_{\ell}(r) \frac{b_{\ell}}{\tilde{C}_{\ell}} a_{\ell m} Y_{\ell m}(\hat{\mathbf{n}}).$$
(7)

Then, we write down the skewness $S(n_{f_{NI}})$:

$$S(n_{f_{\rm NL}}) \equiv \int r^2 dr \int d^2 \hat{\mathbf{n}} A(\hat{\mathbf{n}}, r) B^2(\hat{\mathbf{n}}, r), \qquad (8)$$

which requires $n_{f_{NL}}$ as input (through A), and does not require *a priori* knowledge of f_{NL}^* .

The observed CMB bispectrum is defined as $B_{\ell_1\ell_2\ell_3}^{\text{obs.}} = \langle a_{\ell_1m_1}a_{\ell_2m_2}a_{\ell_3m_3} \rangle$, and $S(n_{f_{\text{NL}}})$ therefore reduces to

$$S = \sum_{\ell_1 \le \ell_2 \le \ell_3} \frac{B_{\ell_1 \ell_2 \ell_3}^{\text{obs}} \tilde{B}_{\ell_1 \ell_2 \ell_3}^{\text{theory}} (f_{\text{NL}} = 1)}{\tilde{C}_{\ell_1} \tilde{C}_{\ell_2} \tilde{C}_{\ell_3}}, \qquad (9)$$

where $\tilde{B}_{\ell_1\ell_2\ell_3}^{\text{theory}} = b_{\ell_1}b_{\ell_2}b_{\ell_3}B_{\ell_1\ell_2\ell_3}^{\text{theory}}$, and b_{ℓ} is the beam transfer function.

We now define $F \equiv F(n_{f_{\rm NL}})$, the Fisher matrix for $f_{\rm NL}^*$, equivalent to the cumulative signal-to-noise squared of the theoretical bispectrum for $f_{\rm NL}^* = 1$

$$F(n_{f_{\rm NL}}) = \sum_{\ell_1 \le \ell_2 \le \ell_3} \frac{(\tilde{B}_{\ell_1 \ell_2 \ell_3}^{\text{theory}}(f_{\rm NL}^* = 1))^2}{\tilde{C}_{\ell_1} \tilde{C}_{\ell_2} \tilde{C}_{\ell_3}}.$$
 (10)

The theoretical expectation for $B_{\ell_1 \ell_2 \ell_3} \propto f_{\rm NL}^*$, so the cubic KSW estimator for $f_{\rm NL}^*$ is:

$$\hat{f}_{\rm NL}^* = \frac{S}{F}.$$
(11)

We used HEALPIX, by way of HealPy, to do the forward and backward spherical harmonic transforms required to obtain the *A* and *B* maps.

Cut-sky maps.—Equation (11) works well for a full-sky map, but a sky cut introduces a spurious non-Gaussian signal. To account for the masking of the CMB sky, we make the substitution $S \rightarrow S_{\text{cut}} = S/f_{\text{sky}} + S_{\text{linear}}$ [35]. S_{linear} is an addition to skewness from Eq. (8), calibrated to account for partial-sky observations:

$$S_{\text{linear}} = -\frac{1}{f_{\text{sky}}} \int r^2 dr \int d^2 \hat{\mathbf{n}} [A(\hat{\mathbf{n}}, r) \langle B_{\text{sim}}^2(\hat{\mathbf{n}}, r) \rangle_{\text{MC}} + 2B(\hat{\mathbf{n}}, r) \langle A_{\text{sim}}(\hat{\mathbf{n}}, r) B_{\text{sim}}(\hat{\mathbf{n}}, r) \rangle_{\text{MC}}].$$
(12)

The subscripted filtered maps A_{sim} and B_{sim} are created from Python-produced Monte Carlo realizations of the cut CMB sky; the brackets $\langle \rangle_{MC}$ indicate an average over all 300 Monte-Python maps. The simulated maps were produced using the prescription laid out in Appendix A of the WMAP5 paper [36]; the only difference (aside from using the WMAP7 cosmological model) is that we used a uniform weighting for the maps, rather than the slightly more complicated weighting given there, since it only gives a marginal improvement in estimating f_{NL} .

Likelihood evaluation.—To find the likelihood, we first find a χ^2 statistic for $f_{\rm NL}^*$, given a value of $n_{f_{\rm NL}}$. Taking the angular-averaged bispectrum $B_{\ell_1\ell_2\ell_3}$ as our observables, we have:

$$\chi^{2}(f_{\rm NL}^{*}, n_{f_{\rm NL}}) = \sum_{\ell_{1}\ell_{2}\ell_{3}} \frac{(B_{\ell_{1}\ell_{2}\ell_{3}}^{\rm obs} - f_{\rm NL}^{*} \tilde{B}_{\ell_{1}\ell_{2}\ell_{3}}^{\rm theory} (n_{f_{\rm NL}}, f_{\rm NL}^{*} = 1))^{2}}{\tilde{C}_{\ell_{1}}\tilde{C}_{\ell_{2}}\tilde{C}_{\ell_{3}}}$$
(13)

(Again, this works because the theoretical expectation for $B_{\ell_1 \ell_2 \ell_3} \propto f_{\rm NL}^*$.) Using Eqs. (9) and (10), we can rewrite χ^2 as

$$\chi^2(f_{\rm NL}^*, n_{f_{\rm NL}}) = F\left(f_{\rm NL}^* - \frac{S}{F}\right)^2 + \chi_0^2 - \frac{S^2}{F},\qquad(14)$$

where $\chi_0^2 \equiv \sum_{\ell_1 \ell_2 \ell_3} (B_{\ell_1 \ell_2 \ell_3}^{\text{obs}})^2 / (\tilde{C}_{\ell_1} \tilde{C}_{\ell_2} \tilde{C}_{\ell_3})$ is the goodnessof-fit parameter for the data with respect to the $f_{\text{NL}}^* = 0$ case. Note that the numerator of χ_0^2 is an observed quantity, and the denominator is based solely on the theoretical prediction for the power spectrum (as well as a few noise and beam parameters of WMAP). Therefore, χ_0^2 does not depend on f_{NL}^* or $n_{f_{\text{NL}}}$ at all. We can use the definition of \hat{f}_{NL}^* in Eq. (11) to rewrite the expression for χ^2 as follows

$$\chi^2(f_{\rm NL}^*, n_{f_{\rm NL}}) = F(f_{\rm NL}^* - \hat{f}_{\rm NL}^*)^2 + \chi_0^2 - (\hat{f}_{\rm NL}^*)^2 F.$$
(15)

For a fixed value of $n_{f_{\rm NL}}$, the χ^2 is, as expected, minimized in $f_{\rm NL}^*$ when $f_{\rm NL}^* = \hat{f}_{\rm NL}^*$, and one obtains $\chi^2_{\rm min}(n_{f_{\rm NL}}) = \chi_0^2 - (\hat{f}_{\rm NL}^*)^2 F$.

A more interesting task is to calculate the constraints when $n_{f_{\rm NL}}$ is allowed to vary. With an expression for $\chi^2(f_{\rm NL}^*, n_{f_{\rm NL}})$ in hand, we can write an expression for the likelihood, $\mathcal{L}(f_{\rm NL}^*, n_{f_{\rm NL}}) \propto \exp(-\chi^2/2)$ (dropping the constant term with χ_0^2)

$$\mathcal{L}(n_{f_{\rm NL}}, f_{\rm NL}^*) \propto \exp\left[-\frac{F(f_{\rm NL}^* - \hat{f}_{\rm NL}^*)^2}{2}\right] \exp\left[\frac{(\hat{f}_{\rm NL}^*)^2 F}{2}\right].$$
(16)

To marginalize over $f_{\rm NL}^*$ is also straightforward

$$\mathcal{L}(n_{f_{\rm NL}}) = \int \mathcal{L}(n_{f_{\rm NL}}, f_{\rm NL}^*) df_{\rm NL}^* \propto \frac{1}{\sqrt{F}} \exp\left[\frac{(\hat{f}_{\rm NL}^*)^2 F}{2}\right], \tag{17}$$

where, recall, $F(n_{f_{\rm NL}})$ is defined in Eq. (10).

WMAP7 constraints on $n_{f_{NL}}$.—Figure 1 shows the likelihood \mathcal{L} in the $n_{f_{NL}}$ - f_{NL}^* plane, as well as the likelihood for $n_{f_{NL}}$ alone, calculated from the WMAP7 temperature maps. We used a weighted and masked combination of the WMAP *V* and *W* band maps with the monopole and dipole subtracted, as recommended by the WMAP team [36]. To extract full information from WMAP maps, we used multipoles out to $\ell_{max} = 800$ for the sums in Eqs. (6), (7), and (10). We did not find a significant improvement between $\ell_{max} = 700$ and $\ell_{max} = 800$; we chose the higher value to be conservative in our analysis.

The quantity χ^2 is independent of our choice for k_{piv} , but the likelihood itself is not, since *F* is inversely proportional to $k_{piv}^{2n_{f_{NL}}}$. The true pivot scale favored by the data is the value of k_{piv} for which the errors in f_{NL}^* are uncorrelated with the errors in $n_{f_{NL}}$. We find this scale by using the likelihood to calculate the covariance matrix **C** between f_{NL}^* and $n_{f_{NL}}$

$$\mathbf{C}_{p_i, p_j} = \langle (p_i - \bar{p}_i)(p_j - \bar{p}_j) \rangle.$$
(18)

We can easily find the pivot value k_{piv} that diagonalizes the covariance matrix **C** (see, e.g., Ref. [27])

$$k_{\rm piv} = k_* \exp\left(-\frac{\mathbf{C}_{f_{\rm NL}^* n_{\rm f_{\rm NL}}}}{f_{\rm NL}^* \mathbf{C}_{n_{f_{\rm NL}}, n_{f_{\rm NL}}}}\right),\tag{19}$$

where k_* is the (arbitrary) pivot used initially, and f_{NL}^* is the corresponding value used in **C**. Despite the fact that k_* and f_{NL}^* show up in the expression, k_{piv} does not depend on them: it is a fixed number telling us roughly where the experiment has greatest power (and where normalization and running of $f_{\rm NL}(k)$ are precisely uncorrelated). We find that $k_{\rm piv}^{\rm WMAP7} \approx 0.064h$ Mpc⁻¹. The 68%, 95%, and 99% constraints on $f_{\rm NL}^*$ and $n_{f_{\rm NL}}$ are shown at the left panel of Fig. 1, assuming flat priors on $f_{\rm NL}^*$ and $n_{f_{\rm NL}}$ and $k_* = k_{\rm piv}^{\rm WMAP7} \approx 0.064h$ Mpc⁻¹.

Dependence on the prior.—As with most present-day cosmological measurements, the precise constraints depend on the prior probability on the parameters we are constraining. Even for a simple flat prior on $f_{\rm NL}^*$ and $n_{f_{\rm NL}}$, the actual effective prior depends on the *a priori* chosen pivot in wave number k_* . For example, a flat prior on $(f_{\rm NL}^*)^{(1)} \equiv f_{\rm NL}(k_{*,1})$ defined at some pivot $k_{*,1}$ corresponds to a nonflat prior on some $(f_{\rm NL}^*)^{(2)} \equiv f_{\rm NL}^*(k_{*,2})$ defined at some other pivot $k_{*,2}$, since $(f_{\rm NL}^*)^{(2)} \equiv (f_{\rm NL}^*)^{(1)} \times (k_{*,2}/k_{*,1})^{n_{f_{\rm NL}}}$. If we assume some alternate pivot $k_{*,2}$ but hold the flat prior in $f_{\rm NL}^*$, the contours in the $n_{f_{\rm NL}}$ - $f_{\rm NL}^*$ plane (left panel of Fig. 1) are stretched vertically by a factor of $(k_{*,2}/0.064h \, {\rm Mpc}^{-1})^{n_{f_{\rm NL}}}$.

We have experimented with different k-pivot values for a flat prior on $f_{\rm NL}^*$ and $n_{f_{\rm NL}}$. We have also investigated other possibilities, such as the prior that assigns equal weight to each decade in $|f_{\rm NL}^*|$ above 0.1 (so uniform in $\log(f_{\rm NL}^*)$, but cut off at the arguably lowest-ever observable value of $|f_{\rm NL}^*| = 0.1$ so that the total integrated likelihood is finite). We present the two aforementioned examples, showing constraints on $n_{f_{\rm NL}}$ marginalized over $f_{\rm NL}^*$, in the right panel of Fig. 1. In the end, we decide to quote results for the flat prior and the uncorrelated $k_{\rm piv}$ value from Eq. (19), which most closely follows priors to both non-Gaussian and other cosmological parameters applied in the literature.

Putting it all together, we can get the estimate for $n_{f_{\rm NL}}$ from the WMAP7 data for a flat prior on $f_{\rm NL}^*$ at the pivot $k_{\rm piv}$ from Eq. (19). The 68% (95%) confidence interval is



FIG. 1 (color online). Likelihood in the $n_{f_{NL}}$ - f_{NL}^* plane (left panel) and marginalized over f_{NL}^* (right panel). The principal constraints, shown in the left panel and with the bold blue curve on the right, correspond to the flat prior on f_{NL}^* at the pivot value where the constraints on f_{NL}^* and $n_{f_{NL}}$ are uncorrelated [see Eq. (19)]. In the right panel we also show the marginalized likelihood for $n_{f_{NL}}$ with a prior on f_{NL}^* that is uniform in $\log(f_{NL}^*)$ for $|f_{NL}^*| > 0.1$ and zero otherwise. The dashed curve in the left panel shows the quantity \hat{f}_{NL}^* , which is the best-fit value of the parameter f_{NL}^* for a fixed $n_{f_{NL}}$. See text for other details.



FIG. 2 (color online). Constraints propagated to $f_{\rm NL}(k)$. We also show several models that are reasonable fits to the data (all within the 99% confidence limit of the left panel of Fig. 1) to guide the eye as to how typical models from our ansatz behave.

$$n_{f_{\rm NL}} = 0.30^{+0.78(1.9)}_{-0.61(1.2)}.$$
 (20)

The current constraints are therefore fully consistent with no running, as Fig. 1 clearly indicates. Figure 2 shows the constraints in the $f_{\rm NL}(k)$ plane together with a few representative models allowed by the data.

Conclusions.—We have presented the first constraints on the scale-dependence of (any form of) non-Gaussianity using the WMAP7 data. The constraints are compatible with zero running, $n_{f_{\rm NL}} = 0$, with very mild (< 1-sigma) preference for a positive value of $n_{f_{\rm NL}}$. We will learn more soon: the Planck data and the data from upcoming large-scale structure surveys should be able to improve constraints on the running of non-Gaussianity by about an order of magnitude [25,28,29], thus shedding important new light on the physics of inflation.

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