Collisional Penrose Process near the Horizon of Extreme Kerr Black Holes

Michał Bejger,[1,*] Tsvi Piran,[2,†] Marek Abramowicz,[1,3,‡] and Frida Håkanson[4]

[1]*Copernicus Astronomical Center, ul. Bartycka 18, 00-716, Warszawa, Poland*
[2]*Racah Institute of Physics, The Hebrew University, Jerusalem 91904, Israel*
[3]*Department of Physics, University of Gothenburg, 412-96, Göteborg, Sweden*
[4]*Department of Physics, Chalmers Technical University, 412-96, Göteborg, Sweden*

Collisions of particles in black hole ergospheres may result in an arbitrarily large center-of-mass energy. This led recently to the suggestion [M. Bañados, J. Silk, and S. M. West, Phys. Rev. Lett. **103**, 111102 (2009)] that black holes can act as ultimate particle accelerators. If the energy of an outgoing particle is larger than the total energy of the infalling particles, the energy excess must come from the rotational energy of the black hole and hence, a Penrose process is involved. However, while the center-of-mass energy diverges, the position of the collision makes it impossible for energetic particles to escape to infinity. Following an earlier work on collisional Penrose processes [T. Piran and J. Shaham, Phys. Rev. D **16**, 1615 (1977)], we show that even under the most favorable idealized conditions the maximal energy of an *escaping particle* is only a modest factor above the total initial energy of the colliding particles. This implies that one should not expect collisions around a black hole to act as spectacular cosmic accelerators.

PACS numbers: 04.70.Bw

In a seminal paper, Penrose [1] suggested the possibility of extracting the rotational energy of a black hole (BH). Penrose, and later Penrose and Floyd [2], considered an infalling particle disintegration in the ergosphere of a Kerr BH. One of the particles produced in this process might be thrown into a negative energy (with respect to infinity) orbit, while the other one will have an energy E_{out} larger than the energy E_{in} of the infalling one. The energy excess arises eventually from the rotational energy of the BH. However, shortly afterwards it was shown [3,4] that in order to have $E_{out} > E_{in}$, the disintegration process must convert most of the rest-mass energy of the infalling particle to kinetic energy. Such a disintegration mechanism does not exist in nature for stable particles, rendering the original Penrose process irrelevant for realistic astrophysical situations.

In a collision between two particles, the center-of-mass (c.m.) frame the collisional energy can be mostly kinetic, and outgoing particles from within the ergosphere might easily have $E_{out} > E_{in}$, resulting in a collisional Penrose process. In fact further studies [5], showed that when two particles collide near the horizon, the c.m. energy $E_{c.m.}$ can be arbitrarily large (see Sec. IIG of Ref. [5]). This fact was recently used by Bañados *et al.* [6] in the context of collisional dark matter. It was shown that for the extremal (spin parameter $a = J/M = 1$, where J and M are BH angular momentum and mass, respectively) Kerr BH, and a collision at the outer horizon ($r \rightarrow r_h = M + \sqrt{M^2 - a^2}$), there are cases (when one of the colliding particles has a critical angular momentum) with $E_{c.m.}(r \rightarrow r_h) \rightarrow \infty$. Unbounded $E_{c.m.}$ was subsequently proposed as an energy source for a Planck-scale particle accelerator.

This idea attracted a lot of attention and resulted in numerous papers repeating the statement that the c.m. energy of colliding particles may grow limitlessly in circumstances different than those considered by Bañados *et al.* [6]. These variations include different colliding particles (e.g., charged, massless, spinning), different spacetimes (e.g., naked singularity, string, nonzero cosmological constant), gravity theories different from Einstein's general relativity, and different collision settings (e.g., plunging from innermost stable circular orbit, collisions at the inner Kerr horizon). There were also several attempts to provide a "simple explanation" for the infinite c.m. energy.

It seems that the main result of Ref. [6] was generally accepted and, when it was criticized, the criticism related to rather irrelevant issues, e.g., gravitational radiation, or self-gravity. Instead, here we present a more substantial criticism of the meaning and physical significance of the result of Ref. [6] based on the statement that, while particles may locally reach huge energies, one has to consider the question of whether they escape to infinity ([5]). This is not trivial: to be energetic, the collision has to take place extremely close to the BH which, however, impedes the particle's escape. In fact, given the geometry of a typical collision, one might expect that the most energetic particles will fall into the BH.

This question was touched on before in a few papers which followed Ref. [6]. These papers attempted to examine the likelihood of the collision products escaping to infinity [7–11]. Here, we go further and examine the maximal energy of particles which actually escape to infinity. These values, rather than the maximal available energy in the c.m. frame, should be the relevant ones for astrophysical considerations.

Specifically, we calculate the upper limit on the energy of an escaping photon which results from a collision between two infalling particles. The parameter phase space

is very large: it involves the energy, angular momentum and Carter's constant of the two infalling particles, the coordinates of the collision point, and the masses and directions of the outgoing particles. Rather then exploring the whole phase space, we examine the special case of two particles falling from rest at infinity and colliding in the ergosphere of a Kerr BH. The collisions take place in the equatorial plane of the Kerr BH and result in two photons which also move in the equatorial plane. Symmetry considerations suggest that collisions which take place in the equatorial plane would result in the most energetic particles and that photons will escape most easily from the vicinity of the BH; hence, the upper limit that we find here is the true upper limit for the energy of an escaping particle from a near-BH collision.

Following Ref. [5] we consider the Kerr metric in the standard Boyer-Lindquist coordinates. In these coordinates, the Kerr metric tensor $g_{\mu\nu}$ depends neither on time t, nor on azimuthal angle ϕ. This implies two Killing symmetries, $\mathcal{L}_\eta g_{\mu\nu} = 0 = \mathcal{L}_\xi g_{\mu\nu}$ given in terms of the two Killing vectors, which in the Boyer-Lindquist coordinates are $\eta^\nu = \delta_t^\nu$ and $\xi^\nu = \delta_\phi^\nu$. The equations of geodesic motion admit two nontrivial constants of motion, which may be expressed by the particle's (or photon's) four-momentum p^ν and the two Killing vectors: energy at infinity $E = -p_\nu \eta^\nu$, and angular momentum parallel to the BH axis $L = p_\nu \xi^\nu$. The third constant of motion, the Carter constant Q, vanishes for particles and photons moving in the equatorial plane $\theta = \pi/2$.

We will make use of two local frames of reference to study the physical properties of the collision. The first one is the locally nonrotating frame (LNRF, also called ZAMO, [12]), and the second one is the c.m. frame (c.m.). Each of these frames defines its own "comoving observer," who is at rest in his corresponding frame.

The family of LNRF observers has its trajectories orthogonal to the spacelike hypersurfaces $t = \text{const}$, and their four-velocities given by

$$N^\nu = e^{-\Phi}[\eta^\nu + \omega\xi^\nu], \tag{1}$$

with $\Phi = \ln[-(\eta^\nu \eta_\nu) - \omega(\xi^\nu \eta_\nu)]^{1/2}$, $\omega = -(\xi^\nu \eta_\nu)/(\xi^\mu \xi_\mu)$.

For two particles with the four momenta $p^\mu_{(1)} = m_{(1)} u^\mu_{(1)}$ and $p^\mu_{(2)} = m_{(2)} u^\mu_{(2)}$, the comoving observer in the c.m. frame has his four-velocity U^μ given by the two conditions: the first one is $U^\mu U_\mu = -1$, and the second one is

$$U^\mu = \frac{1}{E_{\text{c.m.}}}[p^\mu_{(1)} + p^\mu_{(2)}], \tag{2}$$

where the c.m. energy $E_{\text{c.m.}}$ equals

$$E_{\text{c.m.}} = \sqrt{m_{(1)}^2 + m_{(2)}^2 - 2m_{(1)}m_{(2)}g_{\mu\nu}u^\mu_{(1)}u^\nu_{(2)}}. \tag{3}$$

Here $m_{(i)}$ are the masses and $u^\mu_{(i)}$ are the four velocities of the two ($i = 1, 2$) particles.

The two photons resulting from a collision should have their four momenta given in the c.m. frame (2) by

$$p^\mu_{(3)} = \frac{E_{\text{c.m.}}}{2}[U^\mu + \tau^\mu], \tag{4}$$

$$p^\mu_{(4)} = \frac{E_{\text{c.m.}}}{2}[U^\mu - \tau^\mu], \tag{5}$$

where τ^μ is an (arbitrary) direction in the instantaneous 3-space of the c.m. observer (assumed here to be in the equatorial plane, $\theta = \pi/2$). The conserved energies "at infinity" of these photons are,

$$E_{(3)} = -\eta_\mu p^\mu_{(3)}, \qquad E_{(4)} = -\eta_\mu p^\mu_{(4)}, \tag{6}$$

and the respective angular momenta are

$$L_{(3)} = \xi_\mu p^\mu_{(3)}, \qquad L_{(4)} = \xi_\mu p^\mu_{(4)}. \tag{7}$$

The subsequent fate of the photon is decided by the point of collision with respect to the location of the turning points provided by the effective radial potential. In the Boyer-Lindquist coordinates, it reads

$$V_r = E^2 r^4 + (Era)^2 - r(r - 2M)L^2 - 4MrELa + 2Mr(Ea)^2. \tag{8}$$

For $V_r = 0$ one obtains a relation for the photon impact parameter $b = L/E$,

$$b_\pm = \frac{2Ma \pm \sqrt{r^4 - 2Mr^3 + (ar)^2}}{2M - r}. \tag{9}$$

In the case of $a = 1$ and for initially infalling photons, the escape conditions are such that $b > 2$. For initially outgoing photons, b must be greater than the maximum of $b_+ = -7$, located at $r = 4M$ (see Fig. 1).

We examine here a collision in the ergosphere between two massive ($m_{(1)} = m_{(2)} = 1$) test particles infalling from rest at infinity with $E_{(1)} = E_{(2)} = 1$. We first consider an extremal Kerr BH (spin parameter $a = 1$) and we set the angular momentum of one of the particles to $L_{(1)} = 2$, the critical L for this value of a. We then vary the point of collision (distances are measured in units of M, the BH's mass), as well as the initial angular momentum of the second particle, $L_{(2)}$.

We find that photons which actually escape the ergosphere with an energy gain, $E_{\text{out}} > E_{\text{in}}$, i.e., $E_{(3)} > E_{(1)} + E_{(2)}$, are all initially infalling, and are deflected just before plunging into the BH. Figure 2 depicts the maximal energy as a function of the point of collision for various values of $L_{(2)}$. When the collision approaches the horizon, $E_{(3)}^{\text{max}}/(E_{(1)} + E_{(2)})$ peaks at 1.295 at a slight distance from the horizon (which approaches the horizon as $L_{(2)} \to 2$) and then declines asymptotically to 1.093 on the horizon. We stress that in all these
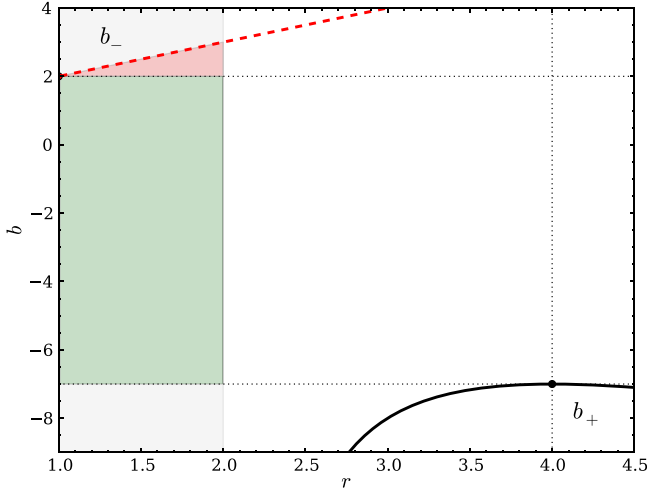
FIG. 1 (color online). Conditions of the photon impact parameter $b = L/E$ in case of $a = 1$, plotted as b_+ (solid curve) and b_- (dashed curve) functions (Eq. (9)). In order to escape the ergosphere, infalling photons must have $b > 2$; for initially-outgoing photons $b > -7$. Gray region denotes the ergosphere.

cases $E_{\text{c.m.}}$ diverges as the collision approaches the horizon. The situation is different at the limit $L_{(2)} = 2$. Now, the ratio $E_{(3)}^{\max}/(E_{(1)} + E_{(2)}) \to (1 + \sqrt{2})/2$ on the horizon. Since both particles fall on the same orbit (there is no real collision here) the situation corresponds to the case of particle decay [4,5]. The maximal energy of initially outgoing photons that escape the ergosphere is presented in Fig. 3. These are not Penrose particles, since $E_{(3)}^{\max}/(E_{(1)} + E_{(2)}) < 1$ (close to the horizon, the value approaches $1/(2 + \sqrt{2})$).

Figure 4 depicts the dependence of $E_{(3)}^{\max}$ on the spin parameter a. We keep $L_{(1)} = 2$ and vary $L_{(2)}$ and the position of the collision; other parameters are the same as for $a = 1$. As expected, the maximal energies are smaller than in the case of $a = 1$. The self-similar behavior shown in Fig. 2 is recovered for $L_{(2)} \to L_{(1)} = 2$ for $a \to 1$. Similarly, lowering the value of $L_{(1)}$ to values smaller than the critical angular momentum results in lowering the $E_{(3)}^{\max}$ (Fig. 5).

We find that, for the specific case considered, a collision between two massive particles falling from rest at infinity and colliding in the equatorial plane, the maximal energy of an escaping photon is ≈ 1.3 times the total initial energy at infinity of the infalling particles. While there is an energy gain and energy is extracted from the BH, the energy gain is very modest, in stark contrast to the diverging c.m. energy. Symmetry arguments suggest that for a configuration of two particles infalling from infinity, this is the maximal possible energy gain even if the collision is not restricted to the equatorial plane. Similarly one expects that the maximal energy gain will be lower if the escaping particle is massive. We don't expect the results to be qualitatively different if we consider different initial conditions for the infalling particles.

In retrospect, it is easy to understand this result-large energy in the c.m. is not sufficient; the energetic particles
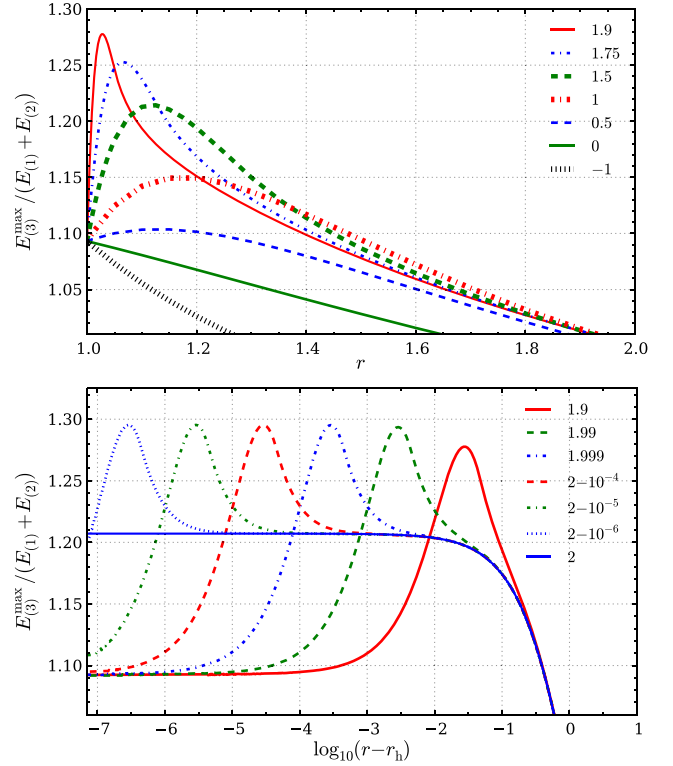

FIG. 2 (color online). Upper panel: The maximal energy of an escaping, initially ingoing photon, resulting from a collision of two massive ($m_{(1)} = m_{(2)} = 1$) particles with $L_{(1)} = 2$ and $L_{(2)} \in (-1, 1.9)$ as a function of the distance, measured in units of the BH's mass, M, from the BH center ($a = 1$). Lower panel: Zoom for $r \to r_h$ and $L_{(2)} \to 2$.

have to escape from the vicinity of the BH to infinity. This is demanding since the overall c.m. system has a diverging (comparable to the large c.m. energy) negative radial momentum. For an outgoing photon to escape, it has to have an outwards-pointing radial momentum (and such an angular momentum that it won't be deflected back into the BH), or a sufficient angular momentum such that it will be
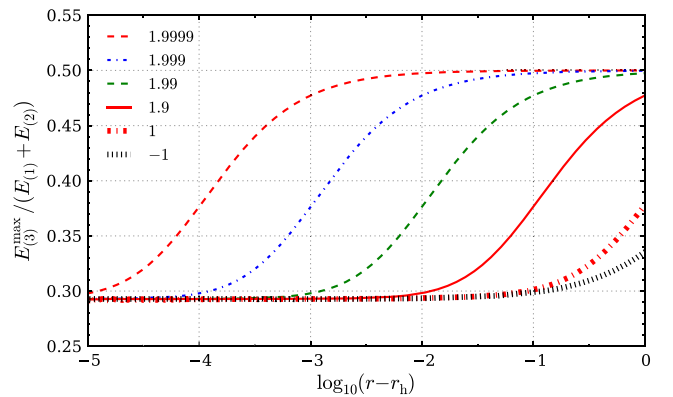

FIG. 3 (color online). Maximal energy of an initially outgoing photon as a function of the distance from the BH center, for $L_{(1)} = 2$, $L_{(2)} \in (-1, 1.9999)$ ($a = 1$).
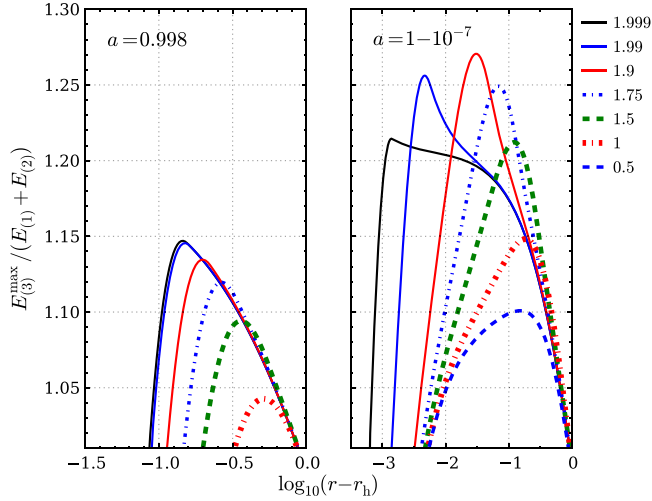
FIG. 4 (color online). The maximal energy for the escaping photon for $a < 1$ [$L_{(1)} = 2$, $L_{(2)} \in (0.5, 1.999)$]. Left panel: Spin parameter $a = 0.998$. Right panel: $a = 1\text{--}10^{-7}$.

deflected before falling into the BH. Given the huge c.m. negative radial momentum, most of the resulting particles will move radially inwards and will be quickly swallowed by the BH. They won't have the angular momentum needed to turn around before reaching the horizon. Only in a very small regime of the parameter phase space do the conditions allow the resulting photons to escape, but in this regime the energy of the escaping photon is not large. Thus, the highly energetic particles simply fall into the BH without any observable effects at infinity.

Are there any caveats that may enable us to avoid this conclusion and to have particles with diverging energies escaping to infinity? As pointed out earlier, the main reason
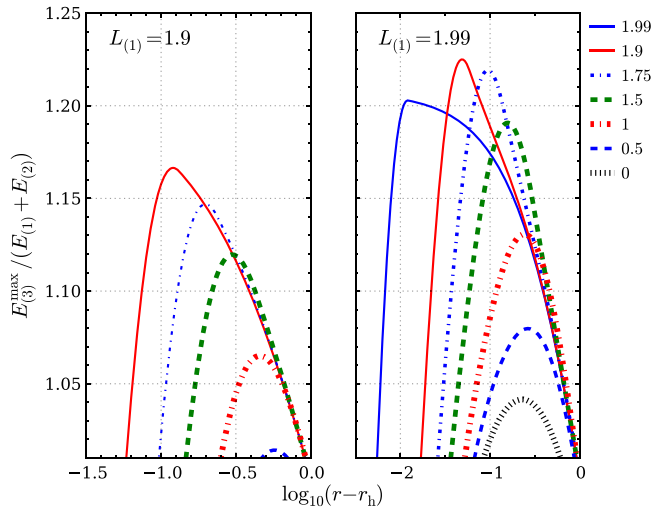

FIG. 5 (color online). Dependence of $E_{(3)}^{\max}$ on the maximal angular momentum of one of the initially infalling particles, $L_{(1)}$. Here, $L_{(1)} = 1.9$ (left panel) and $L_{(1)} = 1.99$ (right panel), whereas $L_{(2)}$ spans values not larger than $L_{(1)}$ ($a = 1$).

that most resulting particles fall into the BH is the huge negative radial momentum of the c.m.. In the optimal case, largest c.m. energies are attained when one of the particles is just turning around near the horizon and the other one falls and collides with it. Practically, the only way to avoid the negative radial momentum is when one of the two colliding particles has a positive radial momentum. This is almost impossible as it would have to come out of the BH. The orbit of an infalling particle that has just turned around would be rather similar to the orbit of the turning particle and the overall c.m. energy won't be large. One can speculate further and consider multiple collisions: the first collision sends a particle outwards and in the second one it will collide with an infalling particle. Alas, the phase space for such a situation is extremely small and the process is simply unlikely.

It seems that from the point of view of a distant observer, the c.m. energy is just an 'illusory' energy. One can imagine extremely energetic collisions very close to a BH, but the results of such collisions plunge quickly below the horizon, prohibiting a distant observer from knowing about them, let alone detecting energetic outgoing particles.

Our results confirm the earlier work of Piran and Shaham [5] that, while collisions in the ergosphere can in principle enable us to extract energy from a rotating BH, it is unlikely that the conditions for a significant energy extraction via a collisional Penrose process would appear in nature. Similar considerations suggest that hydrodynamical flows will not be efficient either. This leaves magnetic processes, such as the Blandford-Znajek mechanism, as the only viable way to extract rotational energy from a rotating BH. Indeed, there are now convincing arguments [13,14] indicating that the power needed to accelerate matter in the relativistic jets (with Lorentz factors up to $\gamma \approx 50$) observed in quasars and microquasars may be powered by the Blandford-Znajek mechanism, an electromagnetic version of the Penrose process.

*bejger@camk.edu.pl
†tsvi.piran@mail.huji.ac.il
‡marek.abramowicz@physics.gu.se

[1] R. Penrose, Riv. Nuovo Cimento **1**, 252 (1969).
[2] R. Penrose and G. R. Floyd, Nature (London), Phys. Sci. **229**, 177 (1971).
[3] A. Kovetz and T. Piran, Nuovo Cimento Lett. **12**, 39 (1975).
[4] R. M. Wald, Astrophys. J. **191**, 231 (1974).
[5] T. Piran, and J. Shaham, Phys. Rev. D **16**, 1615 (1977).
[6] M. Bañados, J. Silk, and S. M. West, Phys. Rev. Lett. **103**, 111102 (2009).

P H Y S I C A L   R E V I E W   L E T T E R S

[7] E. Berti, V. Cardoso, L. Gualtieri, F. Pretorius, and U. Sperhake, Phys. Rev. Lett. **103**, 239001 (2009).

[8] T. Jacobson and T. P. Sotiriou, Phys. Rev. Lett. **104**, 021101 (2010).

[9] M. Bañados, B. Hassanain, J. Silk, and S. M. West, Phys. Rev. D **83**, 023004 (2011).

[10] A. J. Williams, Phys. Rev. D **83**, 123004 (2011).

[11] M. Patil and P. S. Joshi, Classical Quantum Gravity **28**, 235012 (2011).

[12] J. M. Bardeen, W. H. Press, and S. A. Teukolsky, Astrophys. J. **178**, 347 (1972).

[13] A. Tchekhovskoy, R. Narayan, and J. C. McKinney, Mon. Not. R. Astron. Soc. Lett. **418**, L79 (2011).

[14] R. Narayan and J. E. McClintock, Mon. Not. R. Astron. Soc. Lett. **419**, L69 (2012).