Unified Drift-Diffusion Theory for Transverse Spin Currents in Spin Valves, Domain Walls, and Other Textured Magnets

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Spins transverse to the magnetization of a ferromagnet only survive over a short distance. We develop a drift-diffusion approach that captures the main features of transverse spin effects in systems with arbitrary spin textures (e.g., vortices and domain walls) and generalizes the Valet-Fert theory. In addition to the standard characteristic lengths (mean free path for majority and minority electrons, and spin diffusion length), the theory introduces two length scales, the transverse spin coherence length $\ell_\perp$ and the (Larmor) spin precession length ℓ_L. We show how ℓ_L and $\ell_\perp$ can be extracted from *ab initio* calculations or measured with giant magnetoresistance experiments. In long (adiabatic) domain walls, we provide an analytic formula that expresses the so-called ''nonadiabatic'' (or fieldlike) torque in terms of these length scales. However, this nonadiabatic torque is no longer a simple material parameter but depends on the actual spin texture: in thin (< 10 nm) domain walls, we observe very significant deviations from the adiabatic limit.

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Soon after the seminal paper of Slonczewski [1], which computed the spin transfer torque in a noncollinear spin valve, the concept of transverse spin current became widely discussed in the spintronics community. The problematic of transverse spin current can be formulated in a simple setup: by using a ferromagnet, one injects a spin-polarized current into a second ferromagnet whose magnetization is perpendicular to the first. A natural question that arises is, what happens to this spin polarization that is transverse to the magnetization? One can simply surmise that after a short distance the polarization of the current becomes aligned with the magnetization of the second magnet. The corresponding loss of transverse spin current is assumed to be transferred to the magnetic degrees of freedom; hence, a spin torque is exerted on the magnetization. In spin valves, a good understanding of the physics could indeed be reached by assuming a full absorption of the transverse spin current at the interface, i.e., a purely interfacial spin torque [2]. Indeed, quantum calculation has shown [3] that the leading mechanism for the absorption of transverse spin current is of purely ballistic origin: the band structure in general is much different for minority and majority spin in a ferromagnet, and the resulting fast spatial precession of the transverse spin leads, upon averaging the incident directions, to an exponential decay of the transverse spin current as a function of the distance of penetration in the ferromagnetic layer. The transverse spin coherence length $\ell_\perp$ obtained from these calculations is typically rather small, only a few nm, which justifies the interfacial limit. There exist, however, many situations where the interfacial limit $\ell_\perp \to 0$ entirely misses the relevant physics. One of those cases is the current-induced domain wall motion. It was recognized early that in domain wall, the main ''interfacial'' spin torque (known as ''in-plane torque,'' or, in this context, ''adiabatic torque'') is not sufficient to understand how a current can set a domain wall in motion [4,5]. A second spin torque perpendicular to the main one, and usually much weaker (the ''out-of-plane torque,'' also known as ''nonadiabatic'' or ''fieldlike'' torque), is necessary to describe the dynamics. An important theoretical effort has been devoted to the calculation of this nonadiabatic torque using a wide variety of techniques ranging from quantum [6–10] to phenomenological [11] approaches. This effort also includes a large variety of experimental studies [12–16].

In this Letter, we perform four things. First, we develop a drift-diffusion theory, capturing finite transverse spin current effects in systems involving ferromagnetic diffusive metal regions with arbitrary three-dimensional texture. The theory can be thought of as a direct generalization of the Valet-Fert theory [17] to arbitrary noncollinear systems beyond a two-current formulation in the ferromagnetic regions (with the same domain of validity). In the lumped circuit element (discrete) limit, our theory is equivalent to the generalized circuit theory [18,19]. Second, we show how the two new parameters of the theory [spin coherence length $\ell_\perp$ and the (Larmor) spin precession length ℓ_L] can be obtained from *ab initio* quantum calculations. Third, we propose an experimental setup that allows us to measure $\ell_\perp$ and ℓ_L directly using current perpendicular to the plane (CPP) giant magnetoresistance (GMR) measurements. Fourth, we apply our theory to spin torque in domain wall, thereby providing a direct link between CPP GMR and nonadiabatic torque.

Generalized drift diffusion theory.—Our starting point is a set of equations for the current densities in the charge

$j^c_\alpha(\vec{r})$ and spin $\boldsymbol{j}_\alpha(\vec{r})$ sectors and local charge $\mu_c(\vec{r})$ and spin $\boldsymbol{\mu}$ chemical potentials of the system. The entire theory is fully equivalent to the continuous random matrix theory (CRMT) [19,20] developed by some of us, and can be obtained from the latter through a simple change of variables after some straightforward, although somewhat lengthy, calculations [21]. The actual details of the derivation will be postponed to a subsequent publication, but we concentrate here on the physics implications. Similarly to the Valet-Fert theory, the set of equations consists of generalized Ohm's laws and current conservation equations:

$$-\ell_*\partial_\alpha\mu_c = j^c_\alpha - \beta\boldsymbol{m}\cdot\boldsymbol{j}_\alpha \tag{1}$$

$$-\ell_*\partial_\alpha\boldsymbol{\mu} = \boldsymbol{j}_\alpha - \beta j^c_\alpha\boldsymbol{m} + \frac{\ell_*}{\ell_\perp}(\boldsymbol{m}\times\boldsymbol{j}_\alpha)\times\boldsymbol{m} - \frac{\ell_*}{\ell_L}(\boldsymbol{m}\times\boldsymbol{j}_\alpha) \tag{2}$$

$$\sum_\alpha \partial_\alpha j^c_\alpha = 0 \tag{3}$$

$$\sum_\alpha \partial_\alpha \boldsymbol{j}_\alpha = -\frac{\ell_*}{\ell^2_{sf}}\boldsymbol{\mu} - \frac{1}{\ell_\perp}(\boldsymbol{m}\times\boldsymbol{\mu})\times\boldsymbol{m} + \frac{1}{\ell_L}(\boldsymbol{m}\times\boldsymbol{\mu}) \tag{4}$$

where the explicit index $\alpha \in \{x, y, z\}$ stands for the spatial direction with $\partial_\alpha = \partial/\partial\alpha$ and the bold vectors represent the three-dimensional spin space. The unit vector $\boldsymbol{m}(\vec{r})$ points along the local direction of the magnetization. The "charge" and "spin" currents defined above have the dimensions of energy. They are simply related to the (observable) electrical current density

$$\boldsymbol{I}_\alpha = 4j^c_\alpha/(e\mathcal{R}_{\mathrm{Sh}}) \tag{5}$$

and spin current density

$$\boldsymbol{J}_\alpha = 2\hbar\boldsymbol{j}_\alpha/(e^2\mathcal{R}_{\mathrm{Sh}}) \tag{6}$$

where $\mathcal{R}_{\mathrm{Sh}}$ is the Sharvin resistance for a unit surface (typically 0.5 $\mathrm{f\Omega\cdot m^2}$) and $e<0$ the charge of the electron. The theory is parametrized by five independent length scales: the mean free path $\ell_\uparrow$ for majority electrons, the mean free path $\ell_\downarrow$ for the minority electrons, the spin flip diffusion length ℓ_{sf}, the spin coherence length $\ell_\perp$ and the (Larmor) spin precession length ℓ_L. Alternatively, one can introduce the average mean free path ℓ_* ($1/\ell_* \equiv 1/\ell_\uparrow + 1/\ell_\downarrow$) and polarization $\beta \equiv (\ell_\uparrow - \ell_\downarrow)/(\ell_\uparrow + \ell_\downarrow)$. These parameters are totally equivalent to the usual parameters of the Valet-Fert theory [17,20], with the following correspondence: $\ell_\sigma = \mathcal{R}_{\mathrm{Sh}}/\rho_\sigma$ (ρ_σ is the spin-dependent resistivity), same β and ℓ_{sf}, and $\ell_* = \mathcal{R}_{\mathrm{Sh}}/(4\rho_*)$.

The physical meaning of $\ell_\perp$ and ℓ_L is best identified by studying the transverse spin in a nontextured magnet $\partial_\alpha\boldsymbol{m} = 0$; $\boldsymbol{\mu} = \boldsymbol{\mu}_\parallel + \boldsymbol{\mu}_\perp$ (and $\boldsymbol{j}_\alpha$) is decomposed into a longitudinal $\boldsymbol{\mu}_\parallel = (\boldsymbol{\mu}\cdot\boldsymbol{m})\boldsymbol{m}$ and perpendicular $\boldsymbol{\mu}_\perp = (\boldsymbol{m}\times\boldsymbol{\mu})\times\boldsymbol{m}$ contribution. Equations (1)–(4) reduce to Valet-Fert equations for the charge and longitudinal spin part. Expanding $\boldsymbol{\mu} = \mu_\parallel\boldsymbol{m} + \mu_1\boldsymbol{e}_1 + \mu_2\boldsymbol{e}_2$ with $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ two orthonormal unit vectors perpendicular to $\boldsymbol{m}$ we arrive at $\sum_\alpha \partial_{\alpha\alpha}\tilde{\mu} = \tilde{\mu}/l^2_{\mathrm{mx}}$ for $\tilde{\mu} = \mu_1 + i\mu_2$ with

$$\frac{1}{l^2_{\mathrm{mx}}} = \left(\frac{1}{\ell_*} + \frac{1}{\ell_\perp} - \frac{i}{\ell_L}\right)\left(\frac{\ell_*}{\ell^2_{sf}} + \frac{1}{\ell_\perp} - \frac{i}{\ell_L}\right), \tag{7}$$

which for most systems (except Nickel, see below) reduces to $1/l_{\mathrm{mx}} \approx 1/\ell_\perp - i/\ell_L$. In other words, the transverse spin accumulation $\tilde{\mu}(x) \propto \exp(-x/l_{\mathrm{mx}})$ decays over a length scale $\ell_\perp$ and precesses around $\boldsymbol{m}$ over a length ℓ_L. We note that other authors have earlier proposed generalization of the drift-diffusion equations [11,22,23]. While these approaches have captured the precession part of the above theory, they suffer from the absence of the (crucial) terms with $\ell_\perp$ so that the role of absorbing the transverse spin is taken up by a combination of the other length scales ℓ_L, ℓ_* and ℓ_{sf} [see Eq. (7) with $\ell_\perp = \infty$]. Another significant difference is the absence of transverse terms in the generalization of Ohm's law [see Eq. (2)]. Those terms originate from the underlying ballistic origin of the transverse spin precession or absorption [20].

The equations for a nonmagnetic metal are obtained from Eqs. (1)–(4) by setting $\ell_\perp \to \infty$ and $\beta = 0$. The presence of a magnetic field $\vec{B}$ (at the origin of the Hanle effect) is captured using $\ell_L = \hbar v_F/(g\mu_B B)$ and $\boldsymbol{m} = \boldsymbol{B}/|\boldsymbol{B}|$.

Interface and reservoirs boundary conditions.—To complete the theory, we need the boundary conditions between one normal (n) and one ferromagnetic (f) material. Noting n_α the vector normal to the interface and pointing toward the magnetic material, $\Delta\boldsymbol{\mu} = \boldsymbol{\mu}_n - \boldsymbol{\mu}_f$ the difference of chemical potential across the interface, $\epsilon_n = -\epsilon_f = 1$ and $a = n/f$, we get

$$\begin{aligned}\sum_\alpha n_\alpha \boldsymbol{j}^a_\alpha = {}& \sigma^*[(\boldsymbol{m}\cdot\Delta\boldsymbol{\mu}) + \gamma\Delta\mu_c]\boldsymbol{m} \\ & + \Re(\sigma^a_{\mathrm{mx}})\Delta\boldsymbol{\mu}_\perp - \Im(\sigma^a_{\mathrm{mx}})\boldsymbol{m}\times\Delta\boldsymbol{\mu}_\perp \\ & + \epsilon_a[\Re(\eta^a_{\mathrm{mx}})\boldsymbol{\mu}^a - \Im(\eta^a_{\mathrm{mx}})\boldsymbol{m}\times\boldsymbol{\mu}^a],\end{aligned} \tag{8}$$

$$\sum_\alpha n_\alpha j^c_\alpha = \sigma^*[\Delta\mu_c + \gamma\boldsymbol{m}\cdot\Delta\boldsymbol{\mu}], \tag{9}$$

where γ is the (Valet-Fert) polarization of interface resistance and σ^* is related to the Valet-Fert r^*_b as $1/\sigma^* = 2r^*_b(1-\gamma^2)$. The other "mixing" parameters are expressed in terms of the mixing transmission (T_{mx}) and reflection (R_{mx}) parameters [19] of the interface as follows:

$$\sigma^n_{\mathrm{mx}} = \frac{2T^n_{\mathrm{mx}}}{(1+R^n_{\mathrm{mx}})(1+R^f_{\mathrm{mx}}) - T^n_{\mathrm{mx}}T^f_{\mathrm{mx}}}, \tag{10}$$

$$\eta^n_{\mathrm{mx}} = \frac{(1+R^f_{\mathrm{mx}})(1-R^n_{\mathrm{mx}}) + (T^f_{\mathrm{mx}}-2)T^n_{\mathrm{mx}}}{(1+R^f_{\mathrm{mx}})(1+R^n_{\mathrm{mx}}) - T^n_{\mathrm{mx}}T^f_{\mathrm{mx}}}. \tag{11}$$

Last, the boundary conditions between the normal electrode at the potential eV and the system reads (n_α points toward the system) as

$$\sum_\alpha n_\alpha \boldsymbol{j}_\alpha + \boldsymbol{\mu} = 0, \tag{12}$$

$$\sum_\alpha n_\alpha j^c_\alpha + \mu_c = eV. \tag{13}$$

Extracting $\ell_\perp$ and ℓ_L from ab initio calculations.—While an important experimental work has been devoted to the calibration of β, ℓ_* and ℓ_{sf}, very little is known about the actual values of the lengths associated to transverse spins. A first insight is given by *ab initio* calculations that measure the mixing transmission T_{mx} of simple "normal-ferromagnet-normal" metal trilayers [24]. T_{mx} is a complex number whose amplitude measures the probability for a transverse spin to go through the system while its phase measures the angle of precession. Up to tiny corrections (due to multiple reflections of the transverse spin at the interfaces), it is given by

$$T_{mx}(d) = [T^{int}_{mx}]^2 e^{-(d/\ell_\perp)+(id/\ell_L)}, \tag{14}$$

where d is the thickness of the magnetic layer and T^{int}_{mx} the mixing transmission of the normal-ferro interface. Equation (14) allows for a direct extraction of $\ell_\perp$ and ℓ_L. An example of *ab initio* data, taken from Ref. [24], is shown in the upper part of Fig. 1, where we plot the real and imaginary part of T_{mx} for a Cu-Co-Cu trilayer (the inset shows the phase). We find that a good fit with Eq. (14) could be obtained allowing us to extract $\ell_\perp$ and ℓ_L with good precision (the value of the interface mixing transmission being somewhat less accurate). We have repeated the procedure with *ab initio* data available in the literature and collected the results as shown in Table I. We find the typical value $\ell_L \approx 0.3$ nm (corresponding to a full precession of 2π on 2 nm) and $\ell_\perp \approx 2$ nm. Nickel, a somewhat weaker magnet, seems to have a significantly longer transverse coherence length $\ell_\perp \approx 6$ nm than the other materials [25,26]. We note that since $\ell_\perp$ and ℓ_L originate from ballistic effects, they depend, in principle, on the crystalline direction, as seen in Table I. Such an effect could be incorporated into our theory by using tensorial instead of scalar values for these lengths. However, given the lack of information on these lengths, we restrict our study to scalar values at this stage.

Measuring ℓ_L and $\ell_\perp$ from CPP GMR.—An alternative route to the *ab initio* calculations is to actually measure ℓ_L and $\ell_\perp$ in the same way as was done for the other three "Valet-Fert"parameters in the collinear configuration. We propose the setup shown in the lower panel of Fig. 1, which consists of a standard spin valve into the middle of which one inserts the layer that one wants to study (X). The magnetization of the X layer must be perpendicular to the ones of the spin valve (A and P) so

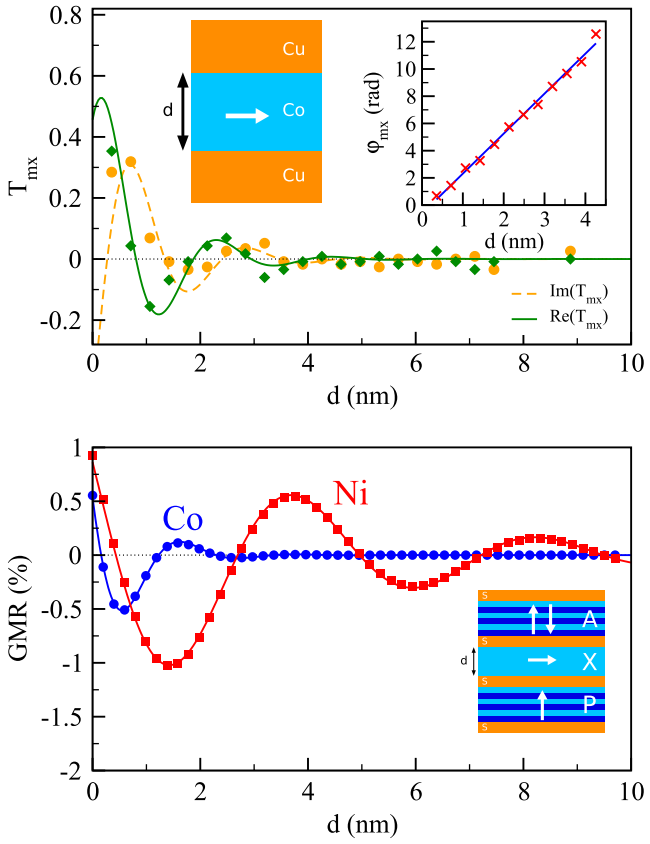

FIG. 1 (color online). Upper panel. Cu-Co-Cu trilayer. Real (circles) and imaginary (diamonds) parts of T_{mx} obtained from the *ab initio* data of Ref. [24] as a function of the thickness d of the Co layer. The lines are the corresponding fits with Eq. (14). Left inset: schematic of the setup. Right inset: same as the main panel for the phase of T_{mx}. Lower panel. Inset: Schematic of the proposed experimental setup for the measurement of $\ell_\perp$ and ℓ_L The studied magnetic layer X is sandwiched between an analysing A and polarizing layer P with normal spacers S. The studied layer must have its magnetization perpendicular to the one of A and P. Main plot: Numerical calculation of the GMR (in %) as a function of the thickness d of the studied layer for a stack with $S = $ Cu and $A = P = (Cu_{0.4} \mid Ni_{0.8})_{\times 3}$ (multilayer with perpendicular anisotropy). The squares (circles) show the numerical data for $X = $ Ni(100) ($X = $ Co(111)), while the lines correspond to the fit with Eq. (15).

that the latter must have strong perpendicular anisotropy if X has planar anisotropy, or vice versa. One performs standard GMR measurement and measures the resistance in the parallel (R_p) and antiparallel (R_{ap}) configurations. The GMR $\equiv (R_{ap} - R_p)/(R_{ap} + R_p)$ signal is proportional to the real part of the mixing transmission of X (and its interface) so that the expected signal reads

$$\mathrm{GMR}\,(d) = \underline{A}\cos\left(\frac{d}{\ell_L} - \delta\right) e^{-(d/\ell_\perp)}, \tag{15}$$

where the two constants $\underline{A}$ and δ depend on the material parameters of the spin valve. An example of the expected

TABLE I. Material parameters for various materials. The reference corresponds to the *ab initio* data from which ℓ_L, $\ell_\perp$ and $T_{\rm mx}^{int}$ have been extracted. A unique value of the Sharvin resistance of $\mathcal{R}_{\rm Sh} = 0.5\ \mathrm{f}\Omega\cdot\mathrm{m}^2$ has been used everywhere. The domain wall nonadiabatic β_τ parameter has been computed using Eq. (19).

Material	$\rho_*(n\Omega\cdot\mathrm{m})$	β	$\ell_{\rm sf}$ (nm)	$\ell_\uparrow$ (nm)	$\ell_\downarrow$ (nm)	Ref	ℓ_L (nm)	$\ell_\perp$ (nm)	β_τ	Interface	$T_{\rm mx}^{\rm int}$
Co(110)	75	0.46	60	24.7	9.13	[3]	0.2 ± 0.05	3 ± 0.1	3.687×10^{-4}	CuCo	/
Co(111)						[3]	0.2 ± 0.05	4 ± 0.1	3.694×10^{-4}	CuCo	/
Co(111)						[24]	0.34	0.75 ± 0.02	5.2×10^{-4}	CuCo	0.28-0.55i
Co(111)						[25]	0.37	0.95 ± 0.05	5.9×10^{-4}	CuCo	/
Fe(001)	80	0.45	60	8.62	22.7	[24]	0.30	1.2 ± 0.05	4.9×10^{-4}	AuFe	0.57-0.18i
Ni(100)	33.6	0.14	21	34.6	26.1	[26]	0.64	10 ± 0.1	2.1×10^{-2}	CuNi	/
Ni(111)						[25]	0.72	4.6 ± 0.1	2.4×10^{-2}	CuNi	/
Py(100)	291	0.76	5.5	14.3	1.95	[26]	1.42	0.9 ± 0.05	2.2×10^{-2}	CuPy	/
Py(111)						[25]	0.7	1 ± 0.1	2.6×10^{-2}	CuPy	/

signal is shown in Fig. 1, where we chose $(\mathrm{Cu}_{0.4} \mid \mathrm{Ni}_{0.8})_{\times 3}$ (indices are in nm) as our polarizing and analyzing layers with perpendicular anisotropy [27]. The symbols show the numerical calculations for $X = \mathrm{Ni}$ and $X = \mathrm{Co}$ together with the fit with Eq. (15). We find that the GMR signal, though a bit smaller than in the standard spin valve, lies around 1% with a very clear oscillating pattern.

Definition of spin torque.—Before turning to a practical calculation of spin torque in a domain wall, we need to identify its proper definition. In the original work of Slonczewski [1], spin torque was defined using a very robust conservation argument: whatever spin current has been lost by the conducting electrons must have been gained by the magnetic degree of freedom (conservation of total spin). However, this is not true in the presence of spin-orbit coupling (which is chiefly responsible for the finite $\ell_{\rm sf}$ in metals) as part of the angular momentum is transferred to the lattice. Hence, in the spin conservation equation [Eq. (4)], the divergence of the spin current is the sum of a (spin-orbit induced) spin flip term and the term corresponding to the exchange coupling to the magnetization. The latter corresponds to the spin torque density $\boldsymbol{\tau}$ which reads as

$$\boldsymbol{\tau} = \frac{2\hbar}{e^2\mathcal{R}_{\rm Sh}}\left(\frac{1}{\ell_\perp}(\boldsymbol{m}\times\boldsymbol{\mu})\times\boldsymbol{m} - \frac{1}{\ell_L}(\boldsymbol{m}\times\boldsymbol{\mu})\right). \quad (16)$$

Fieldlike torque in domain walls.—We now turn to a system with a nontrivial magnetic texture and study spin torque in a one-dimensional domain wall. We redefine the local basis as $\boldsymbol{e}_1 = \partial_x\boldsymbol{m}/|\partial_x\boldsymbol{m}|$ and $\boldsymbol{e}_2 = \boldsymbol{m}\times\boldsymbol{e}_1$. Noting $\theta(x)$ the angle between $\boldsymbol{m}(x)$ and the z axis (with $\dot\theta \equiv \partial_x\theta$), we obtain

$$\partial_{xx}\tilde\mu + \partial_x(\mu_\parallel\dot\theta) = \left(\frac{1}{\ell_*} + \frac{1}{\ell_\perp} - \frac{i}{\ell_L}\right) \times\left[\left(\frac{\ell_*}{\ell_{sf}^2} + \frac{1}{\ell_\perp} - \frac{i}{\ell_L}\right)\tilde\mu + j_\parallel\dot\theta\right]. \quad (17)$$

We now proceed with the adiabatic limit and consider the case of a very long domain wall (so that up to small corrections, the spin accumulation follows adiabatically the magnetization [28]). We obtain

$$\left(\frac{\ell_*}{\ell_{sf}^2} + \frac{1}{\ell_\perp} - \frac{i}{\ell_L}\right)\tilde\mu = -j_\parallel\dot\theta \quad (18)$$

from which we can calculate the torque. The torque τ_1 along $\boldsymbol{e}_1$ is the main contribution predicted by Berger [29] (each up electron leaves the system as a down electron and hence deposits $\hbar$ on the domain wall), $\tau_1 \approx -\beta\dot\theta\hbar I/(2e)$. The torque τ_2 along $\boldsymbol{e}_2$ is the nonadiabatic (or fieldlike) torque, whose importance has been stressed in the Introduction. Introducing the so-called "beta" parameter $\beta_\tau \equiv -\tau_2/\tau_1$, we arrive at

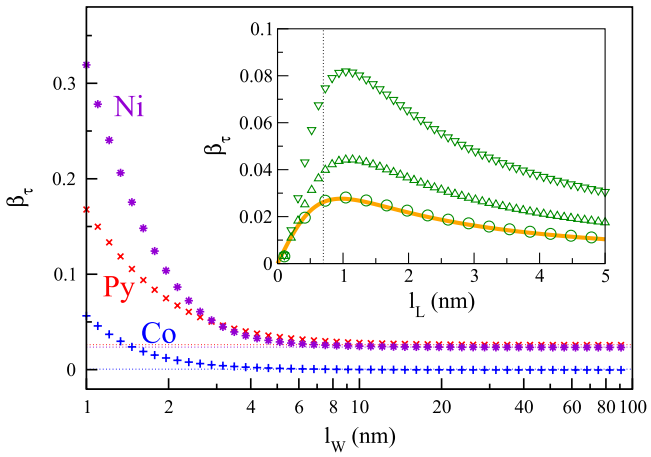

FIG. 2 (color online). Nonadiabatic torque in domain wall. Main: β_τ against domain wall length l_W for Ni, Py, and Co (bottom to top). Horizontal dotted lines correspond to the asymptotic value Eq. (19). Inset : β_τ versus ℓ_L, for Permalloy with $l_W = 20$ nm (○), $l_W = 4$ nm (△), and $l_W = 2$ nm (▽). The full line corresponds to the assymptotic ($l_W \to \infty$) value Eq. (19). The vertical dotted line corresponds to the expected actual value of $\ell_L = 0.7$ nm for Py.

$$\beta_\tau = \frac{\ell_L \ell_*}{\ell_{sf}^2}\left[1 + \frac{\ell_* \ell_L^2}{\ell_\perp \ell_{sf}^2} + \left(\frac{\ell_L}{\ell_\perp}\right)^2\right]^{-1}. \quad (19)$$

The importance of Eq. (19) comes from the fact that it connects two different realms: the domain wall motion on the left-hand sight, and CPP GMR physics on the right-hand side. In the limit of long spin-flip lengths, Eq. (19) reduces to $\beta_\tau \approx \ell_L \ell_* / \ell_{\mathrm{sf}}^2$ (up to a prefactor of order unity), which is similar (but not equivalent) to the one obtained by Zhang and Li [11], $\beta_\tau \approx \ell_L / \ell_{\mathrm{sf}}$. However, in the limit of strong spin-flip scattering, the parameter β_τ saturates toward $\beta_\tau \approx \ell_\perp / \ell_L$. The different values of this parameter have been summarized in Table I. Equation (19) seems to imply that β_τ is a simple combination of material parameters, but it is only valid for very smooth variation of the magnetization. Figure 2 studies numerically (see Ref. [19] for technical details) parameter β_τ at the center of the domain wall as a function of the width l_W of the wall—a simple form $\tan[\theta(x)/2] = \exp(x/l_W)$ was assumed for the wall. We find that the adiabatic limit Eq. (19) works very well for walls thicker than 10 nm, but a very significant increase is observed for thinner walls. Values close to unity are expected for very thin walls (such as those found in systems with strong perpendicular anisotropy) and strong spin-orbit coupling.

Conclusion.—The drift-diffusion approach developed here has the important advantage of hiding many microscopic details, which eventually leads to a renormalization of the parameters of the theory. On the other hand, it does capture the crucial physical ingredient of transverse spin physics—its rapid absorption due to decoherence between different directions of propagation. Once the effective parameters of the theory are measured (as was done in the collinear configuration) or calculated, a large number of predictions and links can be made. For instance, CPP GMR and domain wall motions are usually considered to involve quite different physics, but here we have shown that a direct connection can be made between both.

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[1] J. C. Slonczewski, J. Magn. Magn. Mater. **159**, L1 (1996).
[2] X. Waintal, E. B. Myers, P. W. Brouwer, and D. C. Ralph, Phys. Rev. B **62**, 12 317 (2000).
[3] M. D. Stiles and A. Zangwill, Phys. Rev. B **66**, 014407 (2002).
[4] A. Thiaville, Y. Nakatani, J. Miltat, and Y. Suzuki, Europhys. Lett. **69**, 990 (2005).
[5] D. Claudio-Gonzalez, A. Thiaville, and J. Miltat, Phys. Rev. Lett. **108**, 227208 (2012).
[6] A. Brataas, Y. Tserkovnyak, G. E. W. Bauer, and P. J. Kelly, in *Spin Current*, edited by S. Maekawa, E. Saitoh, S. Valenzuela, and Y. Kimura (Oxford University Press, Oxford, 2011).
[7] G. Tatara, H. Kohno, and J. Shibata, Phys. Rep. **468**, 213 (2008).
[8] J. Xiao, A. Zangwill, and M. D. Stiles, Phys. Rev. B **73**, 054428 (2006).
[9] I. Garate, K. Gilmore, M. D. Stiles, and A. H. MacDonald, Phys. Rev. B **79**, 104416 (2009).
[10] A. K. Nguyen, H. J. Skadsem, and A. Brataas, Phys. Rev. Lett. **98**, 146602 (2007).
[11] S. Zhang and Z. Li, Phys. Rev. Lett. **93**, 127204 (2004).
[12] M. Hayashi, L. Thomas, Y. B. Bazaliy, C. Rettner, R. Moriya, X. Jiang, and S. S. P. Parkin, Phys. Rev. Lett. **96**, 197207 (2006).
[13] G. Meier, M. Bolte, R. Eiselt, B. Krüger, D.-H. Kim, and P. Fischer, Phys. Rev. Lett. **98**, 187202 (2007).
[14] S. Lepadatu, J. S. Claydon, C. J. Kinane, T. R. Charlton, S. Langridge, A. Potenza, S. S. Dhesi, P. S. Keatley, R. J. Hicken, and B. J. Hickey *et al.*, Phys. Rev. B **81**, 020413 (2010).
[15] C. Burrowes, A. P. Mihai, D. Ravelosona, J.-V. Kim, C. Chappert, L. Vila, A. Marty, Y. Samson, F. Garcia-Sanchez, and L. D. Buda-Prejbeanu *et al.*, Nature Phys. **6**, 17 (2009).
[16] G. Malinowski, O. Boulle, and M. Klui, J. Phys. D **44**, 384005 (2011).
[17] T. Valet and A. Fert, Phys. Rev. B **48**, 7099 (1993).
[18] G. E. W. Bauer, Y. Tserkovnyak, D. Huertas-Hernando, and A. Brataas, Phys. Rev. B **67**, 094421 (2003).
[19] S. Borlenghi, V. Rychkov, C. Petitjean, and X. Waintal, Phys. Rev. B **84**, 035412 (2011).
[20] V. S. Rychkov, S. Borlenghi, H. Jaffres, A. Fert, and X. Waintal, Phys. Rev. Lett. **103**, 066602 (2009).
[21] Using Eqs. (11) and (12) in Ref. [20].
[22] N. Strelkov, A. Vedyayev, N. Ryzhanova, D. Gusakova, L. D. Buda-Prejbeanu, M. Chshiev, S. Amara, N. de Mestier, C. Baraduc, and B. Dieny, Phys. Rev. B **84**, 024416 (2011).
[23] S. Zhang, P. M. Levy, and A. Fert, Phys. Rev. Lett. **88**, 236601 (2002).
[24] A. Brataas, G. E. Bauer, and P. J. Kelly, Phys. Rep. **427**, 157 (2006).
[25] I. Turek and K. Carva, J. Phys. Condens. Matter **19**, 365203 (2007).
[26] K. Carva and I. Turek, Phys. Rev. B **76**, 104409 (2007).
[27] W. Chen, G. de Loubens, J.-M. L. Beaujour, J. Z. Sun, and A. D. Kent, Appl. Phys. Lett. **95**, 172513 (2009).
[28] X. Waintal and M. Viret, Europhys. Lett. **65**, 427 (2004).
[29] L. Berger, Phys. Rev. B **54**, 9353 (1996).