Long-Range Magnetic Fields in the Ground State of the Standard Model Plasma

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In thermal equilibrium the ground state of the plasma of Standard Model particles is determined by temperature and exactly conserved combinations of baryon and lepton numbers. We show that at nonzero values of the global charges a translation invariant and homogeneous state of the plasma becomes unstable and the system transits into a new equilibrium state, containing a large-scale magnetic field. The origin of this effect is the parity-breaking character of weak interactions and chiral anomaly. This situation could occur in the early Universe and may play an important role in its subsequent evolution.

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It is generally believed that the ground state of the Standard Model at high temperatures is homogeneous and isotropic. This assumption is important, for example, in the early Universe, where it underlies the description of all of the important processes: baryogenesis, cosmological phase transitions, primordial nucleosynthesis, etc. [1]. In this Letter we demonstrate, however, that at a finite density of lepton or baryon numbers due to the parity-violating nature of the weak interactions this homogeneous "ground state" becomes unstable by developing a long-range magnetic field. The transition to the "true" ground state may depend on the details of the nonequilibrium dynamics, when various violent dissipative processes (e.g., turbulence, radiation emission, and the finite conductivity of the plasma) play an important role.

What are the conditions for the translational invariance to be spontaneously broken by a long-range field? It is sufficient for the free energy of the gauge fields to contain an interaction term that dominates the kinetic energy and can be both positive and negative. An example is provided by a Chern-Simons (CS) term $I_{\rm cs} \propto A\partial A$, which has less derivatives than the kinetic term $(\partial A)^2$ and therefore can dominate it at large scales. The presence of the CS term in the Maxwell equations is known to lead to an instability and to the generation of magnetic fields (see, e.g., Refs. [2–5]).

At zero temperature and densities the CS term for electromagnetic fields is prohibited as a consequence of gauge invariance and Lorentz symmetry (Furry theorem [6]). At finite temperatures and densities the plasma creates a preferred reference frame and the four-dimensional Lorentz invariance is broken down to a three-dimensional one. As a result the free energy of static gauge fields is

$$\mathcal{F}[A] = \int d^3p A_i(\vec{p})\Pi_{ij}(p)A_j(-\vec{p}) + \mathcal{O}(A^3) \qquad (1)$$

with the polarization operator

$$\Pi_{ij}(\vec{p}) = (p^2\delta_{ij} - p_ip_j)\Pi_1(p^2) + i\epsilon_{ijk}p^k\Pi_2(p^2), \qquad (2)$$

where $i, j, k = 1, 2, 3$ are spacial indices; $p^2 = |\vec{p}|^2$; and ϵ_{ijk} is the antisymmetric tensor. Equation (2) is the most general form of Π_{ij} satisfying the gauge-invariance transversality condition $p_i\Pi_{ij} = 0$. In the limit $p^2 \to 0$ a nonzero $\Pi_2(0)$ means that the CS term $\Pi_2(0)\vec{A}\cdot\vec{\nabla}\times\vec{A}$ appears in Eq. (1). The 3×3 matrix of Eq. (2) has then a negative eigenvalue for sufficiently small momenta $p < |\Pi_2(p^2)/\Pi_1(p^2)|$ and the corresponding eigenmode grows larger and larger (until the terms higher order in A stabilize it) [7].

We demonstrate below that in a theory like the Standard Model where fermions are involved in parity-violating weak interactions, the nonzero equilibrium $\Pi_2(0)$ term is generated. We analyze the simplest situation when this effect is present: the case of $T \ll m_W$ (mass of the W-boson) when weak interactions can be described by the Fermi theory

$$\mathcal{L}_{\rm F} = \frac{4G_F}{\sqrt{2}}[(J_\mu^{\rm NC})^2 + 2(J_\mu^{\rm cc})^2]. \qquad (3)$$

The full Hamiltonian of the theory $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\rm F} + \mathcal{H}_{\rm EM}$ has a free part for fermions and photons, $\mathcal{H}_0$, and terms describing electromagnetic ($\mathcal{H}_{\rm EM}$) and Fermi ($\mathcal{H}_{\rm F}$) interactions.

Modified dispersion relation of fermions at finite density of baryon or lepton number.—The equilibrium plasma at $T \ll m_W$ is described by the density matrix, $\hat{\varrho} = \mathcal{Z}^{-1}\exp[-\beta(\mathcal{H} - \sum_\alpha\lambda_\alpha L_\alpha - \lambda_Q Q - \lambda_B B)]$. Five global charges commute with the Hamiltonian $\mathcal{H}$: the baryon (B) and flavor lepton numbers L_α (the index α runs over the flavors) [10] and Q (λ_α, λ_Q, λ_B are the corresponding Lagrange multipliers). $\mathcal{Z}$ ensures that $\operatorname{tr}(\hat{\varrho}) = 1$.

To find the distribution functions of the left- and right-chiral particles we compute the correlators

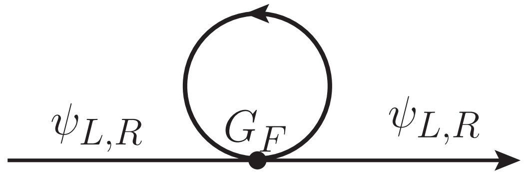

FIG. 1. Fermi corrections to the fermion self-energy. The loop contribution is nonzero only at finite lepton/baryon density.

$\langle \bar\psi P_{\rm L,R} \psi \rangle = {\rm tr}(\hat\varrho\, \bar\psi\, P_{\rm L,R}\psi)$ (where $P_{\rm L,R} = \frac{1}{2}(1 \pm \gamma_5)$ are chiral projectors). We expand the density matrix in the interactions to get

$$\hat\varrho \approx \hat\varrho_0(1 - \beta\mathcal{H}_{\rm F} - \beta\mathcal{H}_{\rm EM} - \frac{\beta^2}{2}\mathcal{H}_{\rm EM}\mathcal{H}_{\rm F} + \ldots). \quad (4)$$

At zeroth order in the interactions one gets $\langle\bar\psi P_L\psi\rangle_0 = \langle\bar\psi P_R\psi\rangle_0 = \frac{1}{2}\langle\bar\psi\psi\rangle_0$. Taking into account the chiral Fermi interactions, $\hat\varrho_F \approx \hat\varrho_0(1-\beta\mathcal{H}_{\rm F})$, one finds that $\langle\bar\psi P_{\rm L}\psi\rangle_{\rm F} \neq \langle\bar\psi P_R\psi\rangle_{\rm F}$. The Green's function is found via $G^{-1} = G_0^{-1} - \Sigma$ where, for example, the self-energy of the left electron, Σ_L, is (see Fig. 1)

$$\Sigma_L = \frac{4G_F}{\sqrt{2}}\Big[2g_L^e(g_L^e\gamma^\mu P_L\langle e\bar e\rangle_0\gamma_\mu P_L + \sum_\psi g_{L,R}^\psi\gamma^\mu P_L\langle\bar\psi\gamma_\mu P_{\rm L,R}\psi\rangle_0) + \frac{1}{2}\gamma^\mu P_L\langle\nu_e\bar\nu_e\rangle_0\gamma_\mu P_L\Big] \equiv \delta\mu_L\gamma_0 P_L. \quad (5)$$

A similar expression for Σ_R defines $\delta\mu_R$. Equation (5) does not depend on momentum. The thermal averages $\langle\psi\bar\psi\rangle_0$ and $\langle\bar\psi\gamma_\mu P_{L,R}\psi\rangle_0$ are proportional to the particle-antiparticle asymmetry (see, e.g., Ref. [11]); the thermal averages are also summarized in Table 1, Appendix B, of that work). As a result, e.g., the electron propagator becomes

$$G = \frac{1}{\gamma_0(\delta\mu_L P_L + \delta\mu_R P_R + \mu_{\rm tree}) + \not p + m_e}; \quad (6)$$

i.e., the dispersion relation of electrons changes when taking into account the Fermi corrections (cf., Refs. [11,12]). Indeed, from $(\not p - m - \Sigma)\psi = 0$ we see that the "on-shell conditions" $(\omega - \mu_{\rm tree})^2 = p^2 + m^2$ get shifted for left (right) particles by $2(\omega - \mu_{\rm tree})\delta\mu_{L,R}$ (in the limit $\delta\mu_{L,R} \ll \omega$), where $\mu_{\rm tree} = (\lambda_Q - \lambda_e)$. Moreover, as $m_e/T \to 0$ the propagator of Eq. (6) can be written as the sum of the free propagators of the left and right fermions, with $\mu_{\rm tree}$ shifted by the different quantities $\delta\mu_L$ or $\delta\mu_R$, correspondingly.

This difference gives rise to a parity-odd term in the polarization operator of the photons. Indeed the polarization operator that was parity-even when computed with respect to the density matrix $\hat\varrho_0$ [Fig. 2(a)] acquires a parity-odd part when averaged with respect to the $\hat\varrho_{\rm F}$. The lowest order weak corrections are represented by two figures, Figs. 2(c) and 2(b). The computation of Fig. 2(b) gives a nonzero $\Pi_2(0) = \frac{\alpha}{2\pi}\sum_f q_f^2(\delta\mu_{f_L} - \delta\mu_{f_R})$

$$\Pi_2(0) = \frac{\alpha}{2\pi}\frac{4G_F}{\sqrt{2}}[c_{L_\alpha}L_\alpha + c_B B] \equiv \frac{\alpha}{2\pi}\Delta\mu, \quad (7)$$

where the coefficients $c_{L_\alpha}, c_B \sim \mathcal{O}(1)$ depend on the fermionic content of the plasma (see Appendix B of Supplemental Material [14]) and $\alpha = \frac{e^2}{4\pi}$ is the fine-structure constant [15]. Notice that, even if the anomalous charge $B + L = 0$, the $\Pi_2(0)$ term [Eq. (7)] remains nonzero.

Figure 2(c) does not contribute to the $\Pi_2(0)$ term as it can be cut into two Fig. 2(a) diagrams along the vertical dotted line, each of which is at least first order in momentum [Eq. (2)] [17].

Although the Fermi theory [Eq. (3)] is not renormalizable, the result of Eq. (7) is given by the nondivergent part of Fig. 1 and is expressed in terms of well-defined physical quantities (cf., Ref. [11]).

The origin of the Π_2 term has its roots in the axial anomaly (cf., Refs. [4,5,18–23]). Indeed, assume $\Delta \ll \mu_{\rm tree}, T$ and consider a correction linear in $\Delta\mu/T$ to the polarization operator. Figure 2(b) then becomes Fig. 2(d)—the famous triangular graph for the axial anomaly [24–26], which gives in the effective action the CS term $\propto \epsilon_{\alpha\beta\mu\nu}X_\alpha A_\beta\partial_\mu A_\nu$ with the "axial vector field" X_α. This term reduces to the parity-odd term in the free energy [Eqs. (1) and (2)] given by Fig. 2(b) if one uses $X_\beta = \Delta\mu\delta^{\beta 0}$ with $\Delta\mu$, defined by Eq. (7).

Axial anomaly means that one can convert left fermions into right ones by exciting the gauge field configurations with a nontrivial CS number $N_{\rm cs} \equiv \int d^3x A\cdot B$ (where $B = \nabla\times A$ is a magnetic field):

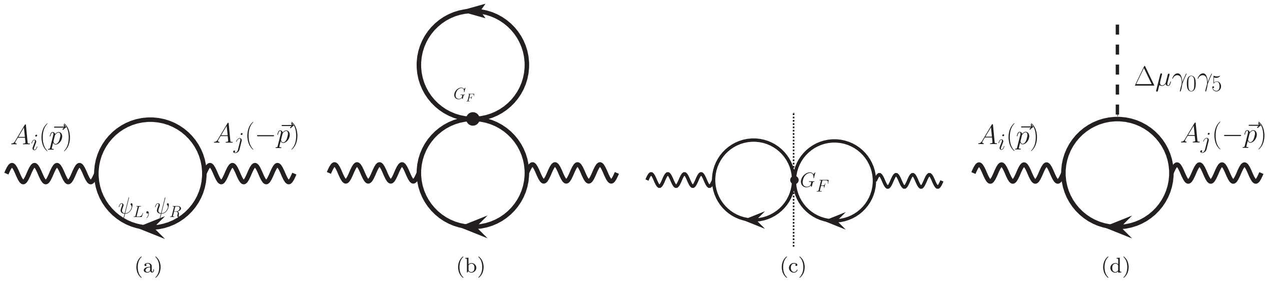

FIG. 2. Polarization operator (a), its one-loop weak corrections [(b) and (c)] and its expansion in $\Delta\mu/T$ (d).

$$\frac{d(n_L - n_R)}{dt} = \frac{e^2}{2\pi^2}\int d^3x E\cdot B = \frac{\alpha}{\pi}\frac{dN_{\rm cs}}{dt} \qquad (8)$$

(here $E = -\dot{A}$ is an electric field). Consider the simplest example of zero temperature and zero mass. The left and right fermion numbers are then (classically) conserved. An infinitesimal change of the gauge field δA destroys (creates) $\delta n_{L,R} = \pm\frac{\alpha}{2\pi}\delta N_{\rm cs}$ real fermions around the Fermi level. If the left and right Fermi energies are different ($\mu_L \neq \mu_R$) the total energy of the system changes by $\delta\mathcal{F} = (\mu_L - \mu_R)\frac{\alpha}{2\pi}\delta N_{\rm cs}$ [19,27], which leads to the parity-odd CS term in the free energy: $\mathcal{F}[A] = \frac{\alpha(\mu_L-\mu_R)}{2\pi}\times\int d^3xA\cdot B$. This remains true also for $T\neq 0$ [18,20].

In our case the fermions are massive and the fermion chiralities are not conserved separately. However, the different dispersion relations (and hence, energies) of the left and right particles play a similar role and make the change of chirality and creation of $N_{\rm cs}\neq 0$ energetically favorable.

Chern-Simons coefficient at two loops and "nonrenormalization theorems."—Figure 2(d) is similar to the triangular diagram responsible, e.g., for $\pi^0\to 2\gamma$ decay (with $\Delta\mu\bar{\psi}\gamma_0\gamma_5\psi$ playing the role of the only nonzero component of the chiral current, describing the pion) [24–26]. It is well known that the axial anomaly should be calculated at one loop only and that it is not renormalized by higher-loop corrections [28–31], also at finite temperature and density. At the same time our result becomes nonzero only at two loops. There is, however, no contradiction. What is nonrenormalized for the chiral anomaly is the numerical coefficient in front of the proper combination of external fields, [e.g., $\frac{\alpha}{2\pi}$ in Eq. (8)]. In our case this coefficient is also not renormalized. The structure of the parity-odd one loop term has the same form at tree level and at one loop in G_F: $\Pi_2(0) = \frac{\alpha}{2\pi}(\Delta\mu_{\rm tree} + \Delta\mu)$. The numerical coefficient is dictated by the axial anomaly; $\Delta\mu_{\rm tree}$ is a possible difference of chemical potentials present at tree level (zero in our case); and $\Delta\mu$ is the shift generated by the diagrams of Fig. 2(b) [Eq. (7)].

Also a four-dimensional theory at finite temperatures can be regarded as a three-dimensional Eucledian model albeit with the infinite number of particles—each Matsubara mode of a fermion becomes a "particle" with mass $\omega_n = \pi(2n+1)T$, $n\in\mathbb{Z}$. Therefore, (as was argued in Ref. [32]) our result may seem to be in contradiction with the "Coleman and Hill theorem" [33] that states that in any Eucledian three-dimensional gauge theory without massless particles $\Pi_2(0) = \sum_f \frac{q_f^2}{4\pi}\frac{m_f}{|m_f|}$ and is exact at one loop. However, the presence of the infinite number of modes changes the situation, as can be seen already in the simplest chiral gauge theory, if one computes the $\Pi_2(0)$ term in the Matsubara formalism [see, e.g., Ref. [20]]. Formally, considering the left- and right-chiral particles as fermions with "complex mass" $m_n \equiv (\omega_n - i\mu_{L,R})$, and applying this directly to the results of Ref. [33] one would arrive at the undefined expression $\Pi_2(0) = \frac{e^2}{4\pi}\sum_{n\in\mathbb{Z}}\frac{\omega_n - i\mu_L}{\sqrt{(\omega_n-i\mu_L)^2}} - \frac{\omega_n - i\mu_R}{\sqrt{(\omega_n-i\mu_R)^2}}$. The reason for this is clear: the degree of divergence of the diagrams is different in three and four dimensions (hence the infinite sum over n). In particular, if we first sum over the Matsubara frequencies and then integrate over momentum (or if one uses dimensional regularization in the three-momentum integral and then takes the limit $d\to 3$, [20]; see also Ref. [22]) one obtains a well-defined answer, Eq. (7).

Moreover, the CH theorem uses the fact that the three-point photon vertex $\Gamma^{(3)}(p_1\ldots) = \mathcal{O}(p_1)$. This is not true in our case, as Fig. 2(d) becomes linearly divergent in four dimensions and therefore the shift of the integration momentum by any fixed vector k changes its parity-odd part by a finite amount $\propto \Delta\mu\epsilon^{ijn}A_jk_nA_j$ [34,35].

Ground state.—The presence of the $\Pi_2(0)\neq 0$ term leads to the generation of magnetic fields. The CS number $Ncs \sim kA^2$ will increase until it reaches $\frac{\alpha}{\pi}Ncs \sim (n_L - n_R) \sim G_F L_{\rm tot}$ (see, e.g., Ref. [19]). At fixed $N_{\rm cs}$ the magnetic field tends to increase its wavelength to decrease the total energy ($B^2\sim kN_{\rm cs}$). As a result, the system does not have a thermodynamic (infinite volume) limit (cf., Ref. [19]); the value of the field and the scale of the inhomogeneity will be determined by the size of the system. It is clear, however, that in realistic systems establishing the long-range field is a complicated process (see, e.g., Ref. [36]), greatly affected by the dissipative processes and by the existence of the different relaxation channels of $N_{\rm cs}$ (resistivity of the plasma, energy radiation, turbulence, etc.; see, e.g., Refs. [37–41]). This may significantly affect the subsequent evolution.

Discussion.—We have demonstrated that the Standard Model plasma at finite densities of lepton and baryon numbers becomes unstable and tends to develop large scale magnetic fields. We considered electrodynamics plus the Fermi theory [Eq. (3)], a description of weak interactions that is valid when $e^{-m_W/T}\lesssim (T/m_W)^2$, i.e., at $T\lesssim$ 40 GeV. At higher temperatures one should consider the full electroweak theory and perform two-loop computations of $\Pi_2(0)$. At even higher temperatures (in the symmetric phase) one should analyze the hypermagnetic fields. We leave these analyses for future work. We expect however that our conclusion about the instability of a homogeneous state will hold.

Below we discuss several realistic systems in which the effects discussed here can become important. First we consider the primordial plasma at the radiation dominated epoch. Equation (7) gives $\Pi_2(0)\sim c\times\alpha(G_FT^3)\eta_{L,B}$ where $\eta_{L,B}<1$ is the ratio of the total lepton (baryon) number to the number of photons; the numerical coefficient $c\approx 2.5\times 10^{-2}$.

The instability starts to develop at scales of $k\sim\Pi_2(0)$ and the magnetic field initially grows as e^{β} where $\beta(t)\equiv k^2t/\sigma$ (see, e.g., Refs. [3,23,36]). The equilibrium considerations of this work apply if the instability develops over less than a Hubble time (i.e., while the temperature does not change significantly). This means that $\beta(t = H^{-1}) \approx 2.0(\frac{T}{m_W})^3(\frac{\eta_{L,B}}{10^{-2}})^2$ should exceed 1 [42] ($T \lesssim$ 40 GeV is required for the applicability of the Fermi theory). These conditions can be realized for $\eta_L \gtrsim$ a few $\times 10^{-2}$. Although this value is much greater than the measured baryon asymmetry $\eta_B \sim 6.0 \times 10^{-10}$ [46], it does not contradict any observations. Indeed, the upper bounds on η_L exist only at the epoch of primordial nucleosynthesis ($|\eta_L| \lesssim$ a few $\times 10^{-2}$ [47]). At earlier epochs even $\eta_L \sim 1$ is possible (if this lepton asymmetry disappears later). Such a scenario is realized, e.g., in the νMSM (see Ref. [48] for a review), where the lepton asymmetry continues being generated below the sphaleron freeze-out temperature and may reach $\eta_L \sim 10^{-2}$–10^{-1} before it disappears at $T \sim$ a few GeV [49]. We see that significant magnetic fields can in principle develop in this case, playing an important role for analysis of the cosmological implications of the νMSM.

We stress that the above condition is sufficient (rather than necessary) for magnetic fields to develop. In principle, one should consider the time-dependent dynamics of the $\Pi_2(0)$ term and $\beta \propto (\int dt \Pi_2(0))^2$. Some part of this dynamics can occur above 40 GeV where the electroweak (rather than the Fermi) interactions should be analyzed. We leave this analysis for future work.

Next, we consider a high density degenerate electron plasma (appearing, e.g., in white dwarfs and neutron stars [50]). Notice that our consideration remains valid in this regime as Eq. (5) makes no assumption about the relation between the mass and temperature of the particles. Only the numerical coefficient in Eq. (7) changes, and we checked that it is $\mathcal{O}(1)$. The same relation $\Pi_2(0) \sim \frac{\alpha}{4\pi} G_F L_{\rm tot}$ holds; however, now $L_{\rm tot} = n_e$ (the density of electrons) and it can be quite essential, reaching 10^{30}–10^{35} cm^{-3} in the crust of neutron stars [51]. The corresponding scale of the instability $k \sim \Pi_2(0)$ is then of (sub)kilometer size and the time of its development is much shorter than the lifetime of the star.

Below, we compare our results to the previous work. In any vectorlike Abelian gauge theory if the numbers of left and right particles (corresponding chemical potentials) are different, the CS term will be generated [2,18,20] (also Refs. [3–5]). In this case (extensively discussed in the literature) the contribution of left and right charged fermions to Fig. 2(a) is different and the term $\Pi_2(0) = \frac{\alpha}{2\pi} \times (\mu_L - \mu_R)$ appears. However, as all charged particles in the Standard Model are massive such quantum numbers are only approximately conserved and chirality flipping reactions [although suppressed as $(\text{mass}/E)^2$] eventually drive $\Delta\mu \to 0$. Therefore one can only speak of nonequilibrium processes when $\Delta\mu \neq 0$ is present as an initial condition and then relaxes. For example, in the expanding Universe, the time scale for relaxation is set by the Hubble parameter $H(t)$. If the chirality-flipping reaction rate is faster than $H(t)$, $\Delta\mu$ is expected to have its equilibrium value of $\Delta\mu = 0$. Despite the smallness of the electron's mass (Yukawa coupling) this happens already at temperatures $T \lesssim 80$ TeV [52] [although $(m_e/80\ \text{TeV})^2 \sim 10^{-17}$]. If the initial conditions had a nonzero μ_{e_R} above 80 TeV, a generation of magnetic fields will occur [3]. However, the horizon size at this epoch is small compared to the scales of the plasma dissipative processes [37,38,40] and the magnetic fields have strongly subhorizon characteristic scales; therefore, they are probably erased during the subsequent evolution [3,53,54].

Contrary to these works we consider an equilibrium state (in particular, where all chirality-flipping reactions are in thermal equilibrium). It is a modification of the dispersion relations due to the Fermi interaction (Fig. 1) that leads to the nonzero coefficient $\Pi_2(0)$ [Eq. (7)] in equilibrium. This effect therefore may take place at much lower temperatures.

After the magnetic fields are generated via the discussed mechanism, their evolution is also strongly affected by the chiral anomaly. As shown in Ref. [36], the self-consistent evolution of a helical magnetic field and of chiral asymmetry, induced by this field due to the chiral anomaly, is very different from conventional MHD, and significantly increases the lifetime of both quantities. It is interesting to combine the results of our work with the description of further evolution, developed in Ref. [36].

The Standard Model at the nonzero density of the anomalous charge $B + L$ was considered, e.g., in Refs. [55–57]. It was shown there that a similar CS term exists for nonAbelian gauge fields and that a homogeneous state becomes classically unstable at large values of the chemical potential, exceeding the mass of weak bosons. Even in the symmetric phase the "magnetic screening" [8] requires $\Delta\mu \gtrsim T$ to overcome the "magnetic mass" $m_{\rm magn} \sim \alpha_W T$. The anomalous nonconservation of the $B + L$ current drives the coefficient of the corresponding CS term to zero [58] and the standing wavelike configurations of the gauge fields are actually metastable (see Ref. [56]). It is important to note that in our case an instability appears even if $B + L = 0$.

Summary.—We have discussed a previously unknown effect that occurs in the Standard Model at finite temperature and density. It implies that a number of processes in the early Universe can be affected, including cosmological phase transitions, baryogenesis, and dark matter production. This effect may in particular lead to the generation of horizon-scale helical cosmic magnetic fields purely within the Standard Model. Such fields may survive until the present and may serve as seeds for the observed magnetic fields in galaxies and clusters. The effect may also be important for the explanation of the physics of compact stars.

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