Searching for Majorana Fermions in 2D Spin-Orbit Coupled Fermi Superfluids at Finite Temperature

Ming Gong,[1,2] Gang Chen,[2,3] Suotang Jia,[3] and Chuanwei Zhang[1,2,*]

[1]*Department of Physics, The University of Texas at Dallas, Richardson, Texas 75080, USA*
[2]*Department of Physics and Astronomy, Washington State University, Pullman, Washington 99164, USA*
[3]*State Key Laboratory of Quantum Optics and Quantum Optics Devices, Laser Spectroscopy Laboratory, Shanxi University, Taiyuan 030006, People's Republic of China*

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Recent experimental breakthroughs in realizing spin-orbit (SO) coupling for cold atoms have spurred considerable interest in the physics of two-dimensional SO coupled Fermi superfluids, especially topological Majorana fermions (MFs) which were predicted to exist at zero temperature. However, it is well known that long-range superfluid order is destroyed in two dimensions by phase fluctuations at finite temperature, and the relevant physics is the BKTs transition. In this Letter, we examine finite temperature effects on SO coupled Fermi gases and show that finite temperature is indeed necessary for the observation of MFs. Majorana fermions are topologically protected by a quasiparticle energy gap which is found to be much larger than the temperature. The restrictions to the parameter region for the observation of MFs have been obtained.

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A new research direction in low dimensional condensed matter physics that has attracted much recent attention is the study of two-dimensional (2D) topological quantum states of matter (e.g., fractional quantum Hall effects, chiral p-wave superfluids or superconductors, etc.) that support exotic quasiparticle excitations (named anyons) with Abelian or non-Abelian exchange statistics [1–8]. For instance, it has been shown recently that a topological cold atom superfluid may emerge from an ordinary 2D s-wave Fermi superfluid in the presence of two additional ingredients: spin-orbit (SO) coupling and a Zeeman field [9]. Such topological superfluids can host Majorana fermions (MFs), non-Abelian anyons which are their own antiparticles, and may have potential applications in fault-tolerant topological quantum computation [1].

On the experimental side, 2D degenerate s-wave Fermi gases have been realized using a highly anisotropic pancake-shaped trapping potential [10–13]. Furthermore, the SO coupling and Zeeman field for cold atoms have been generated in recent pioneering experiments through the coupling between cold atoms and lasers [14–19]. It seems, therefore, that MFs are tantalizingly close to experimental reach in degenerate Fermi gases. Inspired by the experimental achievements, there have been extensive theoretical efforts toward understanding the physics of SO coupled Fermi gases, particularly the mean-field crossover from Bardeen-Cooper-Schrieffer (BCS) superfluids to the Bose-Einstein condensation (BEC) of molecules [20–33]. While the mean-field theory works qualitatively well in three dimensions, it may not yield relevant physics in two dimensions at finite temperature. For instance, differing from the mean-field prediction, there is no long-range superfluid order in two dimensions at finite temperature due to phase fluctuations [34]. At finite temperature, the relevant physics is the Berezinskii-Kosterlitz-Thoules (BKT) transition [35,36] with a characteristic temperature $T_{\rm BKT}$, below which free vortex-antivortex (V-AV) pairs are formed spontaneously. When the temperature is further lowered below another critical temperature $T_{\rm vortex}$, the system forms a square V-AV lattice [37,38]. Because MFs only live in the cores of spatially well separated vortices, it is crucial to study the dependence of $T_{\rm BKT}$ and $T_{\rm vortex}$ on the SO coupling strength in 2D Fermi superfluids.

In this Letter, by taking account of phase fluctuations in finite temperature quantum field theory, we investigate the finite temperature properties of SO coupled degenerate Fermi gases en route to clarifying some crucial issues for the experimental observation of MFs in such systems. Our main results are the following. (i) We show that, quite unexpectedly, the SO coupling reduces both $T_{\rm vortex}$ and $T_{\rm BKT}$, despite enhancing the superfluid order parameter Δ. (ii) In the pseudogap phase ($T > T_{\rm BKT}$), the Fermi gas lacks phase coherence. While in the vortex lattice phase ($T < T_{\rm vortex}$), the short distance between neighboring vortices may induce a large tunneling between vortices that destroys the Majorana zero energy states [39]. Therefore MFs can be observable only in the cores of naturally present V-AV pairs [40] in the temperature region $T_{\rm vortex} < T < T_{\rm BKT}$, instead of the intuitively expected zero temperature. (iii) The MF in a vortex core is protected by a quasiparticle energy gap $\gtrsim \Delta^2/2E_F$ (E_F is the Fermi energy), which is found to be much larger than the experimental temperature. Such a large energy gap greatly reduces the occupation probability of nontopological excited states in the vortex core due to finite temperature, and ensures the topological protection of MFs. (iv) The restrictions to the parameter region for the observation of MFs have been obtained. We also propose a simple strategy for finding the experimental parameters for the observation of MFs.

We consider a 2D degenerate Fermi gas in the presence of a Rashba-type SO coupling and a perpendicular Zeeman field. In experiments, 2D degenerate Fermi gases can be realized using a one-dimensional deep optical lattice with a potential $V_0\sin^2(2\pi z/\alpha_w)$ along the third dimension, where the tunneling between different layers is suppressed completely [10–13]. The Rashba SO coupling and Zeeman field can be realized using the adiabatic motion of atoms in laser fields [14,15]. The Hamiltonian for this system can be written as ($\hbar = K_B = 1$)

$$H = H_F + H_{\rm soc} + H_{\rm I}, \quad (1)$$

where the single atom Hamiltonian $H_F = \sum_{\mathbf{k},\sigma=\uparrow,\downarrow}(\epsilon_{\mathbf{k}} - \mu_\sigma)C^\dagger_{\mathbf{k}\sigma}C_{\mathbf{k}\sigma}$, $C^\dagger_{\mathbf{k}\sigma}$ is the creation operator for a fermion atom with momentum $\mathbf{k}$ and spin σ, $\epsilon_{\mathbf{k}} = k^2/2m$, m is the atom mass, $\mu_\uparrow = \mu + h$, $\mu_\downarrow = \mu - h$, μ is the chemical potential, and h is the Zeeman field. The Hamiltonian for the Rashba-type SO coupling is $H_{\rm soc} = \alpha\sum_{\mathbf{k}}[(k_y - ik_x)C^\dagger_{\mathbf{k}\uparrow}C_{\mathbf{k}\downarrow} + (k_y + ik_x)C^\dagger_{\mathbf{k}\downarrow}C_{\mathbf{k}\uparrow}]$. The interaction between atoms is described by $H_{\rm I} = -g\sum_{\mathbf{k}}C^\dagger_{-\mathbf{k}\uparrow}C^\dagger_{\mathbf{k}\downarrow}C_{\mathbf{k}\downarrow}C_{-\mathbf{k}\uparrow}$, where the effective regularized interaction parameter $1/g = \sum_{\mathbf{k}}[1/(2\epsilon_{\mathbf{k}} + E_b)]$ for a 2D Fermi gas [28,31,32,41]. In experiments, the binding energy E_b can be controlled by tuning the s-wave scattering length or the barrier height V_0 along the z direction. Small and large E_b values correspond to the BCS and BEC limits, respectively [42].

The finite temperature properties of the 2D Fermi gas are obtained using finite temperature quantum field theory, where the action for the Hamiltonian [Eq. (1)] is $S_V = \int_0^\beta d\tau[\sum_{\mathbf{k},\sigma}C^\dagger_{\mathbf{k}\sigma}\partial_\tau C_{\mathbf{k}\sigma} + H]$ with $\beta = 1/T$. Introducing the standard Hubbard-Stratonovich transformation with the mean field superfluid order parameter $\phi = g\sum_{\mathbf{k}}\langle C_{\mathbf{k}\downarrow}C_{-\mathbf{k}\uparrow}\rangle$ and integrating out the fermion degrees of freedom, we have the partition function $Z = \int D\phi D\phi^* \exp(-S_{\rm eff})$ with the effective action $S_{\rm eff} = \int_0^\beta d\tau(g^{-1}|\phi|^2 + \zeta_{\mathbf{k}}) - \frac{1}{2}\mathbf{Tr}[\ln G^{-1}]$. Here

$$G^{-1} = \begin{pmatrix} \partial_\tau + \zeta_{-,\mathbf{k}} & -\alpha k_- & 0 & -\phi \\ -\alpha k_+ & \partial_\tau + \zeta_{+,\mathbf{k}} & \phi & 0 \\ 0 & \phi^* & \partial_\tau - \zeta_{-,\mathbf{k}} & -\alpha k_+ \\ -\phi^* & 0 & -\alpha k_- & \partial_\tau - \zeta_{+,\mathbf{k}} \end{pmatrix},$$

is the inverse Nambu matrix under the Nambu basis $\Psi(\mathbf{k}) = (C_{\mathbf{k}\uparrow}, C_{\mathbf{k}\downarrow}, C^\dagger_{-\mathbf{k}\downarrow}, C^\dagger_{-\mathbf{k}\uparrow})^T$, the symbol $\mathbf{Tr}$ denotes the trace over momentum, imaginary time, and the Nambu indices, $k_\pm = (k_y \pm ik_x)$, and $\zeta_{\pm,\mathbf{k}} = (\epsilon_{\mathbf{k}} - \mu) \pm h = \zeta_{\mathbf{k}} \pm h$. The superfluid order parameter Δ is obtained from the saddle point of the effective action (i.e., $\partial S_{\rm eff}/\partial\phi^*|_{\phi=\Delta} = 0$), which yields the gap equation

$$\sum_{\mathbf{k}}\frac{1}{(2\epsilon_{\mathbf{k}} + E_b)} = \frac{1}{2}\sum_{\mathbf{k},l=\pm}[\lambda_l E^{-1}_{l,\mathbf{k}}\tanh(\beta E_{l,\mathbf{k}}/2)]. \quad (2)$$

Here the quasiparticle energy spectrum $E_{\pm,\mathbf{k}} = \sqrt{E^2_{\mathbf{k}} + \alpha^2k^2 + h^2 \pm 2A}$, $A = \sqrt{\alpha^2\zeta^2_{\mathbf{k}}k^2 + h^2E^2_{\mathbf{k}}}$, $\lambda_\pm = (1/2)(1 \pm h^2/A)$, $E^2_{\mathbf{k}} = \zeta^2_{\mathbf{k}} + |\Delta|^2$, and $k^2 = k_x^2 + k_y^2$.

In 2D Fermi gases, it is well known that long-range superfluid order can be destroyed by phase fluctuations of the order parameter at any finite temperature [34]. To study the phase fluctuations in SO coupled Fermi gases, we set $\phi = \Delta e^{i\theta}$ following the standard procedure, where θ is the superfluid phase around the saddle point [determined by Eq. (2)] that varies slowly in position and time spaces. The superfluid phase can be decoupled from the original Green's function through a unitary transformation $UG^{-1}(\theta)U^\dagger = G_0^{-1} - \Sigma$, where $U = \exp(iM\theta/2)$, and $M = \mathrm{diag}(1, 1, -1, -1)$. $\Sigma = \tau_3\{[(i\partial_\tau\theta)/2] + [(\nabla\theta)^2/8m]\} - I\{[(i\nabla^2\theta)/4m] + [(i\nabla\theta\cdot\nabla)/2m]\} + (\alpha/2)(\tau_3\sigma_x\partial_y\theta - I\sigma_y\partial_x\theta)$ is the corresponding self-energy, σ_i and τ_i are Pauli matrices in the Nambu space, and G_0^{-1} is the Green's function at $\phi = \Delta$. The effective action can be written as $S_{\rm eff} = S_0(\Delta) + S_{\rm fluc}(\nabla\theta, \partial_\tau\theta)$ [43], where $S_{\rm fluc}(\nabla\theta, \partial_\tau\theta) = \mathrm{tr}\sum_{n\geq1}(1/n)(G_0\Sigma)^n$. Expanding Σ up to the leading order ($n = 2$), we have

$$S_{\rm fluc} = \frac{1}{2}\int d^2\mathbf{r}[J(\nabla\theta)^2 + P(\partial_\tau\theta)^2 - Q(i\partial_\tau\theta)], \quad (3)$$

where $J = (1/4m)\sum_{\mathbf{k}}(n_{\mathbf{k}} - \sum_{l=\pm}(\mathcal{J}_{1,l} + \mathcal{J}_{2,l}))$ represents the phase stiffness or superfluid density. $n_{\mathbf{k}} = 1 - \sum_{l=\pm}\{[\xi(1 + l\eta)/2E_{l,\mathbf{k}}]\tanh[(\beta E_{l,\mathbf{k}})/2]\}$, $\mathcal{J}_{1,l} = [((m\alpha^2)/(2E_{l,\mathbf{k}}))((1 - (k^2\alpha^2\xi^2)/2A^2) + l\{((\Delta^2 + \xi^2)/2A) + h^2((\Delta^2 + \xi^2)^2 + (k^2\alpha^2\Delta^2)/2A^3)\}\tanh((\beta E_{l,\mathbf{k}})/2)]$, $\mathcal{J}_{2,l} = \{[(k^2\beta)/(32m^2)][1 + l(m\alpha^2\xi)/A]\mathrm{sech}^2[(\beta E_{l,\mathbf{k}})/2]\}$, $\eta = (h^2 + k^2\alpha^2)/A$, $P = \sum_{\mathbf{k},l=\pm}(\{[-\xi^2(\eta + l)^2 + E^2_{l,\mathbf{k}}(1 + lh^2\Delta^2\eta/A)/(8E^3_{l,\mathbf{k}})]\}\{\mathrm{sech}[(\beta E_{l,\mathbf{k}})/2]\} + \{[\beta\xi^2(\eta + l)^2/(16E^2_{l,\mathbf{k}})]\}\{[\mathrm{sech}^2(\beta E_{l,\mathbf{k}})/2]\})$, and $Q = \sum_{\mathbf{k}}n_{\mathbf{k}}$. We have checked that the expressions for J, P and Q reduce to previous results in different limits [41,44–46].

The phase θ can be decomposed into a static vortex part $\theta_{\rm v}(\mathbf{r})$ and a time-dependent spin-wave part $\theta_{\rm sw}(\mathbf{r}, \tau)$. As a consequence, the action of the phase fluctuation becomes $S_{\rm fluc} = S_{\rm v} + S_{\rm sw}$ with $S_{\rm v} = (1/2)\int d^2\mathbf{r}J[\nabla\theta(\mathbf{r})]^2$ and $S_{\rm sw} = (1/2)\int d^2\mathbf{r}\{J[\nabla\theta_{\rm sw}(\mathbf{r}, \tau)]^2 + P[\partial_\tau\theta_{\rm sw}(\mathbf{r}, \tau)]^2 - Q[i\partial_\tau\theta_{\rm sw}(\mathbf{r}, \tau)]\} = \sum_{\mathbf{k}}\ln[1 - \exp(-\beta\omega_{\mathbf{k}})]$, where $\omega_{\mathbf{k}} = c|\mathbf{k}|$, $c = \sqrt{J/P}$ is the speed of the spin wave [44]. Note that Eq. (3) is exactly the same as the effective action for the 2D Heisenberg XY model [35,36] with different J, P, and Q; therefore, the BKT transition temperature

$$T_{\rm BKT} = \frac{\pi}{2}J(\Delta, \mu, T_{\rm BKT}). \quad (4)$$

Across the BKT temperature, there is a transition from the pseudogap phase (with finite pairing Δ but without phase coherence or superfluidity) to phase coherent V-AV pairs (with both pairing and superfluidity). When the temperature is lowered further, free V-AV pairs form a tightly bounded V-AV lattice below another critical temperature [37,38],

$$T_{\rm vortex} = 0.3J(\Delta, \mu, T_{\rm vortex}). \qquad (5)$$

The atom density equation can be obtained from the total thermodynamic potential $\Omega = TS_{\rm eff}$, yielding [47],

$$n = -\partial\Omega/\partial\mu = \sum_{\mathbf{k}} n_{\mathbf{k}} - \beta^{-1}\partial S_{\rm sw}/\partial\mu, \qquad (6)$$

where the atom density is $n = mE_F/\pi$, and $E_F = \hbar^2K_F^2/2m$ is the Fermi energy without SO coupling and a Zeeman field. The length unit is chosen as the inverse of the Fermi vector K_F^{-1}.

We numerically solve Eqs. (2), (4), and (6) self-consistently, and calculate various physical quantities. In Fig. 1, we plot $T_{\rm BKT}$ with respect to the SO coupling strength αK_F, the Zeeman field h, and the binding energy E_b. We find that, quite surprisingly, $T_{\rm BKT}$ decreases with increasing α, although the superfluid order parameter Δ is enhanced [28]. The unexpected decrease of $T_{\rm BKT}$ has not been found in the previous literature [41] and can be understood as follows. For $h = 0$ and $\alpha K_F \ll 1$, the superfluid density

$$J(\alpha) - J(\alpha = 0) \sim -\sum_{\mathbf{k}}\left[\frac{\Delta^2k^2\alpha^4}{8E_0^5} + \frac{e^{-\beta E_0}\beta\alpha^2}{8}\right], \qquad (7)$$

decreases with increasing α. Here $E_0 = \sqrt{\xi_{\mathbf{k}}^2 + \Delta^2}$. Physically, the increased density of states near the Fermi surface dominates at small α, hence enhances the phase fluctuation and reduces the superfluid density. Note that the next leading term in Eq. (7) is $\sim\alpha^6$ for $T = 0$ and $\sim\alpha^4$ for $T \neq 0$; therefore, the above analytical result is still valid even for $\alpha K_F \sim E_F$. The Zeeman field is detrimental to the superfluid order parameters, and thus further reduces the superfluid density and the BKT temperature, as shown in Figs. 1(b) and 1(d). Perturbation theory near $h \ll 1$ shows that the change of $T_{\rm BKT}$ is $\sim h^2$, with the coefficient depending strongly on α. Similar features are also found for $T_{\rm vortex}$.

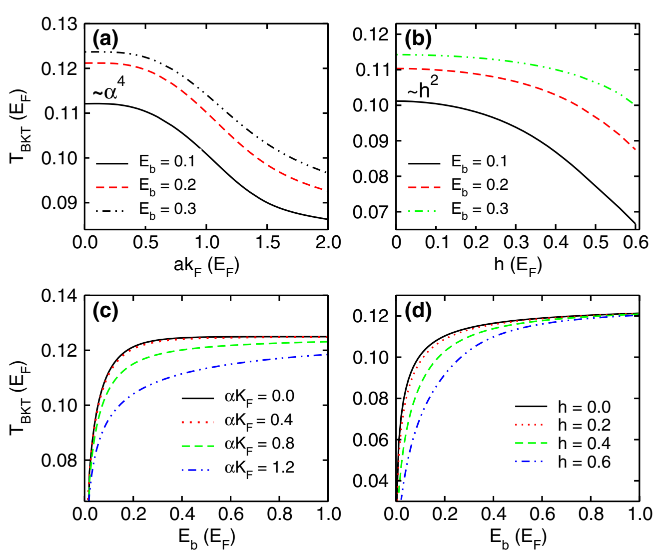

FIG. 1 (color online). (Color online) The dependence of the BKT temperature on (a) SO coupling, (b) Zeeman field, and (c,d) binding energy. $h = 0$ in (a) and (c). $\alpha K_F = 1.0E_F$ in (b) and (d).

In the presence of both SO coupling and a Zeeman field, a topological superfluid can emerge from a regular s-wave interaction when h is larger than a critical value $h_c = \sqrt{\mu^2 + \Delta^2}$ [8,21]. Around h_c, the minimum quasiparticle energy gap occurs at $k = 0$ (i.e., $E_g = E_{-,k=0}$), which first closes and then reopens across h_c, allowing the system to change its topological order from a regular s-wave superfluid to a topological superfluid where MFs exist in vortex cores [8]. At finite temperature, the zero temperature topological phase transition becomes a phase crossover. In Fig. 2(a), we plot the line $E_g = T$ for a finite h, which is obtained by solving Eqs. (2) and (6), and $E_g = T$ self-consistently. The gap closes at a critical E_b^c where $h_c = \sqrt{\mu^2 + \Delta^2} = h$. When $E_b < (>) E_b^c$, $h > (<) h_c$, and the region below the line $E_g = T$ corresponds to the topological (nontopological) superfluid. Above the line $E_g = T$, the temperature is larger than the quasiparticle energy gap and thermal excitations destroy the topological superfluid. We emphasize that at finite temperature there is no sharp phase transition, but only a phase crossover between the topological and nontopological superfluids. The line $E_g = T$ is plotted only as a guide and there is no sharp boundary between the different phases.

The solid and dashed-dotted lines in Fig. 2(a) correspond to $T_{\rm BKT}$ and $T_{\rm vortex}$, respectively. We see that $T_{\rm vortex}$ quickly approaches a constant $T_{\rm vortex} = 3E_F/40\pi$ with increasing binding energy. In the pseudogap region

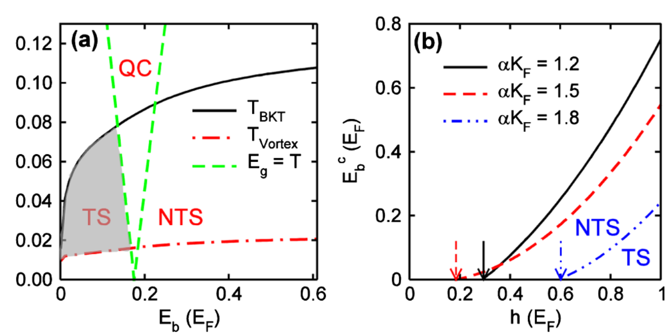

FIG. 2 (color online). (Color online). Parameter region for topological superfluids. (a) QC: quantum critical region; TS: topological superfluids; NTS: nontopological superfluids. The shadow region is the possible parameter region for observing MFs. $h = 0.6E_F$, $\alpha K_F = 1.5E_F$. (b) Phase boundary between TS and NTS at $T = 0$. The arrows mark the required minimum Zeeman fields.

($T > T_{\rm BKT}$), free vortices may exist, but they are not suitable for the observation of MFs due to the lack of phase coherence. Meanwhile, in the vortex lattice region ($T < T_{\rm vortex}$), the zero energy modes may break into two normal states with energy splitting $\propto \Delta e^{-R/\xi}$ [39,48] because of the large tunneling between neighboring vortices in the lattice, where ξ is the coherence length of the superfluid, and R is the intervortex distance. In this region, strong disorder in the tunneling may also lead to Majorana metals [49]. Clearly, the required temperature for observing MFs should be $T_{\rm vortex} < T < T_{\rm BKT}$, instead of the intuitive zero temperature [31,32]. In Fig. 2(a), the shadow regime between $T_{\rm BKT}$ and $T_{\rm vortex}$ gives the possible parameter range for the experimental observation of MFs which exist in the cores of the naturally present V-AV pairs in this region. Note that the intervortex distance R is still essential for the observation of MFs in this region. Such a distance may be estimated using an analogy between V-AV pairs and the 2D Coulomb gases. It has been shown [50] that the mean-square radius $\langle R^2\rangle \sim \xi^2[(\pi\beta J - 1)/(\pi\beta J - 2)]$, which diverges at $T = T_{\rm BKT}$ [see Eq. (4)], and the V-AV pair breaks into free vortices. Therefore there should exist a finite temperature regime below $T_{\rm BKT}$ where $R \gg \xi$ and the splitting of the zero energy Majorana states is vanishingly small. Note here that our theory can only capture the average behavior of V-AV pairs and a more delicate theory is still needed to further understand the detailed structure of V-AV pairs.

In Fig. 2(b), we plot the parameter region for the topological superfluids with respect to the Zeeman field and the binding energy at the zero temperature. Generally, the required critical E_b^c for the topological superfluids increases when h increases. Note that here the phase boundary is determined by $h_c = \sqrt{\mu^2 + \Delta^2}$ at $T = 0$, but does not shift much, even at finite temperature.

Because MFs can only be observed at finite temperature, there exists a nonzero probability $\sim \exp(-\eta)$ for thermal excitations to nontopological excited states in the vortex core, where the ratio $\eta = \epsilon_m/T$, $\epsilon_m \sim \Delta^2/(2E_F)$ is the minimum energy gap (minigap) [51,52] in the vortex core that protects zero energy MFs. For the observation of MFs, it is crucially important to have $\eta > 1$, in addition to the requirement of the temperature $T_{\rm vortex} < T < T_{\rm BKT}$ (i.e., $\eta > \eta_{\rm BKT} \equiv \epsilon_m/T_{\rm BKT}$ and $\eta < \eta_{\rm vortex} \equiv \epsilon_m/T_{\rm vortex}$). When $E_b \gg E_F$, we have $\eta_{\rm BKT} = 8E_b/E_F$ and $\eta_{\rm vortex} = 40\pi E_b/E_F$, which means $\eta \gg 1$ for a large E_b. Remarkably, η is also dramatically enhanced by the SO coupling in the BCS side. In Fig. 3, we plot $\eta_{\rm BKT}$ and $\eta_{\rm vortex}$ with respect to E_b for parameters $h = 0.8E_F$ and $\alpha K_F = 1.6E_F$. The vertical line at $E_b \simeq 0.33E_F$ is the boundary between the topological and nontopological superfluids. There is a broad region in the BCS side with $\eta_{\rm BKT} > 1$ even at the highest temperature $T_{\rm BKT}$. The region can be much larger when the temperature is further lowered to $T = T_{\rm vortex}$. The solid circle represents the

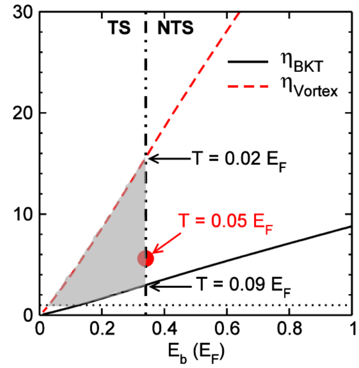

FIG. 3 (color online). (Color online) Plot of $\eta = \varepsilon_m/T$ as a function of E_b. $h = 0.8E_F$ and $\alpha K_F = 1.6E_F$. Dashed-dotted line: $E_b^c = 0.33E_F$ as the corresponding boundary between topological superfluids (TS) and nontopological superfluids (NTS) [see the $E_g = T$ line in Fig. 2(a) for the determination of the phase boundary]. Dotted line: $\eta = 1$. Filled circle corresponds to η at $T = 0.05E_F$. The shadow region is the possible parameter region for observing MFs.

possible temperature $T = 0.05E_F$ that may be accessible in experiments in the near future [12,53,54], yielding $\eta \sim 5.6$ and a thermal excitation probability less than 0.4%. Such a small probability clearly demonstrates that MFs can actually be observed at realistic temperatures in experiments. Note that in the presence of SO coupling ϵ_m may be larger than $\Delta^2/(2E_F)$ used for our estimate [52]; therefore, η could be even larger, which further suppresses thermal excitations.

We illustrate how to find suitable parameters, in particular E_b, for the observation of MFs. Because the topological superfluids only exist in the region $h > \sqrt{\mu^2 + \Delta^2}$, we need $|\Delta|^2 \gg |\mu|^2$ to obtain a small h and a large Δ. Clearly the BEC limit with large E_b does not work because $|\mu| \sim |E_F - E_b/2| \gg \Delta \sim \sqrt{2E_bE_F}$. While in the BCS side, $|\Delta|^2 \gg |\mu|^2$ can be achieved by choosing a suitable SO coupling, Zeeman field, and binding energy [28]. To obtain a large quasiparticle minigap in the vortex core (thus a large η) for MFs, the binding energy should be set to be close to the critical E_b^c (see Fig. 3). At the same time, the bulk quasiparticle gap should also be chosen to be much larger than the temperature. Note that in experiments the bulk quasiparticle gap for the topological superfluid can be detected using the recently experimentally demonstrated momentum-resolved photoemission spectroscopy [55].

Finally we briefly compare the cold atomic gases [9] with the semiconductor-superconductor nanostructure where certain signatures of MFs have been observed in experiments [56–59]. Both systems share the same ingredients for MFs: SO coupling, Zeeman fields, and s-wave pairing; in addition, the s-wave pairing is through an intrinsic s-wave interaction in cold atoms, but is externally induced in the nanostructure. Because of its high controllability and freedom from disorder (lacking in the corresponding solid state systems), the topological cold atomic superfluids provide an ideal and promising platform for observing MFs and the associated non-Abelian statistics, which are both fundamentally and technologically important.

In summary, we study the BKT transition in 2D SO coupled Fermi superfluids and find an unexpected decrease of the BKT transition and vortex lattice melting temperatures with increasing SO coupling. We characterize the finite temperature phase diagram for the experimental observation of MFs in this system. Our work not only provides a basis for the future study of rich and exotic 2D SO coupled Fermi superfluid physics, but also yields realistic parameter regions for experimentally realizing nontrivial topological superfluid states from stable cold atom s-wave superfluids.

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*Corresponding author.
chuanwei.zhang@utdallas.edu

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