

Magnon Mediated Electric Current Drag Across a Ferromagnetic Insulator Layer

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In a semiconductor heterostructure, the Coulomb interaction is responsible for the electric current drag between two 2D electron gases across an electron impenetrable insulator. For two metallic layers separated by a ferromagnetic insulator (FI) layer, the electric current drag can be mediated by a nonequilibrium magnon current of the FI. We determine the drag current by using the semiclassical Boltzmann approach with proper boundary conditions of electrons and magnons at the metal–FI interface.

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The conventional Coulomb drag effect [1–3] occurs in two-dimensional electron gases separated by an insulator barrier. When one of the electron gas carries a current, the momentum transfer due to Coulomb interaction leads to a small current in the other electron gas. Recently, this current drag phenomenon has been discovered in a different system with entirely different physical mechanisms [4]: When an electric current is injected into a Pt bar deposited on a magnetic insulator Yttrium–Iron–Garnet (YIG) film, it is found that a small electric voltage is induced in the other Pt bar, which is also deposited on the same YIG film but is located several millimeters away from the current carrying Pt bar. The authors [4] attributed their finding to the combined effects of spin transfer torque (STT) [5,6] and spin pumping [7,8]: The spin Hall [9] current generated by the electric current in one Pt layer (Pt is a known material with a large spin Hall angle) is absorbed by the ferromagnetic insulator (FI) and for a sufficiently large STT, the magnetic moment of the FI begins precessing. The precessing FI pumps out a spin current to the other Pt layer, resulting in an electron current due to the inverse spin Hall effect [10–15].

In this Letter, we propose a different geometry in which the magnon current flows normal to the plane of the layers throughout the structure. We show that the electron spin current in the metallic layers induces a nonequilibrium magnon current in the FI layer. By using semiclassical Boltzmann approach for electrons and magnons, we are able to self-consistently determine these currents and thereby obtain the drag current for given geometrical and material parameters. The resulting drag current is several orders of magnitude larger than that in the nonlocal geometry in Ref. [4].

To be more specific, we consider a simple trilayer structure, shown in Fig. 1 schematically, where a FI layer is sandwiched by two heavy metal films (NM1 and NM2) such as Pt and Ta. A charge current parallel to the plane of the layers is injected in the layer NM1. To determine the drag current in the layer NM2, we first establish transport equations for each layers and then find proper boundary conditions to solve the transport coefficients.

Electron current and spin accumulation in metallic layers.—For the NM layers, a spin dependent Ohm’s law has been well established and may be written in the following form [16],

$$\hat{\mathbf{j}} = \frac{c}{2} \hat{\mathbf{E}} + \frac{c_h}{4} (\hat{\mathbf{E}} \times \boldsymbol{\sigma} - \boldsymbol{\sigma} \times \hat{\mathbf{E}}), \quad (1)$$

where the spinor current density $\hat{\mathbf{j}}$ and the electric field $\hat{\mathbf{E}}$ are 2×2 vector matrices in spin space, $\boldsymbol{\sigma}$ is a Pauli vector matrix, and c and c_h are the electric conductivity and spin Hall conductivity, respectively. The second term is the spin Hall current whose antisymmetric form is essential for $\hat{\mathbf{j}}$ to be an Hermitian in spin space (also note that $\hat{\mathbf{E}} \times \boldsymbol{\sigma} \neq -\boldsymbol{\sigma} \times \hat{\mathbf{E}}$ due to noncommutivity of the Pauli matrices). The electrical field is related to the spinor chemical potential $\hat{\mu}$ via $\hat{\mathbf{E}} = -(1/e)\nabla\hat{\mu}$, where $e(<0)$ denotes the electron charge. While it is possible to work with an arbitrary choice of the spin quantization, we proceed below to a special case where the magnetic moment of the FI is oriented in the z direction and the electric current flows in the y direction. If we choose the spin quantization axis parallel to the z axis, one can simply work on the two-component (spin up and spin down) form of the Ohm’s law; that is,

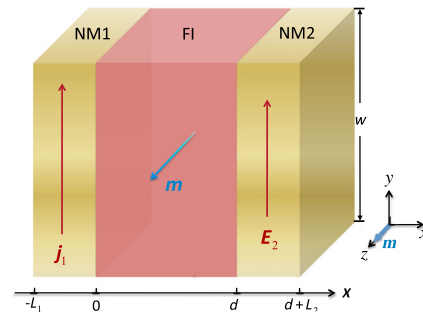


FIG. 1 (color online). Schematics of the NM/FI/NM trilayer structure.

$$j_y^\alpha(x) = \frac{c}{2} E_y^\alpha(x) - \alpha \frac{c_h}{2} E_x^\alpha(x) \quad (2)$$

and

$$j_x^\alpha(x) = \frac{c}{2} E_x^\alpha(x) + \alpha \frac{c_h}{2} E_y^\alpha(x), \quad (3)$$

where $\alpha = \pm 1$ represent spin up and spin down. To determine the spin dependent electric field, we recall the spin diffusion equation [17]

$$\frac{d^2}{dx^2} [\mu^\uparrow(x) - \mu^\downarrow(x)] = \frac{\mu^\uparrow(x) - \mu^\downarrow(x)}{\lambda_{sf}^2} \quad (4)$$

and its solution

$$\mu^\uparrow(x) - \mu^\downarrow(x) = A_i e^{-x/\lambda_{sf}} + B_i e^{x/\lambda_{sf}}, \quad (5)$$

where λ_{sf} is the electron spin diffusion length, and the constants A_i and B_i ($i = 1$ for the NM1 layer and $i = 2$ for NM2) are determined by the boundary conditions. Although these equations apply to both NM1 and NM2 layers, they have different constraints set by experimental measurement. For the NM1 layer, we take $E_y^\alpha(x) = E_{\text{ext}}$ where E_{ext} is the applied electric field in the NM1 layer, while $\int_{\text{NM2}} dx j_y(x) = 0$ in the open circuit of the NM2 layer.

Magnon current and magnon accumulation in the FI layer.—For the FI layer, we start with a general magnon Boltzmann equation in the presence of spatially dependent temperature $T(x)$ and magnetic field $\mathbf{H}(x)$,

$$\begin{aligned} \mathbf{v}_x \frac{\partial N_m}{\partial x} + \mathbf{v}_x \frac{\partial N_m}{\partial T} \frac{dT}{dx} + \mathbf{v}_x \frac{\partial N_m}{\partial \mathbf{H}} \cdot \frac{d\mathbf{H}}{dx} + \dot{\mathbf{q}} \cdot \frac{\partial N_m}{\partial \mathbf{q}} \\ = - \left(\frac{\partial N_m}{\partial t} \right)_{\text{scatt.}}, \end{aligned} \quad (6)$$

where $N_m(x, \mathbf{q}, T(x), \mathbf{H}(x))$ is the magnon distribution. The first term describes magnon diffusion. The second and third terms are responsible for the magnon transport in the presence of temperature and magnetic field gradients, which have been recently studied in the content of spin caloritronics [18–21]. The last term on the left side of Eq. (6) is associated with acceleration of magnons by external forces such as a confining potential at boundary [22]. The scattering term on the right side of the Eq. (6) may be modeled by the relaxation time approximation

$$\left(\frac{\partial N_m}{\partial t} \right)_{\text{scatt.}} = \frac{N_m - \bar{N}_m}{\tau_m} + \frac{N_m - N_m^0}{\tau_{\text{th}}}, \quad (7)$$

where $\bar{N}_m(x) = \int d\mathbf{q} N_m(x, \mathbf{q}) / \int d\mathbf{q}$ is the momentum averaged magnon distribution while $N_m^0(x, \mathbf{q}) = [e^{\epsilon_{\mathbf{q}}/k_B T(x)} - 1]^{-1}$ is the local equilibrium magnon distribution, where $\epsilon_{\mathbf{q}} = D\mathbf{q}^2 + \Delta_g$ is the magnon dispersion, D is the spin wave stiffness, Δ_g is the spin wave gap, and

$\mathbf{v}_x = \frac{1}{\hbar} \frac{\partial \epsilon_{\mathbf{q}}}{\partial q_x}$ is the x component of the magnon velocity. The first relaxation term describes those processes which conserve the number of magnons. For example, magnon scattering by a paramagnetic impurity has the form of $V_{\mathbf{q}\mathbf{q}'} a_{\mathbf{q}}^+ a_{\mathbf{q}'}$; that is, the impurity or surface roughness [23,24] scatters the magnon \mathbf{q}' to the magnon \mathbf{q} . As long as we neglect the wave number dependence of the scattering matrix $V_{\mathbf{q}\mathbf{q}'}$, this process can be modeled by the first term of Eq. (7). The second term of Eq. (7) does not conserve the number of magnons. The magnon absorption and emission relax the nonequilibrium magnons to equilibrium ones, e.g., magnon–phonon interaction [25].

For the present system, we consider uniform temperature and magnetic field, and there is no external force on magnons. Then, Eqs. (6) and (7) reduce to

$$\begin{aligned} \mathbf{v}_x \frac{\partial N_m(x, \mathbf{q})}{\partial x} = - \frac{N_m(x, \mathbf{q}) - \bar{N}_m(x)}{\tau_m} \\ - \frac{N_m(x, \mathbf{q}) - N_m^0(\mathbf{q})}{\tau_{\text{th}}}. \end{aligned} \quad (8)$$

We may proceed to solve N_m by the same way as for the electron distribution in magnetic multilayers [17]. Particularly, one may expand the nonequilibrium distribution by the Legendre polynomials,

$$\begin{aligned} N_m(x, \mathbf{q}) = N_m^0(\mathbf{q}) \\ + \frac{\partial N_m^0(\mathbf{q})}{\partial \epsilon_{\mathbf{q}}} \left[\mu_m(x) + \sum_{n=1}^{\infty} g^{(n)}(x) P_n(\cos\theta) \right], \end{aligned} \quad (9)$$

where $\mu_m(x)$ is the $n = 0$ component of the nonequilibrium distribution and θ is the angle between \mathbf{q} and x axis. By placing the above equation into Eq. (8) and by utilizing the orthogonality property of the Legendre polynomials, one can arrive at a series of algebraic equations for the coefficients $g^{(n)}(x)$. In the Supplemental Material [26], we show the solutions in some limiting cases. Once the distribution functions $N_m(x, \mathbf{q})$ are determined, we can find the magnon accumulation and magnon current via

$$\begin{aligned} j_m(x) = \frac{-2\mu_B}{(2\pi)^3} \int d\mathbf{q} \mathbf{v}_x N_m(\mathbf{q}, x); \\ \delta n_m(x) = \frac{1}{(2\pi)^3} \int d\mathbf{q} [N_m(\mathbf{q}, x) - N_m^0(\mathbf{q})], \end{aligned} \quad (10)$$

where μ_B is the Bohr magneton. Note that a magnon carries spin moment $-\gamma\hbar (= -2\mu_B)$ where γ is the gyromagnetic ratio.

We may further simplify the solution of the nonequilibrium magnon distribution by discarding high orders ($n \geq 2$) of the polynomials. Consequently, we find a local relation between magnon accumulation and magnon current,

$$\frac{d}{dx} j_m(x) = 2\mu_B \frac{\delta n_m(x)}{\tau_{\text{th}}} \quad (11)$$

and

$$j_m(x) = \frac{2\mu_B \tau_m}{3} \frac{I_2}{I_0} \frac{d}{dx} \delta n_m(x), \quad (12)$$

where I_n are integration constants $I_n \equiv \frac{1}{(2\pi)^3} \int d^3q v^n \frac{\partial N_m^0(\mathbf{q})}{\partial \epsilon_q}$. We point out that this local current expression is valid in the limit $\tau_{\text{th}} \gg \tau_m$ which is a good approximation for ferromagnets [25] (see Supplemental Material [26]). By combining Eqs. (11) and (12), we obtain the diffusion equation for nonequilibrium magnons,

$$\frac{d^2}{dx^2} \delta n_m(x) - \frac{\delta n_m(x)}{l_m^2} = 0, \quad (13)$$

where the magnon diffusion length is defined as $l_m = \sqrt{\frac{I_2}{3I_0} \tau_{\text{th}} \tau_m}$. At room temperature ($T = 300$ K), for YIG with $\Delta_g \sim 10^{-5}$ eV, $\tau_m \sim 10^{-7}$ s, and $\tau_{\text{th}} \sim 10^{-6}$ s, l_m is estimated at 0.05 cm, consistent with the measurement [27]. Equation (13) has the general solution,

$$\delta n_m(x) = A_F e^{-x/l_m} + B_F e^{x/l_m} \quad (14)$$

and thus the magnon current density reads

$$j_m(x) = \frac{2\mu_B l_m}{\tau_{\text{th}}} (-A_F e^{-x/l_m} + B_F e^{x/l_m}). \quad (15)$$

Boundary conditions.—The outer-boundary conditions at $x = -L_1$ and $x = d + L_2$, where L_1 , d , and L_2 represent the thicknesses of the layers of NM1, FI, and NM2, are $j_x^{\uparrow\downarrow}(-L_1) = j_x^{\uparrow\downarrow}(d + L_2) = 0$. The boundary conditions at the metal–FI interfaces depend on the interaction between electrons and magnons. Here we assume an s - d type interaction $-J_{sd} \boldsymbol{\sigma} \cdot \mathbf{S}_i$, where $\boldsymbol{\sigma}$ is the itinerant electron spin of the metal layer and \mathbf{S}_i is the local spin of the FI layer at the interfaces. The interaction conserves total angular momentum and thus the first boundary condition is the continuity of total spin current at the interfaces; that is,

$$(-\mu_B/e)[j_x^{\uparrow}(0^-) - j_x^{\downarrow}(0^-)] = j_m(0^+)$$

and

$$j_m(d^-) = (-\mu_B/e)[j_x^{\uparrow}(d^+) - j_x^{\downarrow}(d^+)]. \quad (16)$$

The total angular momentum current conservation simply states that electron spin current in the metals must be converted into magnon current in the FI layer at the interfaces. If the interfaces have magnetic roughness, the spin-flip scattering by magnetic impurities can transfer spin angular momentum to lattice via spin–orbit coupling. In this case, the outgoing spin current would be reduced [28].

The other boundary conditions at the interfaces should relate the electron spin accumulation to the nonequilibrium magnon density. Within the s - d model, one can treat the

electron spin density as an effective magnetic field on the interface spin of the FI layer; that is, $\mathbf{H}_{\text{eff}} = J_{sd} \delta \mathbf{m}_z$ and we find,

$$\mu^{\uparrow}(0^-) - \mu^{\downarrow}(0^-) = \varepsilon \delta n_m(0^+)$$

and

$$\mu^{\uparrow}(d^+) - \mu^{\downarrow}(d^+) = \varepsilon \delta n_m(d^-), \quad (17)$$

where

$$\varepsilon = \frac{4(\pi D)^{3/2}}{J_{sd} D(\epsilon_F) a_0^3 \cdot \sqrt{k_B T} Li_{1/2}(e^{-\Delta_g/k_B T})}, \quad (18)$$

$D(\epsilon_F)$ is the electron density of state at Fermi level, a_0 is the lattice constant of the NM layer, and $Li_s(z) \equiv \sum_{k=1}^{\infty} \frac{z^k}{k^s}$ is the polylogarithm. The detailed derivation of ε is arranged in the Supplemental Material [26]. We note that Takahashi *et al.* [29] have proposed a boundary condition at the interface which relates magnon spin current to spin accumulation, that is, $j_m \propto \mu^{\uparrow} - \mu^{\downarrow}$; such boundary condition is unable to self-consistently determine the magnon current. A rough order of magnitude estimation of ε can be readily obtained by using the following plausible parameters appropriate for Pt/YIG/Pt structure: $D(\epsilon_F) \sim 3n_e/2\epsilon_F$, $n_e = 5 \times 10^{22}$ cm $^{-3}$, $J_{sd} = 1$ eV, $\epsilon_F = 5$ eV, $a_0 = 4$ Å, and $D = 6$ meV · nm 2 , and thus $\varepsilon \sim 0.2$ meV · nm 3 .

With the above boundary conditions, the constants in Eqs. (5) and (14) can be readily determined. If one uses an Ampere meter [30] to measure the average (measured) induced electric current density $j_y^{(2)} = (1/L_2) \int dx j_y^{(2)}(x)$ in NM2 layer, we find that the ratio between the induced current and the injected current magnitude $\eta (\equiv j_y^{(2)}/j_y^{(1)})$ is,

$$\eta = \left(\frac{c_h}{c}\right)^2 \left(\frac{\lambda_{\text{sf}}}{L_2}\right) \times \frac{\text{sech}\left(\frac{L_2}{\lambda_{\text{sf}}}\right) [\cosh\left(\frac{L_2}{\lambda_{\text{sf}}}\right) - 1]}{\left[b^{-1} + b \tanh\left(\frac{L_2}{\lambda_{\text{sf}}}\right)\right] \sinh\left(\frac{d}{l_m}\right) + \left[1 + \tanh\left(\frac{L_2}{\lambda_{\text{sf}}}\right)\right] \cosh\left(\frac{d}{l_m}\right)}, \quad (19)$$

where $b = c\varepsilon\tau_{\text{th}}/(4e^2 l_m \lambda_{\text{sf}})$, and $L_1 \gg \lambda_{\text{sf}}$ is assumed for simplicity. The first prefactor $(c_h/c)^2$ originates from the two successive conversions between electric current and spin Hall current in NM1 and NM2 due to the spin Hall and inverse spin Hall effect, respectively. The second prefactor λ_{sf}/L_2 indicates that the range of the current density in NM2 is λ_{sf} ; that is, if the thickness of NM2 exceeds λ_{sf} , the average current density $j_y^{(2)}$ would be inversely proportional to L_2 . Interestingly, when L_2 is much smaller than λ_{sf} , the induced electric current is also small; this is because the spin current at the surface $x = d + L_2$ is zero and thus the self-consistent calculation demands a small current throughout the NM2 layer. In Fig. 2, we show η as a function of the thickness of the metal layer (NM2) for Pt/YIG/Pt trilayers with several different YIG layer

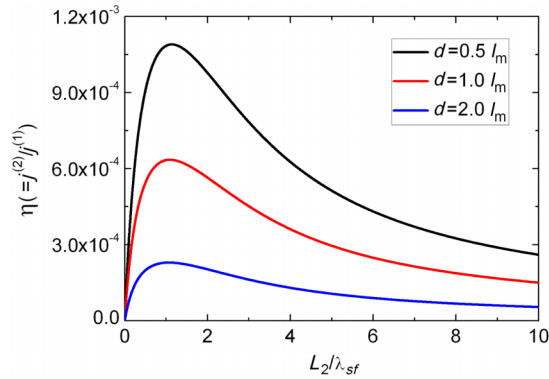


FIG. 2 (color online). The ratio of the average induced current density and the injected current density as a function of the NM2 (Pt) layer thickness for three different thicknesses of the FI (YIG) layer.

thicknesses. We choose the material parameters as follows: Pt layer conductivity $c_{\text{Pt}} \sim 0.1(\mu\Omega \cdot \text{cm})^{-1}$, spin diffusion length $\lambda_{\text{sf}} = 7$ nm, and the spin Hall angle $c_h/c = 0.05$ [31]; magnon diffusion length $l_m = 0.05$ cm and magnon relaxation time $\tau_{\text{th}} = 10^{-6}$ s. We see that η decreases as the thickness of the YIG layer increases due to the decay of magnon diffusion current. Also for fixed YIG layer thickness, η reaches its maximum around $L_2 = \lambda_{\text{sf}}$. The peak value of η is of the order of 10^{-4} . If the injected current density is 10^6 A/cm², the induced voltage of a Pt bar with its length $w = 1$ cm would be $V^{(2)} = w j_y^{(2)}/c \sim 1$ mV. If one replaces Pt with Ta which has larger spin Hall angle of 0.1 [32], $V^{(2)}$ can be further increased by a factor of 4.

Finally we comment on the relation of our calculation with the experimental measurement [4]. In their experiments, when the first Pt layer injects a spin current to the FI layer, the magnons propagate in the plane of the FI layer in order to reach the second Pt. While there is a similar nonequilibrium magnon density buildup near the second Pt layer, the *direction* of the magnon current and the *gradient* of the magnon density are in the plane of the layer. In another word, there is neither magnon current nor magnon density gradient in the direction perpendicular to the layer such that the second Pt layer is unable to receive any spin angular momentum from the FI layer. Thus, we conclude that the nonlocal setup in the experiment [4] is not relevant to our theory. In the conventional nonlocal metallic spin valve, however, one does observe a voltage change of the entire detection bar due to the spin accumulation (not the spin current or gradient of the spin accumulation) in the channel. In the present case, we derive the induced current *in* the second Pt bar which is related to the spin current (or magnon density gradient) in the direction perpendicular to the layer. Furthermore, the observed current in the experiment [4] has been attributed to the STT and spin pumping, which is several orders of magnitude smaller than what we predict in our geometry.

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