Comment on "Impact of a Global Quadratic Potential on Galactic Rotation Curves"

In a recent Letter, Mannheim and O'Brien [1] have presented conformal gravity rotational velocity $v^2_{\rm total}$ fits to the observed data of several galaxy samples, which seem good enough to indicate that conformal gravity could be an interesting alternative to the dark matter hypothesis. An important prediction of the theory is the testable upper limit on the size of the galaxies projected from $v^2_{\rm total} \to 0$ (hence effectively the global limit $R^{\rm global}_{\rm proj} \le \gamma_0/\kappa \approx 100$ kpc). The purpose of this Comment is to correct that this upper limit should be fixed by the criterion of the stability of orbits. If the canonical stable limit is observationally surpassed, conformal theory would be falsified even if the last observed orbit remains within $R^{\rm global}_{\rm proj}$.

Note that emission occurs from stable circular material orbits with information propagating along null geodesics (see, for instance [2]). The stability criterion can severely constrain the extent of the H1 gas and we observe that conformal gravity endows each galaxy with a maximal stable limit $R = R^{\rm max}_{\rm stable}$ that falls within the limit $R^{\rm global}_{\rm proj}$. The two limits often differ significantly, by as much as 20%–30%. Since stability is an essential physical condition, we think that only $R^{\rm max}_{\rm stable}$ should be regarded as the testable upper limit on the size of a galaxy. With their metric ansatz, the geodesic for a single test particle yields the tangential velocity for circular orbits $v^2 = (Rc^2/2)B'$ (primes denote derivatives with respect to R). With approximate $v^2_{\rm total}$, it integrates to

$$B(R) = 1 - \frac{2N^*\beta^*}{R} + (N^*\gamma^* + \gamma_0)R - \kappa R^2 + \frac{3R_0^2N^*\gamma^*}{2R} + \frac{15R_0^4N^*\gamma^* - 24R_0^2N^*\beta^*}{8R^3}. \quad (1)$$

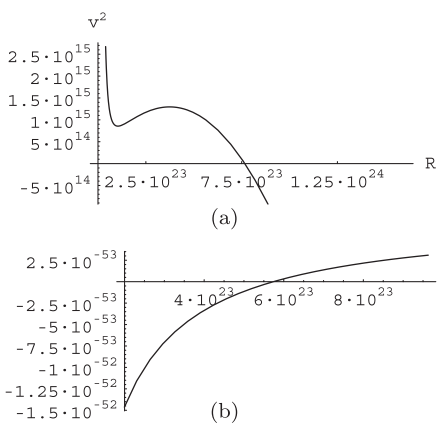

FIG. 1. (a) $v^2_{\rm total}$ vs R and (b) $f(R)$ vs R for UGC2885.

The radial geodesic is given by

$$\left(\frac{dR}{dt}\right)^2 = B^2(R) - a\frac{B^3(R)}{R^2} - bB^3(R), \quad (2)$$

where a and b are constants fixed by the usual conditions for circular orbits. The condition for stability is that the second derivative of the right-hand side ("effective potential") of Eq. (2) with respect to R must be negative, which leads to the generic requirement that

$$f(R) \equiv 2B'^2(R) - B(R)B''(R) - 3B(R)B'(R)/R < 0. \quad (3)$$

We illustrate our comments here only for UGC2885 [Figs. 1(a) and 1(b)]. The rest of the samples yield similar patterns. The very fact that there exists a finite limit $R^{\rm max}_{\rm stable}$ caused *entirely* by the quadratic potential $V_\kappa(r) = -\kappa c^2 r^2/2$, clearly distinguishes conformal theory from some dark matter models because in the latter there is no such limit; see, e.g., [3,4]. Note that we do not know precisely what would happen beyond this special radius $R^{\rm max}_{\rm stable}$, but gas in noncircular motions at larger radii is not certainly excluded. Interestingly, the predicted $R^{\rm max}_{\rm stable}$ does not even much exceed the current $R_{\rm last}$ for many samples, e.g., UGC0128 has $R^{\rm max}_{\rm stable} = 65.6$ kpc, while $R_{\rm last} = 54.8$ kpc, so we might not have to wait too long. The main thing to watch is whether or not any updated $R_{\rm last}$ shoots past $R^{\rm max}_{\rm stable}$, which fortunately has not happened yet. Updated observations on $R_{\rm last}$ would thus provide a nice test of the conformal gravity prediction of $R^{\rm max}_{\rm stable}$ and hence of the global quadratic potential.

Dr. Mannheim [5] has the opinion that the general stability analysis may be performed considering many body dynamics. Observations on where circular orbits might actually terminate could thus be very instructive.

Kamal K. Nandi[1,2] and Arunava Bhadra[3]
[1]Ya. B. Zel'dovich International Center for Astrophysics
Ufa 450000, Russia
[2]Department of Mathematics
University of North Bengal
Siliguri 734013, India
[3]High Energy and Cosmic Ray Research Centre
University of North Bengal
Siliguri 734013, India

Received 17 April 2012; published 16 August 2012
DOI: 10.1103/PhysRevLett.109.079001
PACS numbers: 04.50.Kd, 95.30.Sf

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