Berezinskii-Kosterlitz-Thouless Transition to the Superconducting State of Heavy-Fermion Superlattices

Jian-Huang She[1] and Alexander V. Balatsky[1,2]

[1]*Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA*
[2]*Center for Integrated Nanotechnologies, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA*
(Received 19 March 2012; published 16 August 2012)

We propose an explanation of the superconducting transitions discovered in the heavy-fermion superlattices by Mizukami *et al*. [Nature Phys. **7**, 849 (2011)] in terms of Berezinskii-Kosterlitz-Thouless (BKT) transition. We observe that the effective mass mismatch between the heavy-fermion superconductor and the normal metal regions provides an effective barrier that enables quasi-2D superconductivity in such systems. We show that the resistivity data, both with and without magnetic field, are consistent with BKT transition. Furthermore, we study the influence of a nearby magnetic quantum critical point on the vortex system and find that the vortex core energy can be significantly reduced due to magnetic fluctuations. Further reduction of the gap with decreasing number of layers is understood as a result of pair breaking effect of Yb ions at the interface.

DOI: 10.1103/PhysRevLett.109.077002 PACS numbers: 74.78.Fk, 74.25.fc, 74.62.−c, 74.70.Tx

Thin film growth technology recently has advanced to the point that artificial two-dimensional structures can be fabricated with atomic-layer precision. This has enabled the exploration of novel aspects of emergent phenomena in low dimensional systems with unprecedented control. Using the molecular beam epitaxy technique, Mizukami *et al*. have grown $CeCoIn_5/YbCoIn_5$ superlattices, where superconductivity was found to occur in the two-dimensional Kondo lattice [1]. The combination of f-electron physics, low dimensionality, and interface effects provides a rare opportunity to study new states in strongly correlated electron systems, e.g., unconventional superconductivity, dimensionally tuned quantum criticality [2], interplay of magnetism and superconductivity, Fulde-Ferrell-Larkin-Ovchinnikov phases, and to induce symmetry breaking not available in the bulk like locally broken inversion symmetry [3].

Here, we investigate the mechanism for the onset of superconductivity in such heavy-fermion superlattices. We propose an explanation of the experimental results of Ref. [1] within the framework of Berezinskii-Kosterlitz-Thouless (BKT) transition and further study the interplay of Kondo lattice physics and BKT mechanism. While well established for superfluid films, BKT transition is less convincing for superconductors (See Ref. [4] and references therein). Though implications have been found in numerous thin superconducting films [4–10], highly anisotropic cuprates [11–14], and oxide interfaces [15–17], the results have remained inconclusive (see, e.g., Ref. [18,19]). It is therefore desirable to have a well-controlled, readily tunable system to investigate the BKT physics. The epitaxially grown heavy-fermion superlattices may serve such a role.

Quasi-two-dimensional superconductivity.—First, we discuss why BKT is applicable to HF superlattices. In the $CeCoIn_5/YbCoIn_5$ superlattice, one has a layered structure of alternating heavy-fermion superconductor ($CeCoIn_5$) and conventional metal ($YbCoIn_5$), typically 3.5 nm thick. Proximity effects are expected to happen in such normal metal/superconductor (N, S) junctions. For conventional superconductors, the thickness of the leakage region is on the order of the thermal length $\hbar v_N/2\pi k_B T$, where v_N is the Fermi velocity in the N region (see, e.g., Ref. [20]). At low temperatures, this thickness is typically of order 100 nm, which is much larger than the separation of $CeCoIn_5$ layers. One may thus expect a strong coupling between the superconducting $CeCoIn_5$ layers, and the system would behave as three-dimensional superconductor. However, as we will argue below, the large mismatch of Fermi velocities across the interface changes the story completely and enables quasi-2D superconductivity in $CeCoIn_5$ thin layers.

In normal metal/heavy-fermion superconductor proximity effect studies, it was realized that the large mismatch of effective mass at the interface leads to huge suppression of transmission of electron probability currents [21]. The ratio r_T of the transmitted probability current and the incident current is determined by the ratio of the effective masses, $r_T \simeq 4m_l/m_h$, for $m_h \gg m_l$ [21]. The effective mass of $CeCoIn_5$ is of order $100m_e$. For the more conventional metal $YbCoIn_5$, we take its effect mass to be of order m_e. The transmission is thus on the order of one percent.

This result is intimately related to that of Blonder, Tinkham, and Klapwijk [22,23], where it was shown that the mismatch of Fermi velocities between the N and S regions increases the barrier height between the two, with the effective barrier parameter Z modified to $Z = [Z_0^2 + (1-r)^2/4r]^{1/2}$ where $r = v_S/v_N$ is the ratio of two Fermi velocities. This gives essentially the same result as Ref. [21]. This suppression factor significantly degrades the proximity coupling to the point where a 4 nm normal layer renders heavy-fermion films essentially uncoupled. A direct consequence of the reduced proximity effect is an enhanced c axis resistivity, which can be measured directly in experiment.

More extensive numerical studies of proximity effect in N, S junctions have been carried out recently [24], where it was shown that the proximity effect is substantially suppressed with moderate mismatch of Fermi energies. Another source of suppression of the proximity effect is the pair breaking effects of Yb ions at the interface (see Supplemental Material [25]). It is also expected that a weak magnetic field can destroy the proximity-induced superconductivity in $YbCoIn_5$ layers [1,26].

Suppression of the proximity effect in the $CeCoIn_5/YbCoIn_5$ superlattice and the fact that the thickness of the $CeCoIn_5$ layers is on the order of the perpendicular coherence length $\xi_\perp \sim 20$ Å [1], lead to the conclusion that superconductivity in such systems is essentially two dimensional, and one expects BKT physics to be relevant in such systems.

BKT transition.—The basic experimental fact of Mizukami *et al.* [1] is that when the number of $CeCoIn_5$ layers $n \geq 5$, the upper critical field H_{c2}, both parallel and perpendicular to the *ab* plane retains the bulk value, while the transition temperature T_c decreases with decreasing n (see Fig. 1). H_{c2} in such systems is Pauli limited in both parallel and perpendicular directions [1,27] and is thus a direct measure of the superconducting gap, with $H_{c2}^{\text{Pauli}} \simeq \sqrt{2}\Delta/g\mu_B$, where g is the gyromagnetic factor and μ_B is the Bohr magneton. This means that the gap retains the bulk value for $n \geq 5$. The behavior of the gap and T_c for different number of $CeCoIn_5$ layers is shown in Fig. 1. Our proposal is that such behavior is due to the effect of phase fluctuations, which for the quasi-two-dimensional superconductors considered here is controlled by Berezinskii-Kosterlitz-Thouless physics [28,29].

For two-dimensional systems with continuous Abelian symmetry, despite the lack of broken symmetry due to strong fluctuations, there exists a finite temperature phase

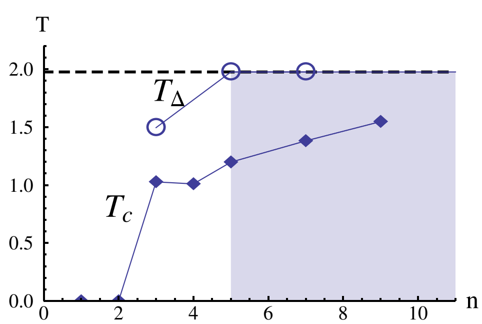

FIG. 1 (color online). Gap and T_c as function of number of $CeCoIn_5$ layers (data from Mizukami *et al.* [1]). For $n \geq 5$ (shaded region), gap retains the bulk value, while T_c decreases with decreasing number of layers.

transition mediated by topological defects, e.g., vortices for superconductors [28,29]. Below the transition temperature T_{BKT}, vortices and antivortices are bound into pairs, and the resistance vanishes. Above T_{BKT}, vortex-antivortex pairs unbind, and the proliferation of free vortices destroys superconductivity. For such systems, one thus has $T_c = T_{\text{BKT}}$.

For layered superconductors, one also needs to include interlayer couplings. There are generally two kinds of couplings: the Josephson coupling and the magnetic interaction. Since the separation of the different $CeCoIn_5$ layers is larger than the perpendicular coherence length, the interlayer Josephson coupling is weak and can be ignored. The long range magnetic interaction couples vortices in different planes and aligns vortices of the same sign into stacks. Since the interlayer coupling is still logarithmic as in two-dimensional superconductors, the phase transition is expected to remain in the same universality class as BKT transition [30]. This has been confirmed by detailed renormalization group studies [31–34] (see also Ref. [35]). It has also been shown in Ref. [34] that T_c is only slightly modified [36]. While such small modification may be detected by future high precision measurements, as first approximation we will ignore it in the following and concentrate on the single-layer problem.

In the following, we are going to check whether the experimental findings of Mizukami *et al.* [1] are consistent with BKT transition: (i) First, we will examine whether resistivity has the right temperature dependence. (ii) Then we extract from the resistivity data the transition temperature T_{BKT}. (iii) Finally, we will check whether T_{BKT} has the right dependence on the number of layers. We find that the observations in Ref. [1] are consistent with BKT transition.

Near T_{BKT}, resistivity behaves as $\rho(T) = \rho_0 e^{-b(T-T_{\text{BKT}})^{-1/2}}$ [37], which gives $[d\ln\rho(T)/dT]^{-2/3} = (2/b)^{2/3}(T - T_{\text{BKT}})$. We plot in Fig. 2 the temperature dependence of $[d\ln\rho(T)/dT]^{-2/3}$ for the four different cases with number of $CeCoIn_5$ layers $n = 4, 5, 7, 9$, where one can see that $[d\ln\rho(T)/dT]^{-2/3}$ is indeed linear in T, and T_{BKT} can be extracted from the intersection points. We also notice that resistivity does not fall to zero at T_{BKT}. It retains a small nonzero value in a temperature region below T_{BKT}. This is generically observed for a BKT transition and is attributed to the temperature difference between the formation of single vortices and the subsequent vortex condensation (see, e.g., Ref. [38] and references therein).

Now, we proceed to study the thickness dependence of the BKT transition temperature. T_{BKT} can be written as [29,37,39,40]

$$k_B T_{\text{BKT}} = \frac{\pi\hbar^2 n_s^{2\text{D}}(T_{\text{BKT}})}{8m\epsilon_c}, \tag{1}$$

with the dielectric constant $\epsilon_c \equiv n_s^{2\text{D}}/n_s^R$, where n_s^R is the renormalized carrier density. The unrenormalized 2D carrier density $n_s^{2\text{D}} = n_s^{3\text{D}} d$ is determined by the 3D carrier

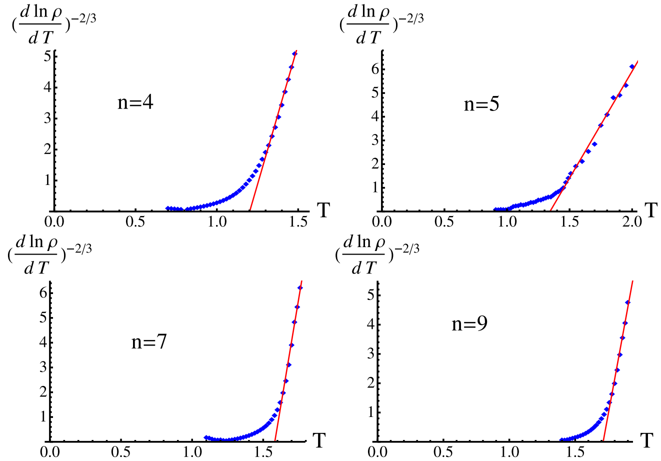

FIG. 2 (color online). Resistivity as function of temperature for $n = 4, 5, 7, 9$ (data from Mizukami *et al.* [1]). The BKT transition temperature is determined from the intersection with the T axis to be $T_{\rm BKT} = 1.202$, 1.344, 1.582, 1.712 K, respectively.

density $n_s^{\rm 3D}(T) = n_s^{\rm 3D}(0)\lambda_b^2(0)/\lambda_b^2(T)$ and the film thickness d. The bulk penetration depth $\lambda_b(T)$ has a temperature dependence of the form $\lambda_b(T) = \lambda_b(0) \times [1 - (T/T_{c0})^\alpha]^{-1/2}$, with bulk mean field transition temperature T_{c0}. In the usual two-fluid picture, the exponent $\alpha = 4$. For cuprates and $CeCoIn_5$, it has been found that $\alpha = 2$ [41,42]. Thus, we have

$$\frac{T_{\rm BKT}}{1 - (T_{\rm BKT}/T_{c0})^2} = \frac{\pi\hbar^2 n_s^{3D}(0)}{8k_B m\epsilon_c} d. \quad (2)$$

Noting that $d = nx - d_0 = (n - n_0)x$, with n the number of $CeCoIn_5$ layers, x the thickness of each layer, and d_0 the thickness of the dead layers on top and bottom, the above result can be written as

$$\frac{T_{\rm BKT}[{\rm K}]}{1 - (T_{\rm BKT}/T_{c0})^2} = \frac{0.98[{\rm cm}]x}{\lambda_b^2(0)} \frac{1}{\epsilon_c}(n - n_0). \quad (3)$$

We plot in Fig. 3 $T_{\rm BKT}$ as function of the number of $CeCoIn_5$ layers. The experimental results are in good agreement with the theoretical prediction determined from Eq. (3). Taking $\lambda_b(0) = 358$ nm [42], $x = \xi_c/4 = 2.1$ nm/4, we get the fitting parameter $\epsilon_c \simeq 90$. With $\lambda^{-2} = \lambda_b^{-2}/\epsilon_c$, our prediction is that the penetration depth of the superlattice is enhanced by about one order of magnitude from the bulk value. Furthermore, another important prediction from BKT transition that can be checked is that the penetration depth of the superlattice λ satisfies the universal relation [39]

$$k_B T_{\rm BKT} = \frac{\Phi_0^2}{32\pi^2}\frac{d}{\lambda^2}, \quad (4)$$

right below the transition temperature, where $\Phi_0 = hc/2e$ is the flux quantum.

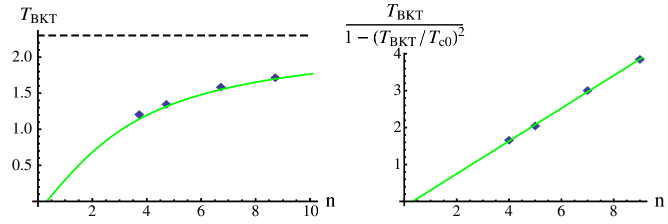

FIG. 3 (color online). The BKT transition temperature $T_{\rm BKT}$ as function of the number of $CeCoIn_5$ layers. The dashed line is $T_{c0} = 2.3$ K. The solid line is a fit to the theoretical result, with $\frac{T_{\rm BKT}}{1-(T_{\rm BKT}/T_{c0})^2} = 0.444(n - 0.317)$.

Antiferromagnetic vortex core.—We extract from the experiment [1] a large dielectric constant ϵ_c, which indicates a large fugacity or a small vortex core energy [29,39]. (See Supplemental Material [25].) Here, we try to understand where such a large renormalization may come from. We find that at the vortex core, where the superconducting gap is suppressed, magnetic ordering can occur locally (see, e.g., Ref. [43]), which reduces the vortex core energy.

A salient feature of the heavy-fermion superconductor $CeCoIn_5$ is the proximity to an antiferromagnetic quantum critical point (QCP). Therefore, one may expect that fluctuating magnetic order may influence the vortex dynamics in the heavy-fermion superlattices. Suppression of the superconductivity in the core can induce the antiferromagnetic state in the cores as opposed to a simple metal in conventional superconductors. To model this effect, we consider magnetic moment that couples to the vortex via a Zeeman term $g\mu_B H_v^z S^z$, where H_v^z is the magnetic field generated by vortices. H_v^z is a superpostion of the magnetic fields generated by vortices at different locations, $H_v^z(\mathbf{r}) = \sum_i n_i H_0(\mathbf{r} - \mathbf{R}_i)$, with n_i the vorticity. $H_0(\mathbf{r})$ can be obtained from its Fourier transform $H_0(\mathbf{k}) = \Phi_0/(1 + \lambda^2 k^2)$, with result $H_0(\mathbf{r}) \sim (\Phi_0/\lambda^2)K_0(r/\lambda)$, where K_0 is the modified Bessel function of the second kind. For $r \ll \lambda$, $K_0(r/\lambda) \sim \ln r$.

Zeeman coupling induces a precession of the magnetic moment perpendicular to the magnetic field, which can be captured by modifying the kinetic energy density to $(\partial_\tau \boldsymbol{\phi} + ig\mu_B \boldsymbol{H} \times \boldsymbol{\phi})^2$, where $\boldsymbol{\phi}$ is the sublattice magnetization density [44–46]. For $\boldsymbol{H}$ in the z direction, one can define $\Phi = (\phi_x + i\phi_y)/\sqrt{2}$. Consider the static limit, its free energy density reads (see Supplemental Material [25]).

$$\mathcal{F}_\Phi = |\nabla\Phi|^2 + [\alpha - g^2\mu_B^2 H^2(r)]|\Phi|^2 + \gamma|\Phi|^4. \quad (5)$$

Near the vortex core, $H \sim \ln|\mathbf{r} - \mathbf{r}_i|$ can be very large. Close to the QCP, α is small. When $\tilde{\alpha} \equiv \alpha - g^2\mu_B^2 H^2 < 0$, the vortex core becomes antiferromagnetic, and qualitatively $|\Phi|^2 = -\tilde{\alpha}/2\gamma$ and the potential energy $V_\Phi = -\tilde{\alpha}^2/4\gamma < 0$. Thus, the vortex core energy is significantly reduced due to magnetic fluctuations.

More precisely, we consider the equation of motion

$$[-\nabla^2 + \alpha - g^2\mu_B^2 H_0^2(\mathbf{r}) + 2\gamma|\Phi(\mathbf{r})|^2]\Phi(\mathbf{r}) = 0, \quad (6)$$

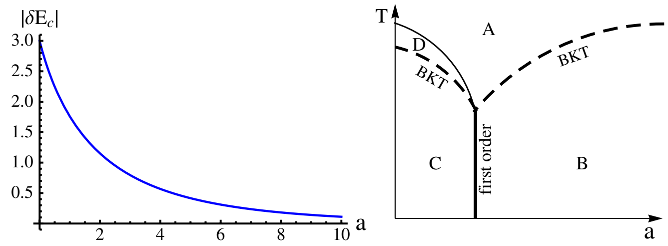

FIG. 4 (color online). Change of vortex core energy as function of distance to the QCP and the resulting *a*-*T* phase diagram as deduced from simulation [49] and theoretical results [47,48] (see also Refs. [50,51]). Phase *A* is gas of free vortices, *B* a gas of bound vortex-antivortex pairs, *C* a crystal of vortices and antivortices, *D* a hexatic phase of vortices and antivortices.

where a vortex of unit vorticity is placed at $\mathbf{r} = 0$. Far away from the vortex core, i.e., $r \sim \lambda$, H_0 decays exponentially, and $\Phi = 0$ is the lowest energy solution. Near the vortex core, we can ignore α and $\Phi(r) \sim \ln(r/\lambda)$ is the lowest energy solution. The change of vortex core energy is $\delta E_c = \int d^2\mathbf{r}\mathcal{F}[\Phi(\mathbf{r})] \sim -g^4\mu_B^4\Phi_0^4/\gamma\lambda^6 \equiv -V_0 < 0$. For small γ, core energy lowering effect can be very large.

We also notice that the vortex core energy depends on α, the distance to the QCP. With the dimensionless quantity $a \equiv \alpha\lambda^4/g^2\mu_B^2\Phi_0^2$, the change of vortex core energy is $\delta E_c \sim -V_0 \int_0^{r^*/\lambda} x dx(\ln^2 x - a)^2$, where $r^* = \lambda e^{-\sqrt{a}}$ is the radius where magnetic condensate vanishes. And we have $\delta E_c \sim -V_0 e^{-2\sqrt{a}}(3 + 6\sqrt{a} + 4a)$ (see Fig. 4). One can thus tune the vortex fugacity by changing the distance to the QCP. It would be interesting to see whether phase diagrams as shown in Fig. 4 can be observed experimentally.

Effect of the magnetic field.—In the presence of a perpendicular magnetic field ($H \perp ab$), there will be an imbalance of vortices parallel to the magnetic field and those antiparallel, with $|n_+ - n_-| > 0$. The unbounded vortices will give rise to finite resistance. When the magnetic field is applied parallel to the *ab* plane, there will be no such effects. This explains the enhanced resistivity when applying perpendicular magnetic field [Fig. 2c in Ref. [1]]. One can also see that a small parallel field will not change T_{BKT}, i.e., $\partial T/\partial H_{c2\parallel} = 0$ near T_{BKT}, while a small perpendicular field will reduce T_{BKT}, i.e., $\partial T/\partial H_{c2\perp} < 0$ near T_{BKT}, as observed in Fig. 4a of Ref. [1]. Near T_{BKT}, where both $H_{c2\parallel}$ and $H_{c2\perp}$ approach zero, the ratio $H_{c2\parallel}/H_{c2\perp} = (\partial T/\partial H_{c2\perp})/(\partial T/\partial H_{c2\parallel})$ thus diverges, as seen in Fig. 3b of Ref. [1].

Conclusions.—In conclusion, we have proposed that superconducting transition in the heavy-fermion superlattice of Mizukami *et al.* [1] is controlled by BKT transition of vortex-antivortex (un)binding. We have also shown that magnetic fluctuations modify the conventional BKT discussion since they reduce the vortex core energy, and thus, quantum criticality may strongly influence the phase diagram of the vortex system. We made suggestions to further test our proposal: the most clear signature of the BKT transition is a jump in the superfluid density at the transition [39], which can be detected by measuring the penetration depth. $CeCoIn_5$ sandwiched with insulating layers may make an even better two-dimensional superconductor. In the opposite limit of a very thin normal $YbCoIn_5$ layer, we expect the crossover to conventional 3D superconducting transitions that also would be interesting to test. In a dense vortex matter, vortex-antivortex pairs may crystallize, and subsequent melting may lead to intermediate hexatic phase [47,48]. It would be interesting to look for such phases in systems close to a magnetic QCP, where vortex core energy can be substantially reduced.

We acknowledge useful discussions with N. Peter Armitage, Lara Benfatto, Lev Bulaevskii, Chih-Chun Chien, Tanmoy Das, Matthias Graf, Jason T. Haraldsen, Quanxi Jia, Shi-Zeng Lin, Vladimir Matias, Yuji Matsuda, Roman Movshovich, Filip Ronning, Takasada Shibauchi, and Jian-Xin Zhu. We are grateful to Yuji Matsuda, Yuta Mizukami, and Takasada Shibauchi for allowing us to use their data. This work was supported, in part, by UCOP-TR01, by the Center for Integrated Nanotechnologies, a U.S. Department of Energy, Office of Basic Energy Sciences user facility and in part by LDRD. Los Alamos National Laboratory, an affirmative action equal opportunity employer, is operated by Los Alamos National Security, LLC, for the National Nuclear Security Administration of the U.S. Department of Energy under Contract No. DE-AC52-06NA25396.

Note added.—While this work was under review, we received a preprint by Fellows *et al.* [52], where they study a related problem of BKT transition in the presence of competing orders, focusing on the behavior near the high symmetry point.

[1] Y. Mizukami, H. Shishido, T. Shibauchi, M. Shimozawa, S. Yasumoto, D. Watanabe, M. Yamashita, H. Ikeda, T. Terashima, and H. Kontani *et al.*, Nature Phys. **7**, 849 (2011).
[2] H. Shishido, T. Shibauchi, K. Yasu, T. Kato, H. Kontani, T. Terashima, and Y. Matsuda, Science **327**, 980 (2010).
[3] D. Maruyama, M. Sigrist, and Y. Yanase, J. Phys. Soc. Jpn. **81**, 034702 (2012).
[4] P. Minnhagen, Rev. Mod. Phys. **59**, 1001 (1987).
[5] A. T. Fiory, A. F. Hebard, P. M. Mankiewich, and R. E. Howard, Phys. Rev. Lett. **61**, 1419 (1988).
[6] L. C. Davis, M. R. Beasley, and D. J. Scalapino, Phys. Rev. B **42**, 99 (1990).
[7] Y. Matsuda, S. Komiyama, T. Onogi, T. Terashima, K. Shimura, and Y. Bando, Phys. Rev. B **48**, 10498 (1993).
[8] R. W. Crane, N. P. Armitage, A. Johansson, G. Sambandamurthy, D. Shahar, and G. Grüner, Phys. Rev. B **75**, 094506 (2007).
[9] M. Mondal, S. Kumar, M. Chand, A. Kamlapure, G. Saraswat, G. Seibold, L. Benfatto, and P. Raychaudhuri, Phys. Rev. Lett. **107**, 217003 (2011).

[10] W. Liu, M. Kim, G. Sambandamurthy, and N. P. Armitage, Phys. Rev. B **84**, 024511 (2011).
[11] H.-H. Wen, P. Ziemann, H. A. Radovan, and S. L. Yan, Europhys. Lett. **42**, 319 (1998).
[12] J. Corson, R. Mallozzi, J. Orenstein, J. N. Eckstein, and I. Bozovic, Nature (London) **398**, 221 (1999).
[13] L. Li, Y. Wang, M. J. Naughton, S. Ono, Y. Ando, and N. P. Ong, Europhys. Lett. **72**, 451 (2005).
[14] L. S. Bilbro, R. V. Aguilar, G. Logvenov, I. Bozovic, and N. P. Armitage, Phys. Rev. B **84**, 100511(R) (2011).
[15] N. Reyren *et al.*, Science **317**, 1196 (2007).
[16] A. D. Caviglia, S. Gariglio, N. Reyren, D. Jaccard, T. Schneider, M. Gabay, S. Thiel, G. Hammerl, J. Mannhart, and J.-M. Triscone, Nature (London) **456**, 624 (2008).
[17] T. Schneider, A. D. Caviglia, S. Gariglio, N. Reyren, and J.-M. Triscone, Phys. Rev. B **79**, 184502 (2009).
[18] V. G. Kogan, Phys. Rev. B **75**, 064514 (2007).
[19] L. Benfatto, C. Castellani, and T. Giamarchi, Phys. Rev. B **80**, 214506 (2009).
[20] G. Deutscher and P. G. de Gennes, in *Superconductivity*, edited by R. D. Parks (Marcel Dekker, Inc., New York, 1969), Vol. 2.
[21] E. W. Fenton, Solid State Commun. **54**, 709 (1985).
[22] G. E. Blonder, M. Tinkham, and T. M. Klapwijk, Phys. Rev. B **25**, 4515 (1982).
[23] G. E. Blonder and M. Tinkham, Phys. Rev. B **27**, 112 (1983).
[24] O. T. Valls, M. Bryan, and I. Žutić, Phys. Rev. B **82**, 134534 (2010).
[25] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.109.077002 for a more detailed analysis.
[26] A. Serafin, J. D. Fletcher, S. Adachi, N. E. Hussey, and A. Carrington, Phys. Rev. B **82**, 140506(R) (2010).
[27] A. D. Bianchi *et al.*, Science **319**, 177 (2008).
[28] V. L. Berezinskii, Zh. Eksp. Teor. Fiz. **59**, 907 (1970).
[29] J. M. Kosterlitz and D. J. Thouless, J. Phys. C **6**, 1181 (1973).
[30] S. E. Korshunov, Europhys. Lett. **11**, 757 (1990).
[31] B. Horovitz, Phys. Rev. B **45**, 12632 (1992).
[32] S. Scheidl and G. Hackenbroich, Phys. Rev. B **46**, 14010 (1992).
[33] B. Horovitz, Phys. Rev. B **47**, 5947 (1993).
[34] K. S. Raman, V. Oganesyan, and S. L. Sondhi, Phys. Rev. B **79**, 174528 (2009).
[35] C. Timm, Phys. Rev. B **52**, 9751 (1995).
[36] With $s \ll \lambda_{\parallel}$, the transition temperature now reads $T_c = (\pi/2)\rho_s(1 - \frac{s}{2\lambda_{\parallel}})$, where s is the layer spacing, $\lambda_{\parallel}$ is the in-plane penetration depth, and $\rho_s = \Phi_0^2 s/(16\pi^3\lambda_{\parallel}^2)$ is the in-plane superfluid stiffness, which can be measured directly. In the experiment of Mizukami *et al.* [1], $s \sim 3.7$ nm, $d \sim 5$ nm. Taking $\lambda \sim \lambda_b \sim 358$ nm, we have $\lambda_{\parallel} \sim 308$ and $s/2\lambda_{\parallel} \sim 0.006$.
[37] B. I. Halperin and D. R. Nelson, J. Low Temp. Phys. **36**, 599 (1979).
[38] J. Pereiro, A. Petrovic, C. Panagopoulos, and I. Božović, Phys. Express **1**, 208 (2011).
[39] D. R. Nelson and J. M. Kosterlitz, Phys. Rev. Lett. **39**, 1201 (1977).
[40] M. R. Beasley, J. E. Mooij, and T. P. Orlando, Phys. Rev. Lett. **42**, 1165 (1979).
[41] D. A. Bonn *et al.*, Phys. Rev. B **47**, 11314 (1993).
[42] V. G. Kogan, R. Prozorov, and C. Petrovic, J. Phys. Condens. Matter **21**, 102204 (2009).
[43] D. P. Arovas, A. J. Berlinsky, C. Kallin, and S.-C. Zhang, Phys. Rev. Lett. **79**, 2871 (1997).
[44] I. Affleck, Phys. Rev. B **41**, 6697 (1990).
[45] I. Affleck, Phys. Rev. B **43**, 3215 (1991).
[46] I. Fischer and A. Rosch, Phys. Rev. B **71**, 184429 (2005).
[47] M. Gabay and A. Kapitulnik, Phys. Rev. Lett. **71**, 2138 (1993).
[48] S. C. Zhang, Phys. Rev. Lett. **71**, 2142 (1993).
[49] J.-R. Lee and S. Teitel, Phys. Rev. B **46**, 3247 (1992).
[50] G. Orkoulas and A. Z. Panagiotopoulos, J. Chem. Phys. **104**, 7205 (1996).
[51] J. Lidmar and M. Wallin, Phys. Rev. B **55**, 522 (1997).
[52] J. M. Fellows, S. T. Carr, C. A. Hooley, and J. Schmalian, arXiv:1205.1333v1.